Lindenwold
Public Schools

Pre-Calculus Unit 3: Analytic Geometry *February-April (45 instructional days)*
Targeted Standards

Cluster: Analyze the characteristics and properties of conic figures and conic sections. Understand polar coordinates and polar equations. Understand the complex number system and how it is applied to conics.	**G-CO.A4**: Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G-CO.A5**: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **G-GPE.A1**: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G-GPE.A2**: Derive the equation of a parabola given a focus and directrix. **G-GPE.A3**: (+)Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. **N-CN.B4**: (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

Rationale and Transfer Goals:

This unit introduces students to analytic geometry through analysis of conic figures. Students continue building on Algebra II knowledge and review key areas of that course. Students begin to understand how to apply knowledge and skills in algebra, specifically involving exponents into the calculus-based application of logarithms. This bridge helps transition students to Calculus.

Enduring Understandings:

Shapes have certain properties that govern how they function in the real world.
Mathematicians can determine best-fit models for given sets of data.
Math can be used to explain, understand, and predict real-world situations.

Essential Questions:

What is analytical geometry? How can technology be used to enhance the understanding of conics? How are the shapes, known as conic sections, created? How are equations of conic sections analyzed and graphed? What are the similarities and differences in the equations of conic sections and the key features of their graphs? How can the general form of a conic section equation be converted to standard form? What real-world issues could be analyzed and solved using equations and graphs of conic sections?

Content/Objectives		**Instructional Actions**	
Content ***What students will know***	**Skills** ***What students will be able to do***	**Activities/Strategies** ***How we teach content and skills***	**Evidence (Assessments)** ***How we know students have learned***
• Write equations of ellipses in standard form. • Graph ellipses centered at the origin. • Graph ellipses not centered at the origin. • Solve applied problems involving ellipses. • Locate a hyperbola's vertices and foci. • Write equations of hyperbolas in standard form. • Graph hyperbolas centered at the origin. • Graph hyperbolas not centered at the origin.	• An ellipse is the set of all points (x,y) in a plane such that the sum of their distances from two fixed points is a constant. Each fixed point is called a focus • When given the coordinates of the foci and vertices of an ellipse, we can write the equation of the ellipse in standard form. • When given an equation for an ellipse centered at the origin in standard form, we can identify its vertices, co-vertices, foci,	Math practice individually, whole group, and small group. Peer group leadership Student presentations of concepts and demonstration of skills Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) Open Source activities below from Illustrative Math, Desmos, Geogebra: • Slopes and Circles	• Written section assessments • Review Games • Practice exercises and assignments • White board demonstrations • Desmos Activities • Written Topic Assessments • Technology Assessments • Benchmark 3 Assessment

• Solve applied problems involving hyperbolas. • Graph parabolas with vertices at the origin. • Write equations of parabolas in standard form. • Graph parabolas with vertices not at the origin. • Solve applied problems involving parabolas. • Identify non degenerate conic sections given their general form equations. • Use rotation of axes formulas. • Write equations of rotated conics in standard form. • Identify conics without rotating axes. • Locate points in a polar coordinate system • convert between rectangular and polar systems • Create graphs of equations in polar coordinates	and the lengths and positions of the major and minor axes in order to graph the ellipse. • When given the equation for an ellipse centered at some point other than the origin, we can identify its key features and graph the ellipse. • Real-world situations can be modeled using the standard equations of ellipses and then evaluated to find key features, such as lengths of axes and distance between foci. • A hyperbola is the set of all points (x,y) in a plane such that the difference of the distances between (x,y) and the foci is a positive constant. • The standard form of a hyperbola can be used to locate its vertices and foci. • When given the coordinates of the foci	• Explaining the equation for a circle • Defining Parabolas Geometrically • Building Conic Sections • Polygraph: Conics • Polygraph: Circles and Ellipses • Creative Conics • Geogebra - Conic Section • Geogebra - Hyperbolas • Geogebra - Ellipses • Geogebra - Parabolas • Geogebra - Conics in Polar Coordinates • Geogebra - Intro to Polar Coordinates • Geogebra - Battleship (polar)	

• Identify a conic in polar form. • Graph the polar equations of conics. • Define conics in terms of a focus and a directrix.	and vertices of a hyperbola, we can write the equation of the hyperbola in standard form. • When given an equation for a hyperbola, we can identify its vertices, co-vertices, foci, asymptotes, and lengths and positions of the transverse and conjugate axes in order to graph the hyperbola. • Real-world situations can be modeled using the standard equations of hyperbolas. For instance, given the dimensions of a natural draft cooling tower, we can find a hyperbolic equation that models its sides. • A parabola is the set of all points (x,y) in a plane that are the same distance from a fixed line, called the directrix, and a fixed point (the focus) not on the directrix.		

