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Pre-Calculus Unit 2: Trigonometric and Periodic Functions
November-January (45 instructional days)

Targeted Standards

Cluster: Model periodic phenomena with trigonometric functions. Extend the domain of trigonometric functions using the unit circle. Analyze functions using different representations. Apply trigonometry to general triangles. Define trigonometric ratios, and solve problems involving right triangles.	• **F-IF.C.7E**: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. • **F-TF.A1**: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. • **F-TF.A2**: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. • **F-TF.A3**: (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for πx, π+x, and 2π–x in terms of their values for x, where x is any real number. • **F-TF.A4**: (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. • **F-TF.B5**: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ • **F-TF.B6**: (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. • **F-TF.B7**: (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★ • **F-TF.C8**: Prove the Pythagorean identity sin2 (θ) + cos2 (θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. • **F-TF.C9**: (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. • **G-SRT.C6**: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. • **G-SRT.C7**: Explain and use the relationship between the sine and cosine of complementary angles.

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	• **G-SRT.C8**: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. • **G-SRT.D10**: (+) Prove the Laws of Sines and Cosines and use them to solve problems. • **G-SRT.D11**: (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Rationale and Transfer Goals:

This unit introduces students to trigonometric and periodic functions. Students continue building on Algebra II knowledge and review key areas of that course. Students begin to understand how to apply knowledge and skills in algebra, specifically involving exponents into the calculus-based application of logarithms. This bridge helps transition students to Calculus.

Enduring Understandings:

Sine, Cosine, and Tangent all have intrinsic connections to each other and can be used to predict each other.
Sine, Cosine, and Tangent can be used to describe and predict real-world phenomena.
Mathematicians can determine best-fit models for given sets of data.
Math can be used to explain, understand, and predict real-world situations.

Essential Questions:

How can you model periodic behavior?
What are periodic functions?
What information does a trigonometric function provide of its graph, and vice versa?
If you know the value of sin θ, how can you find the cos θ, tan θ, csc θ, sec θ, and cot θ?
How are translated sine and cosine graphs created on graph paper?
How is the domain and range of the six Trigonometric functions determined?
How do amplitudes, periods, phase shifts, vertical shifts and co-functions relate to the graphs of translated sine and cosine functions?
What are the basic properties of tangent, cotangent, cosecant and secant graphs?
Which trigonometric functions have asymptotes and why?

Content/Objectives		Instructional Actions	
Content	**Skills**	**Activities/Strategies**	**Evidence (Assessments)**

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What students will know	*What students will be able to do*	*How we teach content and skills*	*How we know students have learned*
• The unit circle allows all real numbers to work in trigonometric functions. • Trigonometric identities can be proven and used to solve problems with specified context. • Key features in graphs and tables shed light on relationships between two quantities with trig • Trigonometric functions can be represented by a table, graph, verbal description or equation, and each representation can be transferred to another representation • Specific transformations occur to trigonometric functions based on a value k and its manipulation to the function • The trigonometric functions sin(x) and cos(x) can be used to model real-life situations that exhibit periodic behavior.	• Rewrite trigonometric expressions/equations to solve trigonometric identities. Prove the Pythagorean Identity. • Graph a trigonometric function • Extract information from trigonometric equations and graphs • Convert radians to degree and degrees to radians • Find reference angles, and use them to calculate trig values • Complete a unit circle and use it to determine trigonometric values of special angles. • Find all six trig function values • Translate among representations of trigonometric functions including tables, graphs, equations, and real-life situations.	Math practice individually, whole group, and small group. Peer group leadership Student presentations of concepts and demonstration of skills Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) Open Source activities below from Illustrative Math and Desmos: • Trigonometric Graphing: An Introduction to Amplitude and Vertical Shift • Graphing the Sine Function using Amplitude, Period and Vertical Translation • Bicycle Wheel • What exactly is a radian? • Trigonometric Functions for arbitrary angles	• Written section assessments • Review Games • Practice exercises and assignments • White board demonstrations • Desmos Activities • Written Topic Assessments • Technology Assessments • Benchmark 2 Assessment

