Lindenwold Curricular Framework Mathematics-Grade 8 *Updated 2020

Overview	Standards for Mathematical Content		Unit Focus	Standards for Mathematical Practice
Unit 1 **Exponents, Expressions, and Equations**	8.EE.A.1 8.EE.A.3 8.EE.A.4 ❑ 8.NS.A.1 ❑ 8.NS.A.2 8.EE.B.5	8.EE.B.6 ❑ 8.SP.A.1 ❑ 8.SP.A.2 ❑ 8.SP.A.3 ❑ 8.SP.A.4	• Work with integer exponents • Know that there are numbers that are not rational, and approximate them by rational numbers • Understand the connections between proportional relationships, lines, and linear equations • Investigate patterns of association in bivariate data	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 1: ***Suggested Open Educational Resources***	**8.EE.A.1 Extending the Definitions of Exponents** **8.EE.A.3 Ant and Elephant** **8.EE.A.4 Giantburgers** **8.NS.A.1 Converting Decimal Representations of Rational Numbers to Fraction Representations** **8.NS.A.2 Irrational Numbers on the Number Line**		**8.EE.B.5 Who Has the Best Job?** **8.EE.B.6 Slopes Between Points on a Line** **8.SP.A.1 Texting and Grades 1** **8.SP.A.2 Animal Brains** **8.SP.A.3 US Airports** **8.SP.A.4 What's Your Favorite Subject** **8.SP.A.4 Music and Sports**	
Unit 2 **Functions, Equations, and Solutions**	8.F.A.1 8.F.A.2 8.F.A.3 8.F.B.4*	8.F.B.5 8.EE.C.7 8.EE.C.8*	• Define, evaluate, and compare functions • Use functions to model relationships between quantities • Analyze and solve linear equations and simultaneous linear equations	
Unit 2: ***Suggested Open -Educational Resources***	**8.F.A.1 Function Rules** **8.F.A.2 Battery Charging** **8.F.A.3 Introduction to Linear Functions** **8.F.B.4 Chicken and Steak, Variation 1** **8.F.B.4 Baseball Cards**		**8.EE.C.7 The Sign of Solutions** **8.EE.C.7 Coupon versus discount** **8.EE.C.8a Intersection of Two Lines** **8.EE.C.8 How Many Solutions**	

Key: Major Cluster | ❑ Supporting | -◎ Additional Cluster | * Benchmarked

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Overview	Standards for Mathematical Content	Unit Focus	Standards for Mathematical Practice
Unit 3 **Geometry: Pythagorean Theorem, Congruence and Similarity Transformations**	8.EE.A.2 8.G.B.6 8.G.B.7* 8.G.B.8* 8.G.A.1 8.G.A.2 8.G.A.3 8.G.A.4 8.G.A.5	• Work with radicals and integer exponents • Understand and apply the Pythagorean Theorem • Understand congruence and similarity using physical models, transparencies, or geometry software	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.
Unit 3: ***Suggested Open Educational Resources***	**8.G.B.6 Converse of the Pythagorean Theorem** **8.G.B.7 Running on the Football Field** **8.G.B.8 Finding isosceles triangles** **8.G.A.1 Reflections, Rotations, and Translations** **8.G.A.2 Congruent Triangles**	**8.G.A.3 Effects of Dilations on Length, Area, and Angles** **8.G.A.4 Are They Similar** **8.G.A.5 Street Intersections** **8.G.A.5 Similar Triangles II** **8.G.A.5 Triangle's Interior Angles**	
Unit 4 **Volume and Linear Functions**	◎8.G.C.9 8.F.B.4* 8.G.B.7* 8.G.B.8* 8.EE.C.8c*	• Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres • Use functions to model relationships between quantities • Understand and apply the Pythagorean Theorem • Analyze and solve linear equations and simultaneous linear equations	
Unit 4: ***Suggested Open Educational Resources***	**8.F.B.4 Delivering the Mail** **8.G.B.8 Finding the distance between points** **8.EE.C.8 Kimi and Jordan** **8.G.C.9 A Canister of Tennis Balls**		

