Fractions, Volume, and Expressions

Content Area: **Math**
Course(s):
Time Period: **MP1**
Length: **45**
Status: **Published**

Unit Overview

Unit Summary	Unit Rationale
In this unit learners extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. Building on their grade 3 work with area, they find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths. They show that the area is the same as would be found by multiplying the side lengths and represent fraction products as rectangular areas. Learners build upon earlier work with multiplication and division. They interpret multiplication as scaling and compare the size of the product to the size of the factors. They come to understand and explain that multiplying a given factor by a number greater than 1 leads to a product that is greater than the given factor. Learners solve real world problems involving multiplication of fractions and represent problems using visual fraction models and equations. Learners are introduced to a new interpretation of fraction. They interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). They solve word problems involving division of whole numbers that lead to answers in fraction form. Learners then extend these previous understandings of division to divide unit fractions by whole numbers and to divide whole numbers by unit fractions Learners build upon earlier work in grade 3 tiling rectangular figures to develop the concept of area. Now in grade 5, learners pack rectangular prisms with unit cubes to develop the concept of volume. They recognize volume as an attribute of solid figures, understand foundational concepts of volume measurement, and measure volumes by counting unit cubes of various standard and non-standard units. They relate volume to the operations of	Using fractions, scaling, and volume are key in 2-D and 3-D measurement and design in the real world, and are fundamental components of some careers. They are also necessary for future math studies.

multiplication and addition and solve real world and mathematical problems by applying volume formulas V = l × w × h and V = B × h to rectangular prisms with whole number edge lengths. To conclude the unit, learners recognize volume as additive and use the concept to determine volumes of composite solid figures composed of right rectangular prisms. The additional and supporting concepts and skills engage learners in analyzing the structure of numerical expressions. Learners evaluate and write numerical expressions with grouping symbols, write numerical expressions from a description, and interpret numerical expressions.	

NJSLS

MATH.5.OA.A.1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
MATH.5.OA.A.2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.
MATH.5.NF.B.3	Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
MATH.5.NF.B.4.a	Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$.
MATH.5.NF.B.4.b	Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
MATH.5.NF.B.5.a	Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
MATH.5.NF.B.5.b	Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than

	1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
MATH.5.NF.B.6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
MATH.5.NF.B.7.a	Interpret division of a unit fraction by a non-zero whole number, and compute such quotients.
MATH.5.NF.B.7.b	Interpret division of a whole number by a unit fraction, and compute such quotients.
MATH.5.NF.B.7.c	Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.
MATH.5.M.B.2.a	A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.
MATH.5.M.B.2.b	A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.
MATH.5.M.B.3	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.
MATH.5.M.B.4.a	Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
MATH.5.M.B.4.b	Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
MATH.5.M.B.4.c	Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Standards for Mathematical Practice

MATH.K-12.1	Make sense of problems and persevere in solving them
MATH.K-12.2	Reason abstractly and quantitatively
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning

Unit Focus

Enduring Understandings	Essential Questions

• Models can be used to show that the product of a fraction and a whole number can be interpreted as repeated addition. • Multiplying a whole number by a fraction involves both multiplication and division. Models can be used to represent multiplying a whole number by a fraction. • To multiply a whole number and a fraction, write a fraction a/b as the product a x 1/b, multiply the whole numbers, and write the product as a fraction or mixed number. • The meaning of multiplying a whole number by a fraction can be extended to multiplying a fraction by a fraction. Different models can be used to show this connection. • To find the product of two fractions, multiply the numerators, and then multiply the denominators. Recognize when the product is less than or greater than 1. • An area model can be used to represent the product of two fractions. • Multiplying mixed numbers is an extension of multiplying fractions. • The relative size of the factors can be used to determine the relative size of the product. • Good math thinkers make sense of problems and think of ways to solve them. If they get stuck, they don't give up. • A fraction can be interpreted as division of the numerator by the denominator. • A fraction or mixed number can represent the quotient of two whole numbers. • Models can be used to show how dividing a whole number by a fraction relates to multiplication. • Visual fraction models can be used to represent and solve problems involving whole numbers divided by unit fractions. • Dividing a unit fraction by a non-zero whole number can be modeled by showing part of a whole divided into equal parts. • Area models and number lines can be used to represent and solve division problems involving whole numbers and unit fractions. • Volume can be measured by counting the number of cubic units needed to fill a three-dimensional figure. • Formulas can be used to find the volume of rectangular prisms and cubes. • The volume of a solid figure composed of	• What are some ways to multiply a fraction by a whole number? • How can you multiply a whole number by a fraction? • How can you multiply fractions and whole numbers? • How can you use a model to multiply fractions? • How can you find the product of two fractions? • How can you find the area of a rectangle with fractional side lengths? • How can you find the product of mixed numbers? • How can you use number sense to evaluate the size of a product? • How can you make sense of problems and persevere in solving them? • How are fractions related to division? • How can you show a quotient using a fraction of mixed number? • How is dividing by a fraction related to multiplication? • How can you divide by a unit fraction? • How can you model dividing a unit fraction by a whole number? • How can you divide with unit fractions and whole numbers? • How can you measure space inside a solid figure? • How can you use a formula to find the volume of a rectangular prism? • How can you find the volume of a solid figure composed of two rectangular prisms? • What order should you use when evaluating an expression? • How can you write a numerical expression to record calculations

rectangular prisms can be found by adding the volumes of the rectangular prisms. • There is an agreed-upon order in which operations are carried out in a numerical expression. • Numerical expressions can represent the calculations needed to solve the problem.	