• Changing parameters such as amplitude, period, and midline of a function will alter its graph and that these parameters are related to the context or phenomena being modeled. • Understand there are trigonometric/geometric relationships, and apply them • Manipulate trig equations to prove identities by using formulas • Solve with inverse concept • Identify even/odd functions	• Calculate the area of a sector, and linear and angular velocities. • Calculate or interpret the rate of change of a trig function • Find inverse trig values • Modify f(x)=Asin[B(x+C)] +D to model various real life situations and contexts. They should also be able to explain the contextual meaning of points on these functions and the significance of values and solutions. • For a trig function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is	• Trigonometric Identities and Rigid Motions • Properties of Trigonometric Functions • Trig Functions and the Unit Circle • Special Triangles 1 • Equilateral triangles and trigonometric functions • Special Triangles 2 • As the Wheel Turns • Foxes and Rabbits 2 • Foxes and Rabbits 3 • Hours of Daylight 1 • Trigonometric Ratios and Pythagorean Theorem • Finding Trig Values • Calculations with Sine and Cosine • Sum and Difference angle formulas • Coordinates of Equilateral Triangles • Defining Trigonometric Ratios • Tangent of Acute Angles • Sine and Cosine of Complementary Angles • Trigonometric Function Values	

	increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. ● Calculate and interpret the average rate of change of a trig function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ● Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the trig functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear,	● Setting up Sprinklers ● Seven Circles III ● Coins in a circular pattern ● Shortest Line Segment from point P to a line L ● Neglecting the Curvature of the Earth ● Ask the Pilot ● Constructing Special Angles	

	polynomial, rational, absolute value, exponential, and logarithmic functions. ● Define appropriate quantities for the purpose of descriptive modeling and create equations in one, two, or more variables ● Model real-world applications including navigation, aviation, and surveying ● Sketch appropriate trigonometric models ● Restrict a trig function to a domain that allows for an inverse. ● Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context ● Use special triangles to determine geometrically the values of sine, cosine,		

	tangent for ð/3, ð/4 and ð/6, and use the unit circle to express the values of sine, cosine, and tangent for ð–x, ð+x, and 2ð–x in terms of their values for x, where x is any real number.		

Spiraling for Mastery

Content or Skill for this Unit	Spiral Focus from Previous Unit	Instructional Activity
● Define a function and the different parts in expressions, equations, and functions. ● Define trigonometric ratios and solve problems involving right triangles. ● Graph , evaluate, and solve trigonometric functions. ● Should be able to work in degrees and radians.	Algebra II and Geometry knowledge and skills ● HS.G-C.B.5 ● HS.G-CO.A.1 ● HS.F-IF.A.1 ● HS.F-IF.A.3 ● 8.G.B.7 ● 8.G.B.6 ● HS.F-BF.B.3 ● HS.F-IF.B.4 ● HS.F-IF.B.5 ● HS.F-IF.B.6 ● 8.G.B.7 ● HS.G-SRT.B.5	Students given handouts of powerpoint notes Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) IXL Review Materials: ● Trigonometric Ratios: Sine, Cosine, and Tangent ● Trigonometric Ratios: csc, sec, and cot ● Find Trigonometric ratios using a calculator ● Inverses of trigonometric ratios using a calculator ● Solving a right triangle ● Law of Sines ● Law of Cosines ● Area of a Triangle ● Find Trigonometric Ratios using the unit circle ● Sin, Cos, and Tan of special angles ● Csc, sec, and cot of special angles

		• Convert between radians and degrees • Radians and Arc Length

21st Century Skills:

CRP2. Apply appropriate academic and technical skills.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11. Use technology to enhance productivity.

Career and Technical Education

9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth

Key resources:

Pre-Calculus: A Graphing Approach, Holt, Rinehart and Winston 2007, Chapters 6 - 10
Desmos Activity Builder
Desmos Graphing Calculator Explorations
Geometer's Sketchpad Explorations/Geogebra

Interdisciplinary Connections

NJSLS ELA

NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

NJSLA Science

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.
HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.