Key: Major Cluster | ❑ Supporting | -◎ Additional Cluster | * Benchmarked

Unit 1 Grade 8
45 Instructional Days

Unit 1 introduces learners to the concept of irrational numbers, requiring them to classify numbers as either rational or irrational and approximate irrational expressions using rational numbers. The unit continues with the understanding and application of integer exponents and scientific notation. Learners not only know the properties of exponents, but also apply those properties to efficiently simplify and/or rewrite exponential expressions. With respect to scientific notation, learners perform simple mathematical operations with numbers written in scientific notation and make comparisons between two quantities by estimating numbers written in scientific notation. Learners solve linear equations in one variable. Learners continue with their analysis of linear models by shifting to how they pertain to bivariate data. Learners investigate patterns of association in bivariate data using scatter plots and two-way tables, including informally fitting and assessing the fit of a linear model for a scatter plot, interpreting the slope and intercept of a linear model in the context of bivariate data, and using joint and relative frequencies of a two-way table to describe possible association between two variables.

Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Concept(s):** ● Exponents as simplified representation of repeated multiplication. **Students are able to:** ● apply properties of exponents to numerical expressions. ● generate equivalent numerical expressions using positive and negative integer exponents. · **Learning Goal 1:** Apply the properties of integer exponents to write equivalent numerical expressions.
8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as* 3×10^8 *and the population of the world as* 7×10^9*, and determine that the world population is more than 20 times larger.*	MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Concept(s):** ● Very large and very small quantities can be approximated with numbers expressed in the form of a single digit times an integer power of 10. **Students are able to:** ● estimate very large and very small quantities with numbers expressed in the form of a single digit times an integer power of 10. ● compare numbers written in the form of a single digit times an integer power of 10 and express how many times as much one is than the other. **Learning Goal 2:** Estimate and express the values of very large or very small numbers with numbers expressed in the form of a single digit times an integer power of 10. Compare numbers expressed in this form, expressing how many times larger or smaller one is than the other.

8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	MP. 2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Concept(s)**: No new concept(s) introduced **Students are able to:** ● multiply and divide numbers expressed in scientific notation, including problems in which one number is in decimal form and one is in scientific notation. ● add and subtract numbers expressed in scientific notation, including problems in which one number is in decimal form and one is in scientific notation. ● use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. ● interpret scientific notation that has been generated by technology (e.g. recognize 4.1E-2 and 4.1e-2 as 4.1×10^{-2}). **Learning Goal 3:** Perform operations using numbers expressed in scientific notation, including problems where both decimals and scientific notation are used. In real-world problem-solving situations, choose units of appropriate size for measurement of very small and very large quantities and interpret scientific notation generated when technology has been used for calculations.
❑ 8.NS.A.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	MP. 2 Reason abstractly and quantitatively.	**Concept(s):** ● Numbers that are not rational are irrational. ● Every number has a decimal expansion. **Students are able to:** ● compare decimal expansions of rational and irrational numbers. ● represent a rational number with its decimal expansion, showing that it repeats eventually. ● convert a decimal expansion (which repeats eventually) into a rational number. **Learning Goal 4:** Represent a rational number with its decimal expansion, showing that it eventually repeats, and convert such decimal expansions into rational numbers.
❑ 8.NS.A.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., p^2). *For example, by truncating the decimal expansion of Ö2, show that Ö2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*	MP.1 Make sense of problems and persevere in solving them. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically.	**Concept(s):** ● Rational approximation of irrational numbers **Students are able to:** ● compare irrational numbers by replacing each with its rational approximation. ● locate rational approximations on a number line. ● estimate the value of expressions containing irrational numbers. **Learning Goal 5:** Use rational numbers to approximate irrational numbers, locate irrational numbers on a number line, and estimate the value of expressions containing irrational numbers.

