Lindenwold Public Schools

# *Calculus Unit 3: Integrals and Applications of Integrals*

## *February - March (45 instructional days)*

**Targeted Standards**

| # | STUDENT LEARNING OBJECTIVES | CORRESPONDING NJSLS-Math |
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| **1** | Recognize anti-derivatives of basic functions. | A.SSE.4<br>**A.SSE.2**<br>A.APR.3<br>**A.SSE.2**<br>A.APR.3<br>F.1F.7c |
| **2** | Interpret the definite integral as the limit of a Riemann Sum. | |
| **3** | Express the limit of a Riemann sum in integral notation. | |
| **4** | Approximate a definite integral geometrically. | |
| **5** | Calculate a definite integral using areas and properties of definite integrals. | |
| **6** | Analyze functions defined by an integral. | |
| **7** | Calculate antiderivatives. | |
| **8** | Evaluate definite integrals. | |
| **9** | Interpret the meaning of a definite integral within a problem. | |

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| **10** | Apply definite integrals to problems involving the average value of a function, motion, area, and volume. | |
| **11** | Use the definite integral to solve problems in various contexts. | |
| **12** | Analyze differential equations to obtain general and specific solutions. | |
| **13** | Interpret, create and solve differential equations from problems in context. | |

**Rationale and Transfer Goals**:
Integrals are used in a wide variety of practical and theoretical applications. Calculus students should understand the definition of a definite integral involving a Riemann Sum, be able to approximate a definite integral using different methods, and be able to compute definite integrals using geometry. They should be familiar with basic techniques of integration and properties of integrals. The interpretation of a definite integral is an important skill and students should be familiar with area, volume, and Motion applications, as well as with the use of the definite integral as an accumulation function. It is critical that students grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus - a central idea in Calculus. Students should be able to work with and analyze functions defined by an integral.

**Enduring Understandings:**

The process of evaluating a limit allows us to make calculations pertaining to dynamic situations that are impossible were we armed only with algebraic, geometric, and trigonometric techniques.

Physical relationships and phenomena can be represented graphically (geometrically).
The sum of an infinite series is sometimes finite - and sometimes has important applications.

Integration and differentiation are inverse operations and can be used to evaluate quantities and solve problems.

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| **Essential Questions**:<br>What can we do with calculus that we couldn't do before?<br>How do we measure total change?<br>Who needs to know calculus? | | | |
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| **Content/Objectives** | | **Instructional Actions** | |
| **Content**<br><br>***What students will know*** | **Skills**<br><br>***What students will be able to do*** | **Activities/Strategies**<br><br>***How we teach content and skills*** | **Evidence (Assessments)**<br><br>***How we know students have learned*** |

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| ➔ The use of sigma (summation) notation of the form $\sum_{i=1}^{n} a_i$ is useful for expressing long sums of values in compact form.<br>➔ For a continuous function defined over an interval $[a,b]$, the process of dividing the interval into n equal parts, extending a rectangle to the graph of the function, calculating the areas of the series of rectangles, and then summing the areas yields an approximation of the area of that region.<br>➔ The width of each rectangle is $\Delta x = \frac{b-a}{n}$.<br>➔ Riemann sums are expressions of the form $\sum_{i=1}^{n} f(x_i^*)\Delta x$, and can be used to estimate the area under the curve $y=f(x)$. Left- and right-endpoint approximations are special kinds of Riemann sums | ➔ Use sigma (summation) notation to calculate sums and powers of integers.<br>➔ Use the sum of rectangular areas to approximate the area under a curve.<br>➔ Use Riemann sums to approximate area.<br>➔ State the definition of the definite integral.<br>➔ Explain the terms integrand, limits of integration, and variable of integration.<br>➔ Explain when a function is integrable.<br>➔ Describe the relationship between the definite integral and net area.<br>➔ Use geometry and the properties of definite integrals to evaluate them.<br>➔ Calculate the average value of a function.<br>➔ Describe the meaning of the Mean Value Theorem for Integrals.<br>➔ State the meaning of the Fundamental Theorem of Calculus. | Math practice individually, whole group, and small group.<br>Peer group leadership<br><br>Student presentations of concepts and demonstration of skills<br><br>Students given access to online textbook<br><br>Partners or group work (groups formed heterogeneously according to ability)<br><br>Open Source activities below from Illustrative Math, Desmos, Geogebra:<br><br>● Graphing an antiderivative<br>● Upper and Lower Sum Activities<br>● The Fundamental Theorem of Calculus<br>● Integration by Substitution<br>● Numerical Integration<br>● Area of a Region between two Curves<br>● The Disk Method | **Formative/Summative:**<br>● Written section assessments<br>● Review Games<br>● Practice exercises and assignments<br>● White board demonstrations<br>● Desmos Activities<br>● Written Topic Assessments<br>● Technology Assessments<br>● Benchmark 3 Assessment |

