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Pre-Calculus Unit 4 Introduction to Discrete and Continuous Mathematics
April-June (45 instructional days)

Targeted Standards

Cluster: Apply the probability properties to solve various problems. Understand the concepts of limits and how they are used in solving higher-level problems. Analyze sequences and sums.	**S-CP.A1**: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **S-CP.A2**: Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **S-CP.A3**: Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. **S-MD.A1**: (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **S-MD.A2**: (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **S-MD.A3**: (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. **S-MD.A4**: (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? **S-MD.B6**: (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). **S-CP.B9**: (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

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	F-IF.A3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. **F-IF.B6**: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ **F-LE.A2**: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Rationale and Transfer Goals:

This unit introduces students to discrete and continuous variables through examination of probability. Students continue building on Algebra II knowledge and review key areas of that course. Students begin to understand how to apply knowledge and skills in algebra, specifically involving exponents into the calculus-based application of logarithms. This bridge helps transition students to Calculus.

Enduring Understandings:

Probability and stochasticity are governed by rules and tendencies that can help us predict and anticipate patterns in the real world.
Mathematicians can determine best-fit models for given sets of data.
Math can be used to explain, understand, and predict real-world situations.

Essential Questions:

How do you distinguish between discrete and continuous random variables?
How do you identify the sample space of a probability experiment?
How do you find the probability of an event given that another event has occurred?
How do you use counting principles to find probabilities?
How are sequences and series used to model many mathematical ideas and realistic situations?
How does the concept of a limit lead to a derivative?
How are integrals used to measure changing quantities?

Content/Objectives	Instructional Actions

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Content ***What students will know***	**Skills** ***What students will be able to do***	**Activities/Strategies** ***How we teach content and skills***	**Evidence (Assessments)** ***How we know students have learned***
• Solve counting problems using the Addition Principle. • Solve counting problems using the Multiplication Principle. • Solve counting problems using permutations involving n distinct objects. • Solve counting problems using combinations. • Find the number of subsets of a given set. • Solve counting problems using permutations involving n non-distinct objects. • Apply the Binomial Theorem. • Construct probability models. • Compute probabilities of equally likely outcomes. • Compute probabilities of the union of two events. • Use the complement rule to find probabilities.	• Apply the general Multiplication Rule in a uniform probability model, P(A and B), and interpret the answer in terms of the model. • Use permutations and combinations to compute probabilities of compound events and solve problems. • Use the mean and standard deviation for a data set to fit the data to a normal curve (normal distribution) and estimate population percentages. Use Z scores. • Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. • Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given function or	Math practice individually, whole group, and small group. Peer group leadership Student presentations of concepts and demonstration of skills Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) Open Source activities below from Illustrative Math, Desmos, Geogebra: • The Titanic 1 • The Titanic 2 • Geogebra - Conditional Probabilities and Independence • Finding Probabilities of Compound Events • Geogebra - Conditional Probabilities and Independence • Bob's Bagel Shop	• Written section assessments • Review Games • Practice exercises and assignments • White board demonstrations • Desmos Activities • Written Topic Assessments • Technology Assessments • Benchmark 4 Assessment

• Compute probability using counting theory. • Standard Deviation • Normal distributions and Z scores • Scatterplots • Normal distributions are only appropriate for some data. • Scatter plots can only be used to represent quantitative variables. • The role of randomization in sample surveys, experiments, and observational studies. • The difference between variables as quantitative or categorical • Understand limit notation. • Find a limit using a graph. • Find a limit using a table. • Find the limit of a sum, a difference, and a product. • Find the limit of a polynomial. • Find the limit of a power or a root.	choose a function suggested by the context. Emphasize linear, quadratic, and exponential models • Estimate the area under a normal curve using technology and explain the significance of this value in terms of probability and the original context. • Sketch the function of best fit on a scatter plot and find the function using technology when necessary. • Choose a probability model for a problem context. • Conduct a simulation of a model and determine which results are typical or considered outliers. • Calculate a sample mean or population. • Determine how often the true population mean or proportion is within the margin of error of each	• Fred's Fun Factory • Sounds Really Good! (sort of…) • Random Walk III • Random Walk IV • Alex, Mel, and Chelsea Play a Game • Return to Fred's Fun Factory • The High School Gym • Mathemafish Population • Introduction to Limits • Graphical Limits • One-Sided Limits • Desmos - Limits and Continuity • Desmos - Average Value of a Function	