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Concept(s):** ● Quantitative relationships can be represented in different ways. **Students are able to:** ● graph proportional relationships. ● interpret unit rate as the slope of a graph. ● compare two different proportional relationships that are represented indifferent ways (table of values, equation, graph, verbal description). **Learning Goal 6**: Graph proportional relationships, interpreting slope as unit rate, and compare two proportional relationships, each represented in different ways.
8.EE.B.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Concept(s):** No new concept(s) introduced **Students are able to:** ● show, using similar triangles, and explain why the slope, m, is the same between any two distinct points on a non-vertical line. ● derive, from two points, the equation $y = mx$ for a line through the origin. ● derive, from two points, the equation $y = mx + b$ for a line intercepting the vertical axis at b. **Learning Goal 7**: Derive the equation of a line ($y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b) and use similar triangles to explain why the slope (m) is the same between any two points on a non-vertical line in the coordinate plane.
❑ 8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	MP.3 Construct viable arguments and critique the reasoning. of others. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	**Concept(s):** ● Association in data (bivariate measurement data) **Students are able to:** ● construct and interpret scatter plots. ● analyze patterns of association between the two quantities represented in a scatter plot. ● describe clustering, outliers, positive or negative association, linear or non-linear association when explaining patterns of association in a scatter plot. **Learning Goal 8**: Construct and interpret scatter plots for bivariate measurement data and describe visual patterns of association (clusters, outliers, positive or negative association, linear association and nonlinear association, strong, weak, and no association).

❑ 8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit (e.g. line of best fit) by judging the closeness of the data points to the line.	MP.2 Reason abstractly and quantitatively. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	**Concept(s):** ● Straight lines are used to model *approximately* linear relationships between quantitative variables. **Students are able to:** ● informally fit a line (of best fit) to a scatter plot that suggests a linear association. ● informally assess the model's fit by judging the closeness of the data points to the line (line of best fit). **Learning Goal 9**: For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model's fit.
❑ 8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*	MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.6 Attend to precision. MP.7 Look for and make use of structure.	**Concept(s):** No new concept(s) introduced **Students are able to:** ● given the equation for a linear model (line of best fit), interpret the slope and intercept. ● given the equation for a linear model, solve problems in the context of measurement data. **Learning Goal 10**: Use a linear model (equation) representing measurement data to solve problems, interpreting the slope and intercept in the context of the situation.
❑ 8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*	MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.7 Look for and make use of structure.	**Concept(s):** ● Categorical data: patterns of association can also be observed in bivariate categorical data through analyzing two-way tables containing frequencies or relative frequencies. **Students are able to:** ● construct and interpret a two-way frequency table containing data on two categorical variables. ● construct and interpret a two-way relative frequency table containing data on two categorical variables. ● describe any association between the two categorical variables using relative frequencies calculated for rows or columns. **Learning Goal 11**: Construct two-way frequency tables and two-way relative frequency tables, and describe possible associations between two variables.

Unit 1 Grade 8 What This May Look Like	
District/School Formative Assessment Plan	**District/School Summative Assessment Plan**
Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards. PDF Test Banks Envisions Alignment & Sample Pacing	*Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.* Benchmark Scoring Guide: MC OE Common Reasons for Incorrect Answers
Focus Mathematical Concepts	
Prerequisite skills: Parts of a power. Show proportional relationships among variables in a variety of ways. Identify the constant of proportionality. Identify variables to determine appropriate range. Understand the place value system. Simplify algebraic expressions. The subsets of rational numbers. **Common Misconceptions:** Students commonly misinterpret exponents as multiplication instead of repeated multiplication. It may be difficult for students to remember how to apply properties of exponents and that the square root of a number has 2 possible solutions. For Real Number Subgroups, students often think integers are irrational. Also, that the most restrictive subgroup is the only classification of a number. Students may think that all linear relationships are proportional leading them to incorrectly find the slope or equation of the relationship. Students reverse the coordinates when representing slope as well as when plotting ordered pairs. Students forget to look at the scale of a graph before solving a problem assuming it is 1. Often graph labels are interchanged.	
District/School Tasks	**District/School Primary and Supplementary Resources**
Exemplar tasks or illustrative models could be provided. Vocabulary List Manipulative Ideas Manipulative Kit Standards UnPacking Coherence Map	*District/school resources and supplementary resources that are texts as well as digital resources used to support the instruction.* **SavvasRealize** **BrainPop** **Illustrative Mathematics** **Illuminations NCTM** **Estimation 180** **Georgia Standards OER** **NJ Center for Teaching & Learning** **Achieve the Core** **Khan Academy** **Hinge Questions** **Teacher Education by Design** **EngageNY** Interactive Notebook

