Lindenwold Public Schools

Calculus Unit 4: Transcendental Functions

April - June (45 instructional days)

Targeted Standards

#	STUDENT LEARNING OBJECTIVES	CORRESPONDING NJSLS-Math
1	Definition of a natural logarithm function.	A.SSE.4
2	Properties of logarithmic functions.	A.SSE.2
3	The base of the natural logarithm (‘discovering” e)	A.APR.3
4	Inverse functions and their derivatives.	HSF.LE.A.1
5	Exponential functions as inverses of logarithmic functions.	ID.A.4
6	Differentiation and Integration with logarithms and exponential functions.	F.IF.7
7	Logarithmic differential techniques.	F.IF.8
8	Exponential growth and decay problems.	F.IF.9
9	inverse trigonometric functions and their derivatives.	ID.A.1 F.IF.4

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Rationale and Transfer Goals:
Up to this point, students of study two types of elementary functions - algebraic functions and trigonometric functions. This unit concludes the introduction of elementary functions. As each new type is introduced, students will study its properties, its derivative, and its anti-derivative.

Enduring Understandings:

- Natural growth/decay is continuous.
- Logarithms can make differentiation simpler.

Essential Questions:

- What can we do with calculus that we could not do with algebra or geometry or trigonometry?
- Who needs to know calculus?
- What situations can be modeled by exponential or logarithmic functions?

Content/Objectives		Instructional Actions	
Content ***What students will know***	**Skills** ***What students will be able to do***	**Activities/Strategies** ***How we teach content and skills***	**Evidence (Assessments)** ***How we know students have learned***
➔ On the basis of the assumption that the exponential function $y = b^x,\ b > 0$ is continuous everywhere and differentiable at 0, this function is differentiable	➔ Find the derivative of exponential functions. ➔ Find the derivative of logarithmic functions. ➔ Use logarithmic differentiation to determine the derivative of a function.	Math practice individually, whole group, and small group. Peer group leadership Student presentations of concepts and demonstration of skills	**Formative/Summative:** ● Written section assessments ● Review Games ● Practice exercises and assignments ● White board demonstrations

everywhere and there is a formula for its derivative. ➔ We can use a formula to find the derivative of $y = ln\, x$, and the relationship $log_b x = \frac{ln\, x}{ln\, b}$ allows us to extend our differentiation formulas to include logarithms with arbitrary bases. ➔ Logarithmic differentiation allows us to differentiate functions of the form $y = g(x)^{f(x)}$ or very complex functions by taking the natural logarithm of both sides and exploiting the properties of logarithms before differentiating. ➔ The inverse function theorem allows us to compute derivatives of inverse functions without using the limit definition of the derivative. ➔ We can use the inverse function theorem to develop differentiation formulas for	➔ Calculate the derivative of an inverse function. ➔ Recognize the derivatives of the standard inverse trigonometric functions. ➔ Integrate functions involving exponential functions. ➔ Integrate functions involving logarithmic functions. ➔ Integrate functions resulting in inverse trigonometric functions ➔ Write the definition of the natural logarithm as an integral. ➔ Recognize the derivative of the natural logarithm. ➔ Integrate functions involving the natural logarithmic function. ➔ Define the number e through an integral. ➔ Recognize the derivative and integral of the exponential function. ➔ Prove properties of logarithms and exponential functions using integrals.	Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) Open Source activities below from Illustrative Math, Desmos, Geogebra: ● derivatives of exponential functions ● derivatives of logarithmic functions ● integrals of exponential functions ● integrals of logarithmic functions ● derivatives of inverse trigonometric functions ● integrals of inverse trigonometric functions	● Desmos Activities ● Written Topic Assessments ● Technology Assessments ● Benchmark 4 Assessment

the inverse trigonometric functions. ➔ Exponential and logarithmic functions arise in many real-world applications, especially those involving growth and decay. ➔ Substitution is often used to evaluate integrals involving exponential functions or logarithms. ➔ Formulas for derivatives of inverse trigonometric functions developed in Derivatives of Exponential and Logarithmic Functions lead directly to integration formulas involving inverse trigonometric functions. ➔ Use the formulas listed in the rule on integration formulas resulting in inverse trigonometric functions to match up the correct format and make alterations as necessary to solve the problem.	➔ Express general logarithmic and exponential functions in terms of natural logarithms and exponentials.		

➔ Substitution is often required to put the integrand in the correct form. ➔ The earlier treatment of logarithms and exponential functions did not define the functions precisely and formally. This section develops the concepts in a mathematically rigorous way. ➔ The cornerstone of the development is the definition of the natural logarithm in terms of an integral. ➔ The function e^x is then defined as the inverse of the natural logarithm. ➔ General exponential functions are defined in terms of e^x, and the corresponding inverse functions are general logarithms. ➔ Familiar properties of logarithms and exponents still hold in this more rigorous context.			

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Spiraling for Mastery

Content or Skill for this Unit	Spiral Focus from Previous Unit	Instructional Activity
• Apply appropriate mathematical rules or procedures, with and without technology. • Identify an appropriate mathematical rule or procedure based on the classification of a given expression. • Knowledge of logarithmic, exponential, and inverse trigonometric functions.	• Average Rate of Change • Find instantaneous rates of change • Velocity as a rate of change • Find values of derivatives using limits • Find the slope of a tangent line using limits • Sum and Difference Rules • Product Rule • Quotient Rule • Power Rule • Chain Rule • Derivatives of Polynomials • Derivatives using Implicit Differentiation	Students given handouts of powerpoint notes Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) iXL Review Sections: • Find derivatives of exponential functions • Find derivatives of logarithmic functions • Find derivatives of inverse trig functions

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21st Century Skills:

CRP2. Apply appropriate academic and technical skills.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

Career and Technical Education

9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth

Key resources:

Calculus, by Larson, 9e

TI-84Plus Graphing Calculators

www.khanacademy.org

Test Prep materials from the College Board and other publishers

Teacher created worksheets and activities

Interdisciplinary Connections

NJSLS ELA

NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

NJSLA Science

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.