Instructional Focus

Learning Targets

- multiply multi-digit whole numbers with accuracy and efficiency.
- represent a fraction as a division statement (a/b = a ÷ b).
- divide whole numbers in order to solve real world problems, representing the quotient as a fraction or a mixed number.
- represent word problems involving division of whole numbers using visual fraction models and equations.
- for whole number or fraction q, represent (a/b) × q as a parts of a partition of q into b equal parts [e.g. using a visual fraction model, (3/4) x 5 can be represented by 3 parts, after partitioning 5 objects into 4 equal parts].
- for whole number or fraction q, represent (a/b) × q as a × q ÷ b [e.g. showing that (2/5) x 3 is equivalent to (2 x 3) ÷ 5].
- from a story context, interpret (a/b) × q as a parts of a partition of q into b equal parts.
- tile a rectangle having fractional side lengths using unit squares of the appropriate unit fraction [e.g. given a 3 ¼ inch x 7 ¾ inch rectangle, tile the rectangle using ¼ inch tiles].
- show that the area found by tiling with unit fraction tiles is the same as would be found by multiplying the side lengths.
- multiply fractional side lengths to find areas of rectangles.
- represent fraction products as rectangular areas.
- multiply a fraction by a whole number.
- multiply a fraction by a fraction, in general, if q is a fraction c/d, then (a/b) x (c/d) = a(1/b) × c(1/d) = ac × (1/b)(1/d) = ac(1/bd) = ac/bd.
- compare the size of a product to the size of one of its factors, considering the size of the other factor (at least one factor is a fraction).
- explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number.
- explaining why multiplying a given number by a fraction less than 1 results in a product smaller than

the given number.

- explain that multiplying a given number by a fraction equivalent to 1 does not change the product.
- multiply fractions and mixed numbers in order to solve real world problems.
- represent the solution to these real world problems with visual fraction models and equations.
- use a story context to interpret division of a unit fraction by a whole number.
- divide of a unit fraction by a whole number and represent with visual fraction models.
- use a story context to interpret division of a whole number by a unit fraction.
- divide of a whole number by a unit fraction and represent with visual fraction models.
- divide unit fractions by whole numbers to solve real-world problems, using visual fraction models and equations to represent the problem.
- divide whole numbers by unit fractions to solve real-world problems, using visual fraction models and equations to represent the problem.
- count unit cubes in order to measure the volume of a solid.
- use unit cubes of centimeters, inches, and/or other units to measure volume.

Prerequisite Skills

Multiplication of whole numbers

Representations of fractions on a number line

Use the four operations with whole numbers to solve problems

Build fractions from unit fractions

Understanding and applying place value for multi digit whole numbers.

Common Misconceptions

Students may believe the order in which a problem with mixed operations is written is the order to solve the problem. Students may believe that multiplication always results in a larger number and division results in a smaller number. Students may interchange volume, area, and perimeter. Students might have difficulty seeing or describing how they could break apart a composite shape.

Spiraling For Mastery

Current Unit Content/Skills	Spiral Focus	Activity
• multiply a fraction by a whole number • use a model to multiply fractions • find the product of mixed numbers • use number sense to evaluate the size of a product. • interpret a fraction as the numerator divided by the denominator • find quotients that are mixed numbers • use models and the relationship between multiplication and division to divide a whole number by a unit fraction • use the meanings of division and models to find the quotient of a whole number and a unit fraction • use the meanings of division and models to divide a unit fraction by a whole number. • find the volume of a solid figure • use the volume formula to find the volume of a rectangular prism • find the volume of a solid figure composed of two rectangular prisms. • evaluate an expression using the order of operations • write numerical expressions that record calculations.	• multiply a fraction by a whole number using addition • use a model to multiply a fraction by a whole number • add fractions with like denominators • use equivalent fractions to add and subtract mixed numbers. • interpret remainders in division of whole numbers • decompose mixed numbers and change between fractions and mixed numbers • multiply a fraction by a whole number using a model and repeated addition • multiply whole numbers, fractions, and mixed numbers • model multiplication of whole numbers, fractions, and mixed numbers. • make and use 1-D and 2-D geometric measurements • use formulas to find rectangular area • apply area formula to solve real-world and mathematical problems involving the area of a shape composed of two rectangles. • use the Distributive Property and partial products to multiply whole numbers, breaking apart a calculation into multiple steps using parentheses • solve multi-step problems, using an equation to represent each step.	• Math Diagnostic and Intervention System Activities • IXL

Assessment

Formative Assessment	Summative Assessment
• Homework • Lesson Checks • MathXL • Quizzes • Exit Tickets • Lesson Reflections • Performance Tasks	• Topic Tests (Common Assessments) • Unit 3 Benchmark (Link-It)

Resources

Key Resources	Supplemental Resources
• Savvas EnVision Grade 5 • Pacing Guide	• IXL • Delta Math • Desmos • Khan Academy

Career Readiness, Life Literacies, and Key Skills

WRK.9.2.5.CAP.1	Evaluate personal likes and dislikes and identify careers that might be suited to personal likes.
TECH.9.4.2.CT.3	Use a variety of types of thinking to solve problems (e.g., inductive, deductive).
TECH.9.4.2.IML.2	Represent data in a visual format to tell a story about the data (e.g., 2.MD.D.10).

Interdisciplinary Connections

ELA.L.RF.5.4.A	Read grade-level text with purpose and understanding.
ELA.L.KL.5.1.A	Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases.
ELA.L.VL.5.2.C	Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation and determine or clarify the precise meaning of key words and phrases.
SCI.5-ESS2	Earth's Systems

Mathematical and computational thinking in 3–5 builds on K–2 experiences and progresses to extending quantitative measurements to a variety of physical properties and using computation and mathematics to analyze data and compare alternative design solutions.

Standard units are used to measure and describe physical quantities such as weight and volume.