Instructional Best Practices and Exemplars
ELLS: EnVisions Language Support Handbook Resource Folder **Number/Pattern talks:** Resource Folder **Which One Doesn't Belong:** Resource Folder **Error Analysis:** Resource Folder **Performance Tasks:** Resource Folder **3-ACT tasks:** Resource Folder **Launch – Explore – Summarize Tasks:** Resource Folder **3-Reads:** Resource Folder **Contemplate then Calculate:** Resource Folder **Connecting Representations:** Resource Folder **Capturing Quantities:** Resource Folder

21st Century Skills	Career and Technical Education
CRP2. Apply appropriate academic and technical skills. CRP4. Communicate clearly and effectively and with reason. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.	9.2.8.B.3 Evaluate communication, collaboration, and leadership skills that can be developed through school, home, work, and extracurricular activities for use in a career. 9.2.8.B.4 Evaluate how traditional and nontraditional careers have evolved regionally, nationally, and globally.

Interdisciplinary Connections

NJSLS ELA

- NJSLSA.R4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
- NJSLSA.W7. Conduct short as well as more sustained research projects, utilizing an inquiry based research process, based on focused questions, demonstrating an understanding of the subject under investigation.

NJSLS Science

- MS-PS3-1. Construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.
- MS-PS3-5. Construct, use, and present arguments to support the claim that when the kinetic energy of an object changes, energy is transferred to or from the object.

Unit 2 Grade 8
45 Instructional Days

Unit 2 introduces learners to the idea of a function as a precursor to concepts about functions that are included in the high school standards. Learners begin the unit describing qualitatively the relationship between two quantities by analyzing a graph as an informal introduction to functions. They describe a function more formally by identifying it as a rule that assigns to each input exactly one output. In this unit, the concepts developed in grades 6 and 7 such as modeling relationships with variables and equations and ratio and proportional reasoning, are used to make connections between proportional relationships, lines, and linear equations. Learners graph linear functions, construct a function to model a linear relationship, interpret the rate of change and initial value of a linear function in a real-world context, and compare linear functions presented in different ways. The unit concludes with analyzing and solving pairs of simultaneous linear equations. Learners will begin solving systems of linear equations algebraically, and solve real-world mathematical problems leading to two linear equations in two variables.

Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	MP.2 Reason abstractly and quantitatively. MP.5 Use appropriate tools strategically	**Concept(s)**: ● A function is a rule. ● If a rule is a function, then for each input there is exactly one output. **Students are able to:** ● use function language. ● describe a function as providing a single output for each input. ● determine whether non-numerical relationships are functions. ● describe a function as a set of ordered pairs. ● read inputs and outputs from a graph. ● describe the ordered pairs as containing an input, and the corresponding output. **Learning Goal 1**: Define a function as a rule that assigns one output to each input and determine if data represented as a graph or in a table is a function.
8.F.A.2 Compare properties (e.g. rate of change, intercepts, domain and range) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic*	MP.5 Use appropriate tools strategically. MP.8 Look for and express regularity in repeated reasoning.	**Concept(s):** ● Functions (quantitative relationships) can be represented in different ways. ● Functions have properties; properties of linear functions. **Students are able to:** ● analyze functions represented algebraically, as a table of values, and as a graph. ● interpret functions represented by a verbal description. ● given two functions, each represented in a different way, compare their properties. ● **Learning Goal 2:** Compare two functions each represented in a different way (numerically, verbally, graphically, and algebraically) and draw conclusions about their properties (rate of change and intercepts).