	- The standard form of a parabola with vertex (0,0) and the x-axis as its axis of symmetry can be used to graph the parabola. If $p>0$, the parabola opens right. If $p<0$, the parabola opens left. - The standard form of a parabola with vertex (0,0) and the y-axis as its axis of symmetry can be used to graph the parabola. If $p>0$, the parabola opens up. If $p<0$, the parabola opens down. - When given the focus and directrix of a parabola, we can write its equation in standard form. - The standard form of a parabola with vertex (h,k) and axis of symmetry parallel to the x-axis can be used to graph the parabola. If $p>0$, the parabola opens right. If $p<0$, the parabola opens left.		

	• The standard form of a parabola with vertex (h,k) and axis of symmetry parallel to the y-axis can be used to graph the parabola. If $p>0$, the parabola opens up. If $p<0$, the parabola opens down. • Real-world situations can be modeled using the standard equations of parabolas. For instance, given the diameter and focus of a cross-section of a parabolic reflector, we can find an equation that models its sides. • Four basic shapes can result from the intersection of a plane with a pair of right circular cones connected tail to tail. They include an ellipse, a circle, a hyperbola, and a parabola. • A nondegenerate conic section has the general form		

	$Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ where A,B and C are not all zero. The values of $A,B,$ and C determine the type of conic. ● Equations of conic sections with an xy term have been rotated about the origin. ● The general form can be transformed into an equation in the x' and y' coordinate system without the $x'y'$ term. ● An expression is described as invariant if it remains unchanged after rotating. Because the discriminant is invariant, observing it enables us to identify the conic section. ● Any conic may be determined by a single focus, the corresponding eccentricity, and the directrix. We can also define a conic in terms of a fixed point, the focus $P(r,\theta)$ at the pole, and a line, the directrix, which		

	is perpendicular to the polar axis. • A conic is the set of all points $e=\frac{PF}{PD}$, where eccentricity e is a positive real number. Each conic may be written in terms of its polar equation. • The polar equations of conics can be graphed. • Conics can be defined in terms of a focus, a directrix, and eccentricity. • We can use the identities $r=\sqrt{x^2+y^2}$, $x=r\cos\theta$, and $y=r\sin\theta$ to convert the equation for a conic from polar to rectangular form. • Convert between rectangular and polar coordinates • Input data into graphing calculator for parametric problems		

Spiraling for Mastery

Content or Skill for this Unit	Spiral Focus from Previous Unit	Instructional Activity

• Should be able to determine if a relation is a function • Find the domain and range of functions • Evaluate piecewise-defined and greatest integer functions • Determine whether a graph represents a function • Analyze graphs to determine domain, range, local maxima and minima, inflection points, and intervals there they are increasing, decreasing, concave up, and concave down. • Graph parametric equations • Define quadratic equations • Find the vertex and intercepts of a quadratic function and sketch its graph. • Convert one form of a quadratic function to another. • Evaluate Trigonometric Ratios • Solve Triangles using Trig • Define Trig ratios in the coordinate plane	Algebra II and Trigonometry Sections • 8.G.A.1 • 8.G.A.2 • 8.G.A.3 • 8.G.B.8 • HS.G-CO.A.1 • HS.G-CO.A.2 • HS.G-CO.A.4 • HS.G-SRT.A.1 • HS.A-REI.B.4a • HS.A-REI.D.10 • HS.N-CN.A.1 • HS.N-CN.A.3	Students given handouts of powerpoint notes Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) IXL Remediation: • Convert between Radians and degrees • Solve a Right Triangle - Trig • Graph Sine and Cosine Functions • Find Properties of parabolas • Find properties of circles • find properties of hyperbolas • find properties of ellipses • Convert between rectangular and polar
21st Century Skills: CRP2. Apply appropriate academic and technical skills. CRP8. Utilize critical thinking to make sense of problems and persevere in solving them. CRP11. Use technology to enhance productivity.		
Career and Technical Education		

9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs. 9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth
<u>Key resources:</u> *Pre-Calculus: A Graphing Approach,* Holt, Rinehart and Winston 2007, Chapter 11 Desmos Activity Builder Desmos Graphing Calculator Explorations Geometer's Sketchpad Explorations/Geogebra
<u>Interdisciplinary Connections</u> **NJSLS ELA** NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words. **NJSLA Science** HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media. HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.