Lindenwold Public Schools

Pre-Calculus Unit 1: Algebraic, Exponential, and Logarithmic Functions
September-November (45 instructional days)

Targeted Standards

Cluster: Analyze functions using different representations. Analyze regression equations and their behavior. Understand polynomial and rational functions and their behavior.	• **F-BF.B.5**: (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents • **A-REI.A.2**: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. • **A-REI.D11**: Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ • **S-ID.B.6B**: Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology • **F.IF.A.3**: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. • **F-IF.C.7D**: (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. • **F-IF.C.7E**: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Rationale and Transfer Goals:

This unit introduces students to algebraic, exponential, and logarithmic functions. Students build on Algebra II knowledge and review key areas of that course. Students begin to understand how to apply knowledge and skills in algebra, specifically involving exponents into the calculus-based application of logarithms. This bridge helps transition students to Calculus.

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Enduring Understandings:

Algebraic functions model real-world problems and solutions.
Exponential functions and logarithms are mathematical ways of explaining real-world phenomena.
Mathematicians can determine best-fit models for given sets of data.
Math can be used to explain, understand, and predict real-world situations.

Essential Questions:

How do algebraic functions model real-world problems and their solutions?
How do exponential functions model real-world problems and their solutions?
How do logarithmic functions model real-world problems and their solutions?
How are expressions involving exponents and logarithms related?
How do you determine the best-fit model for a given set of data?

Content/Objectives		Instructional Actions	
Content ***What students will know***	**Skills** ***What students will be able to do***	**Activities/Strategies** ***How we teach content and skills***	**Evidence (Assessments)** ***How we know students have learned***
● Algebraic and Geometric Sequences and Series ● Distance/midpoint formulas ● Vertical/horizontal line test ● Linear/quadratic/cubic/quartic functions ● Absolute value/radical functions ● Greatest integer function ● Piecewise functions ● Circle equations ● Parent functions	● Derive the formula for the sum of a finite geometric series ● Identify functions from a variety of representations. ● Distinguish between linear, exponential and quadratic functions from multiple representations ● Relate the domain of a function to its graph and to the context.	Math practice individually, whole group, and small group. Peer group leadership Student presentations of concepts and demonstration of skills Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability)	**Formative/Summative:** ● Written section assessments ● Review Games ● Practice exercises and assignments ● White board demonstrations ● Desmos Activities ● Written Topic Assessments ● Technology Assessments ● Benchmark 1 Assessment

• Horizontal and Vertical shrinks and stretches • Increasing/decreasing functions • Asymptotes and Domain restrictions with rational functions • Relative maximum/minimums • Synthetic substitution/division • Rational zero test • Factor/remainder theorems • Intermediate value theorem • Leading coefficient test • Upper and Lower bound rules. • Asymptotes • Sign Analysis • Properties of exponents • The definition of a logarithm. • The graph of f(x) = log (x) has a domain of x>0, a vertical asymptote at x=0, and an x- intercept at x = 1. • Change of base formula.	• Relate values of a function back to the original context. • For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts, intervals, where the function is increasing, decreasing, positive, or negative, relative maximums function in increasing, decreasing, positive, or negative, relative maximums and minimums, symmetries, end behavior, and periodicity • Transform graphs based on changes in equations and write equations	Open Source activities below from Illustrative Math and Desmos: • Snake on a Plane - Sequences • Two Squares are Equal - Solving Quadratics • Will it hit the Hoop? - Graphing Quadratics • Introduction to Polynomials • Graphing Rational Functions • Polygraph: Rational Functions • Exponential Kiss • Identifying Exponential Functions • Exponentials and Logarithms I • Radical Equations • Basketball - Rational Equations • Who wins the race? - Radical and Rational • Canoe Trip - Rational • An Extraneous Solution	

• Exponential functions • Growth and decay formula • Common/natural logarithms • Base e	based on a transformed parent graph. • Explain the features of a function in relation to its context and to its mathematical structure. • Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative), and find the value of k given the graphs. • Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. • Identify asymptotes and relate them to the restrictions of a function in algebraic form. • Graph a function using basic transformations.		

	• Compare transformations that preserve distance and angle to those that do not. • Create inverse functions, and solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. and composite functions • Create compositions of functions by accurately using function notation. Use compositions to prove inverse relationships. • Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. • Create equations in two or more variables to represent relationships		

	between quantities; graph equations on coordinate axes with labels and scales ● Prove and use polynomial identities ● Graph a polynomial given in factored form, indicating all intercepts and directions of end behaviors. ● Use the structure of an expression to identify ways to rewrite it. ● Create equations in one, two, or more variables and use them to solve problems. ● Construct viable arguments to justify a solution method. ● Write the equation of a polynomial function given its graph or defining characteristics of its graph. ● Solve rational equations in one variable, checking for extraneous solutions.		