 Key: Major Cluster | Supporting | Additional Cluster | * Benchmarked

expression, determine which function has the greater rate of change.		
8.F.A.3 Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically.	**Concept(s):** ● A linear function is defined by the equation $y = mx + b$. ● The graph of a linear function is a straight line. **Students are able to:** ● analyze tables of values, graphs, and equations in order to classify a function as linear or non-linear. ● determine if equations presented in forms other than y = mx + b (for example 3y – 2x = 7) define a linear function. ● give examples of equations that are non-linear functions. ● show that a function is not linear using pairs of points. **Learning Goal 3**: Classify functions as linear or non-linear by analyzing equations, graphs, and tables of values; interpret the equation $y = mx + b$ as defining a linear function.
8.F.B.4* Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	MP.6 Attend to precision. MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	**Concept(s):** ● As with equations, two (x,y) values can be used to construct a function. **Students are able to:** ● determine the rate of change and initial value of a function from a description of a relationship. ● determine the rate of change and initial value of a function from two (x, y) values by reading from a table of values. ● determine the rate of change and initial value of a function from two (x, y) values by reading these from a graph. ● construct a function in order to model a linear relationship. ● interpret the rate of change and initial value of a linear function in context. **Learning Goal 4:** Model a linear relationship by constructing a function from two (x,y) values. Interpret the rate of change and initial value of the linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically.	**Concept(s)**: No new concept(s) introduced **Students are able to:** ● analyze a graph. ● provide qualitative descriptions of graphs (e.g. where increasing or decreasing, linear or non-linear). ● given a verbal description, sketch a graph of a function based on the qualitative features described. **Learning Goal 5**: Sketch a graph of a function from a qualitative description and give a qualitative description of a graph of a function.
8.EE.C.7 Solve linear equations in one variable. 8EE.C.7a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). 8.EE.C.7b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms	MP.5 Use appropriate tools strategically. MP.6 Attend to precision.	**Concept(s):** ● Linear equations may have an infinite number of solutions. ● Linear equations may have no solution or a single solution. **Students are able to:** ● give examples of linear equations in one variable with one solution *(x = a),* infinitely many solutions *(a = a),* or no solutions *(a = b.)* ● transform a given equation, using the properties of equality, into simpler forms. ● transform a given equation until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results *(a and b are different numbers).* ● solve linear equations that have fractional coefficients; include equations requiring use of the distributive property and collecting like terms. **Learning Goal 6:** Apply the distributive property and collect like terms to solve linear equations in one variable that contain rational numbers as coefficients. Use an equivalent equation of the form $x = a$, $a = a$, or $a = b$ (where a and b are different numbers) to describe the number of solutions.

8.EE.C.8* Analyze and solve pairs of simultaneous linear equations. 8.EE.C.8a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. 8.EE.C.8b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.* 8.EE.C.8c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	MP.1 Make sense of problems and persevere in solving them. MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure.	**Concept(s):** ● Simultaneous linear equations may have an infinite number of solutions. ● Simultaneous linear equations may have no solution or a single solution. ● Solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs. **Students will be able to:** ● solve systems of two linear equations in two variables algebraically. ● estimate solutions of a linear system of two equations by graphing. ● solve simple cases of a linear system of two equations by inspection. ● solve real-world and mathematical problems leading to two linear equations in two variables. **Learning Goal 7:** Solve systems of linear equations in two variables algebraically and by inspection. Estimate solutions by graphing, explain that points of intersection satisfy both equations simultaneously, and interpret solutions in context.

Unit 2 Grade 8 What This May Look Like	
District/School Formative Assessment Plan	**District/School Summative Assessment Plan**
Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards. PDF Test Banks Envisions Alignment & Sample Pacing	*Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.* Benchmark Scoring Guide: MC OE Common Reasons for Incorrect Answers
Focus Mathematical Concepts	
Prerequisite skills: Represent proportional relationships by equations. Graphing multiple lines in the coordinate plane. Tables. Distributive Property. **Common Misconceptions**: Students may interchange x and y coordinates or the x and y axis. Students commonly think that vertical lines are linear functions. Students have a difficult time getting past the idea that every equation have to end as x=a. Students believe the variable is always on the left side of the equation. As students begin to build and work with expressions containing more than two operations, students tend to set aside the order of operations. Students commonly forget that the negative sign in front of a variable is really a coefficient of -1. Students confuse one-variable and two-variable equations. Students mistakenly believe that linear functions (with a constant rate of change) are the only type of functions. Students commonly do not recognize a constant rate of change when entries in a table are absent. Students frequently attempt to "solve" expressions.	
District/School Tasks	**District/School Primary and Supplementary Resources**
Exemplar tasks or illustrative models could be provided. Vocabulary List Manipulative Ideas Manipulative Kit Standards UnPacking Coherence Map	*District/school resources and supplementary resources that are texts as well as digital resources used to support the instruction.* **SavvasRealize** **BrainPop** **Illustrative Mathematics** **Illuminations NCTM** **Estimation 180** **Georgia Standards OER** **NJ Center for Teaching & Learning** **Achieve the Core** **Khan Academy** **Hinge Questions** **Teacher Education by Design** **EngageNY** Interactive Notebook