• Find the limit of a quotient. • Determine whether a function is continuous at a number. • Determine the numbers for which a function is discontinuous. • Determine whether a function is continuous.	sample mean or proportion. • Conduct a simulation for each group using sample results as the parameters for the distributions. • A function has a limit if the output values approach some value L as the input values approach some quantity a. • A shorthand notation is used to describe the limit of a function according to the form $\lim x \to a f(x)=L$, which indicates that as x approaches a, both from the left of $x=a$ and the right of $x=a$, the output value gets close to L. • A function has a left-hand limit if $f(x)$ approaches L as x approaches a where $x<a$. A function has a right-hand limit if $f(x)$ approaches L as x approaches a where $x>a$. • A two-sided limit exists if the left-hand limit and the right-hand limit of a		

	function are the same. A function is said to have a limit if it has a two-sided limit. ● A graph provides a visual method of determining the limit of a function. ● If the function has a limit as x approaches a, the branches of the graph will approach the same y-coordinate near x=a from the left and the right. See ● A table can be used to determine if a function has a limit. The table should show input values that approach a from both directions so that the resulting output values can be evaluated. If the output values approach some number, the function has a limit. ● A graphing utility can also be used to find a limit. ● The properties of limits can be used to perform operations on the limits		

	of functions rather than the functions themselves. ● The limit of a polynomial function can be found by finding the sum of the limits of the individual terms. ● The limit of a function that has been raised to a power equals the same power of the limit of the function. Another method is direct substitution. ● The limit of the root of a function equals the corresponding root of the limit of the function. ● One way to find the limit of a function expressed as a quotient is to write the quotient in factored form and simplify. ● Another method of finding the limit of a complex fraction is to find the LCD. ● A limit containing a function containing a root		

	may be evaluated using a conjugate. ● The limits of some functions expressed as quotients can be found by factoring. ● One way to evaluate the limit of a quotient containing absolute values is by using numeric evidence. Setting it up piecewise can also be useful. ● A continuous function can be represented by a graph without holes or breaks. ● A function whose graph has holes is a discontinuous function. ● A function is continuous at a particular number if three conditions are met: ○ Condition 1: $f(a)$ exists. ○ Condition 2: $\lim_{x \to a} f(x)$ exists at $x=a$. ○ Condition 3: $\lim_{x \to a} f(x)=f(a)$		

	● A function has a jump discontinuity if the left- and right-hand limits are different, causing the graph to “jump.” ● A function has a removable discontinuity if it can be redefined at its discontinuous point to make it continuous. ● Some functions, such as polynomial functions, are continuous everywhere. Other functions, such as logarithmic functions, are continuous on their domain. ● For a piecewise function to be continuous each piece must be continuous on its part of the domain and the function as a whole must be continuous at the boundaries.		

Spiraling for Mastery

Content or Skill for this Unit	Spiral Focus from Previous Unit	Instructional Activity
● Represent sample spaces. ● Apply basic properties of probability.	Algebra II knowledge and skills ● 7.SP.C.8	Students given handouts of powerpoint notes

• Use two-way frequency tables. • Use P(A∩B)as the probability of A and B occurring together. • Create visual displays of data sets. • Analyze data using statistical measures • The definition of event, sample space, union, intersection, and complement. • Identify independent events. • The definition of dependent events and conditional probability • Addition Rule with unions • Permutations and Combinations • Average Rate of Change • Functions	• F.IF.A.2	Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) iXL Extra Practice and help: • Introduction to probability • Calculate probabilities of events • combinations and permutations • Identify independent events • variance and standard deviation • find limits using graphs • find one-sided limits using graphs • determine if a limit exists

21st Century Skills:

CRP2. Apply appropriate academic and technical skills.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11. Use technology to enhance productivity.

Career and Technical Education

9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth

Key resources:

Pre-Calculus: A Graphing Approach, Holt, Rinehart and Winston 2007, Chapters 13-14
Desmos Activity Builder
Desmos Graphing Calculator Explorations
Geometer's Sketchpad Explorations/Geogebra

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Interdisciplinary Connections

NJSLS ELA

NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

NJSLA Science

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.