	• Perform the long division algorithm for polynomials in order to rewrite simple rational expressions in different forms; use computer algebra systems to perform the same on complicated examples. • Perform the synthetic division method to solve for zeros, rewrite polynomials to find binomial factors. • Use direct/synthetic substitution to evaluate polynomials • Use technology (graphs, tables) to solve the equation f(x) = g(x), where f(x) and/or g(x) are polynomial or rational functions. • Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).		

	• Apply the Remainder/Rational Root Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). • Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.		

	• Simplify complex numbers by using magnitudes and conjugates. • Recognize visual representations of higher degree polynomials and their characteristics. Apply Descartes Rule of Signs to determine upper and lower bound. • Design polynomial functions to represent real-world applications • Analyze graphs of rational functions Solve polynomial/rational inequalities Write a polynomial equation and/or function to model a real-life situation. Use a model of a polynomial function to interpret information about a real-life situation. • Translate among representations of polynomial functions including tables, graphs, equations and real-life		

	situations. Rewrite polynomial and rational equations to reveal new information. ● Create a scatterplot from data and interpret the relationship of the quantities represented. ● Appropriately fit a model to data ● Review exponential laws and simplifications, including integral and rational exponents ● Define and identify transcendental numbers ● Use logarithms to solve exponential equations in base 2, 10, or e. ● Evaluate logarithms based on the definition for simple cases. ● Evaluate logarithms using the change of base formula with technology. ● Graph exponential functions, identifying		

	intercepts and end behavior. ● Graph logarithmic functions, identifying intercepts and end behavior. ● Construct a viable argument to justify a solution method when solving exponential equations ● Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. ● Determine the best function to fit a certain situation or set of data. ● Use technology to fit exponential models to data. ● Model applied situations using exponential and logarithmic functions and answer questions using those models.		

	• Apply exponents to model population growth, business/finance, health/medicine, and physics/science • Reason quantitatively and use units to solve problems. • Interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.		

Spiraling for Mastery		
Content or Skill for this Unit	**Spiral Focus from Previous Unit**	**Instructional Activity**
Students should already be able to: • Graph linear equations and inequalities • Systems of Equations • Basic Complex Number Operations • Simple Log Operations • Solving Quadratic Equations • Factoring Quadratics • Graphing Quadratics • Domain and Range • Simple Operations on Functions • Factor a quadratic expression over the integers.	Algebra II knowledge and skills • HS.F-IF.A.1 • HS.F-IF.A.2 • 8.EE.A.1 • HS.F-BF.B.4 • HS.F-LE.A.4 • HS.F-TF.B.6 • HS.F-TF.B.7 • HS.A-REI.A.1 • HS.N-RN.A.2 • HA.A-REI.D.10 • 8.SP.A.2 • 8.F.B.4	Students given handouts of powerpoint notes Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) iXL Review Sections: • Graphing Linear Inequalities • System of Equations • Operations with Complex Numbers • Basic Log Operations • Solving Quadratic Equations

• Determine zeros from a factored form of a quadratic. • Manipulate a quadratic function between various forms to determine key features, including zeros, the vertex, maximum/minimum value, and end behavior. • Determine domain and range of functions involving simple rational expressions. • Write, graph, and interpret exponential functions. • Solve equations with unknowns in the exponent by inspection or use of exponent rules. • Identify the effects of the transformations f(x) + k, k f(x), f(kx), and f(x + k).	• HS.F-BF.B.3 • HS.F-LE.A.2 • HS.S-ID.B.6.a	○ Completing the Square ○ Using Square Roots ○ by Factoring ○ Quadratic Formula • Simple Rational Expressions • Exponential Functions • Transformations of Functions

21st Century Skills:

CRP2. Apply appropriate academic and technical skills.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

Career and Technical Education

9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth

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Key resources:

Pre-Calculus: A Graphing Approach, Holt, Rinehart and Winston 2007, Chapters 1 - 5
Desmos Activity Builder
Desmos Graphing Calculator Explorations
Geometer's Sketchpad Explorations/Geogebra

Interdisciplinary Connections

NJSLS ELA

NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

NJSLA Science

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.
HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.