Instructional Best Practices and Exemplars
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21st Century Skills	Career and Technical Education
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- NJSLSA.R4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
- NJSLSA.W7. Conduct short as well as more sustained research projects, utilizing an inquiry based research process, based on focused questions, demonstrating an understanding of the subject under investigation.

NJSLS Science

- MS-PS3-1. Construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.
- MS-PS3-5. Construct, use, and present arguments to support the claim that when the kinetic energy of an object changes, energy is transferred to or from the object.

Unit 3 Grade 8
45 Instructional Days

Unit 3 provides a continuation of solving linear equations including using square root and cube root symbols to represent solutions to simple equations. Learners apply the Pythagorean Theorem to find unknown side lengths of right triangles in both two- and three-dimensional figures, and to find distances between coordinate points on a coordinate plane. The unit continues with an analysis of transformations (i.e. reflections, rotations, translations, and dilations) in which learners should develop an understanding of congruence and similarity. They understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. They understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. These understandings are then used to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	MP.2 Reason abstractly and quantitatively. MP.4 Model with mathematics. MP.5 Use appropriate tools strategically. MP.6 Attend to precision. MP.7 Look for and make use of structure. MP.8 Look for and express regularity in repeated reasoning.	**Concept(s):** ● Square root and cube roots; perfect squares and perfect cubes ● Inverse relationship between powers and square roots **Students are able to:** ● give the value of square roots of small perfect squares. ● solve equations of the form $x^2 = p$, where p is a positive rational number. ● use the square root symbol to represent solutions to equations of the form $x^2 = p$. ● give the value of cube roots of small perfect cubes. ● solve equations of the form $x^3 = p$, where p is a positive rational number. ● use the cube root symbol to represent solutions to equations of the form $x^3 = p$. ● show or explain that Ö2 is an irrational number. ● use volume formulas to find a single unknown dimension of cones, cylinders and spheres when solving real world problems. **Learning Goal 1**: Evaluate square roots and cubic roots of small perfect squares and cubes respectively and use square and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ where p is a positive rational number; identify $\sqrt{2}$ as irrational.

8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse.	MP.2 Reason abstractly and quantitatively.	**Concept(s):** ● Pythagorean Theorem ● If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle (Pythagorean theorem converse). **Students are able to:** ● given a proof of the Pythagorean theorem, explain the proof. ● given a proof of the converse of the Pythagorean theorem, explain the proof. **Learning Goal 2:** Explain a proof of the Pythagorean Theorem and its converse.
8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	**Concept(s):** No new concept(s) introduced **Students are able to:** ● determine side lengths of right triangles by applying the Pythagorean Theorem to solve real world and mathematical problems involving two dimensional spaces. ● determine side lengths of right triangles by applying the Pythagorean Theorem to solve real world and mathematical problems involving three dimensional spaces. **Learning Goal 3:** Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in two and three dimensional cases when solving real-world and mathematical problems.
8.G.B.8* Apply the Pythagorean Theorem to find the distance between two points in a coordinate system	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	**Concept(s)**: No new concept(s) introduced **Students are able to:** ● determine the distance between two points in a coordinate plane by drawing a right triangle and applying the Pythagorean Theorem. **Learning Goal 4**: Use the Pythagorean Theorem to determine the distance between two points in the coordinate plane.

8.G.A.1 Verify experimentally the properties of rotations, reflections, and translations: 8.G.A.1a. Lines are transformed to lines, and line segments to line segments of the same length. 8.G.A.1b. Angles are transformed to angles of the same measure. 8.G.A.1c. Parallel lines are transformed to parallel lines.	MP.3 Construct viable arguments and critique the reasoning of others. MP.5 Use appropriate tools strategically. MP.8 Look for and express regularity in repeated reasoning.	**Concept(s):** ● A property of rigid motion transformations (rotation, reflection, and translation) is that the measure of a two-dimensional object under the transformation remains unchanged. **Students are able to:** ● show and explain that performing rotations, reflections, and translations on lines results in a line. ● show and explain that performing rotations, reflections, and translations on line segments results in a line segment and does not alter the length of the line segment. ● show and explain that performing rotations, reflections, and translations on angles results in an angle and does not alter the measure of the angle. ● show and explain that performing rotations, reflections, and translations on parallel lines results in parallel lines. ● explain that a property of rigid motion transformations (rotation, reflection, and translation) is that the measure of a two-dimensional object under the transformation remains unchanged. **Learning Goal 5:** Explain and model the properties of rotations, reflections, and translations with physical representations and/or geometry software using pre-images and resultant images of lines, line segments, and angles.
8.G.A.2 Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	**Concept(s)**: ● A two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations. **Students are able to**: ● given two congruent figures, describe a transformation or sequence of transformations that shows the congruence between them. **Learning Goal 6:** Describe and perform a sequence of rotations, reflections, and/or translations on a two dimensional figure in order to prove that two figures are congruent.

8.G.A.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning. of others. MP.5 Use appropriate tools strategically.	**Concept(s):** No new concept(s) introduced **Students are able to:** ● describe, using coordinates, the resulting two-dimensional figure after applying dilations with scale factor greater than, less than, and equal to 1. ● describe, using coordinates, the resulting two-dimensional figure after applying translation, rotation, and reflection. **Learning Goal 7:** Use the coordinate plane to locate images or pre-images of two-dimensional figures and determine the coordinates of a resultant image after applying dilations, rotations, reflections, and translations.
8.G.A.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	**Concept(s):** ● A two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations. ● Congruent figures are also similar. **Students are able to:** ● describe a transformation or sequence of transformations that show the similarity between them given two similar two-dimensional figures. ● **Learning Goal 8:** Apply an effective sequence of transformations to determine that figures are similar when corresponding angles are congruent and corresponding sides are proportional. Write similarity statements based on such transformations.
8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	MP.2 Reason abstractly and quantitatively. MP.3 Construct viable arguments and critique the reasoning. of others.	**Concept(s):** No new concept(s) introduced **Students are able to:** ● give informal arguments to establish facts about the angle sum of triangles. ● give informal arguments to establish facts about exterior angles of triangles. ● give informal arguments to establish facts about the angles created when parallel lines are cut by a transversal. ● give informal arguments to establish the angle-angle criterion for similarity of triangles. **Learning Goal 9**: Give informal arguments to justify facts about the exterior angles of a triangle, the sum of the measures of the interior angles of a triangle, the angle-angle relationship used to determine similar triangles, and the angles created when parallel lines are cut by a transversal.

Unit 3 Grade 8 What This May Look Like	
District/School Formative Assessment Plan	**District/School Summative Assessment Plan**
Formative assessment informs instruction and is ongoing throughout a unit to determine how students are progressing against the standards. PDF Test Banks Envisions Alignment & Sample Pacing	*Summative assessment is an opportunity for students to demonstrate mastery of the skills taught during a particular unit.* Benchmark Scoring Guide: MC OE Common Reasons for Incorrect Answers
Focus Mathematical Concepts	
Prerequisite skills: Multiplication of fractions. Square roots. Parts of a graph. Slope. Properties of parallel and perpendicular lines. Angle relationships from previous years. **Common Misconceptions:** Students may only express the positive square root in the solution. Students often think cube roots also have 2 solutions. Cube and square root problems are division by 2 or 3. Students mislabel the legs and hypotenuse of a right triangle or incorrectly substitute values to use the Pythagorean Theorem. When using PT in a coordinate plane, students often find the slope of the line instead of the length of the line. Students attempt to memorize Transformation rules instead of learning how to use them. Students may forget to move each point using the same rule.	
District/School Tasks	**District/School Primary and Supplementary Resources**
Exemplar tasks or illustrative models could be provided. Vocabulary List Manipulative Ideas Manipulative Kit Standards UnPacking Coherence Map	*District/school resources and supplementary resources that are texts as well as digital resources used to support the instruction.* **SavvasRealize** **BrainPop** **Illustrative Mathematics** **Illuminations NCTM** **Estimation 180** **Georgia Standards OER** **NJ Center for Teaching & Learning** **Achieve the Core** **Khan Academy** **Hinge Questions** **Teacher Education by Design** **EngageNY** Interactive Notebook

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- NJSLSA.R4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
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- MS-PS3-1. Construct and interpret graphical displays of data to describe the relationships of kinetic energy to the mass of an object and to the speed of an object.
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Unit 4 Grade 8
45 Instructional Days

Unit 4 continues with the understanding and application of properties of integer exponents by applying them to volume and surface area. Learners will use Pythagorean Theorem inside of the coordinate plane. Learners will solve systems of linear equations algebraically, and solve real-world mathematical problems leading to two linear equations in two variables.

Content Standards	Suggested Standards for Mathematical Practice	Critical Knowledge & Skills
◎8.G.C.9 Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.		**Concept(s):** **Students are able to:** ● use volume formulas to find a single unknown dimension of cones, cylinders and spheres when solving real world problems. ● find volume of cones, cylinders and spheres using to solve real world problems. **Learning Goal 1**: Apply the formula for the volume of a cone, a cylinder, or a sphere to find a single unknown dimension when solving real-world and mathematical problems.
8.F.B.4* Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.7 Look for and make use of structure.	**Concept(s):** ● As with equations, two (x,y) values can be used to construct a function. **Students are able to:** ● construct a function in order to model a linear relationship. ● interpret the rate of change and initial value of a linear function in context. **Learning Goal 2**: Model a linear relationship by constructing a function from two (x,y) values. Interpret the rate of change and initial value of the linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.G.B.7* Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8.G.B.8* Apply the Pythagorean Theorem to find the distance between two points in a coordinate system	MP.2 Reason abstractly and quantitatively. MP.7 Look for and make use of structure.	**Concept(s):** No new concept(s) introduced **Students are able to:** ● determine side lengths of right triangles by applying the Pythagorean Theorem to solve real world and mathematical problems in two and three dimensions. ● determine the distance between two points in a coordinate plane by applying the Pythagorean Theorem. **Learning Goal 3**: Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in two and three dimensions to solve real-world and mathematical problems and to determine the distance between two points in the coordinate plane
8.EE.C.8c* Analyze and solve pairs of simultaneous linear equations. 8.EE.C.8c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*	MP.2 Reason abstractly and quantitatively. MP.6 Attend to precision. MP.1 Make sense of problems and persevere in solving them. MP.7 Look for and make use of structure.	**Concept(s):** ● Simultaneous linear equations may have an infinite number of solutions. ● Simultaneous linear equations may have no solution or a single solution. ● Solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs. **Students will be able to:** ● solve systems of two linear equations in two variables algebraically. ● estimate solutions of a linear system of two equations by graphing. ● solve simple cases of a linear system of two equations by inspection. ● solve real-world and mathematical problems leading to two linear equations in two variables. **Learning Goal 4**: Solve real world and mathematical problems leading to two linear equations in two variables, interpreting solutions in context.

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Focus Mathematical Concepts	
Prerequisite skills: Graphing. Solving equations in 1 and 2 variables. Substituting values into equations and expressions. **Common Misconceptions**: Students believe that volume is a number that results from substituting other numbers into a formula rather than a measure related to the amount of space occupied. When solving systems of linear equations, students may make terms "go away" incorrectly.	
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Exemplar tasks or illustrative models could be provided. Vocabulary List Manipulative Ideas Manipulative Kit Standards UnPacking Coherence Map	*District/school resources and supplementary resources that are texts as well as digital resources used to support the instruction.* **SavvasRealize** **BrainPop** **Illustrative Mathematics** **Illuminations NCTM** **Estimation 180** **Georgia Standards OER** **NJ Center for Teaching & Learning** **Achieve the Core** **Khan Academy** **Hinge Questions** **Teacher Education by Design** **EngageNY** Interactive Notebook

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