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| where the values of $\{x_i^*\}$ are chosen to be the left or right endpoints of the subintervals, respectively.<br>➔ The definite integral can be used to calculate net signed area, which is the area above the x-axis less the area below the x-axis. Net signed area can be positive, negative, or zero.<br>➔ The component parts of the definite integral are the integrand, the variable of integration, and the limits of integration.<br>➔ Continuous functions on a closed interval are integrable. Functions that are not continuous may still be integrable, depending on the nature of the discontinuities.<br>➔ The properties of definite integrals can be used to evaluate integrals.<br>➔ The area under the curve of many functions can be | ➔ Explain the relationship between differentiation and integration.<br>➔ Apply the basic integration formulas.<br>➔ Explain the significance of the net change theorem.<br>➔ Use the net change theorem to solve applied problems.<br>➔ Use substitution to evaluate indefinite and definite integrals.<br>➔ Determine the area of a region between two curves by integrating with respect to the independent variable.<br>➔ Find the area of a compound region .<br>➔ Determine the area of a region between two curves by integrating with respect to the dependent variable. | ● The Shell Method<br>● Arc Length and Surfaces of Revolution | |

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| calculated using geometric formulas.<br>➔ The average value of a function can be calculated using definite integrals.<br>➔ The Mean Value Theorem for Integrals states that for a continuous function over a closed interval, there is a value c such that $f(c)$ equals the average value of the function.<br>➔ The Fundamental Theorem of Calculus, Part 1 shows the relationship between the derivative and the integral.<br>➔ The Fundamental Theorem of Calculus, Part 2 is a formula for evaluating a definite integral in terms of an antiderivative of its integrand. The total area under a curve can be found using this formula.<br>➔ Substitution is a technique that simplifies the integration of functions that are the result of a chain-rule derivative. The term | | | |

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| 'substitution' refers to changing variables or substituting the variable u and du for appropriate expressions in the integrand.<br>➔ When using substitution for a definite integral, we also have to change the limits of integration.<br>➔ Just as definite integrals can be used to find the area under a curve, they can also be used to find the area between two curves.<br>➔ To find the area between two curves defined by functions, integrate the difference of the functions.<br>➔ If the graphs of the functions cross, or if the region is complex, use the absolute value of the difference of the functions. In this case, it may be necessary to evaluate two or more integrals and add the results to find the area of the region.<br>➔ Sometimes it can be easier to integrate with respect to y | | | |

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| to find the area. The principles are the same regardless of which variable is used as the variable of integration.<br>➔ Definite integrals can be used to find the volumes of solids. Using the slicing method, we can find a volume by integrating the cross-sectional area.<br>➔ For solids of revolution, the volume slices are often disks and the cross-sections are circles. The method of disks involves applying the method of slicing in the particular case in which the cross-sections are circles, and using the formula for the area of a circle.<br>➔ If a solid of revolution has a cavity in the center, the volume slices are washers. With the method of washers, the area of the inner circle is subtracted from the area of the outer circle before integrating. | | | |

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| ➔ The method of cylindrical shells is another method for using a definite integral to calculate the volume of a solid of revolution. This method is sometimes preferable to either the method of disks or the method of washers because we integrate with respect to the other variable. In some cases, one integral is substantially more complicated than the other.<br>➔ The geometry of the functions and the difficulty of the integration are the main factors in deciding which integration method to use. | | | |

| Spiraling for Mastery | | |
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| **Content or Skill for this Unit** | **Spiral Focus from Previous Unit** | **Instructional Activity** |
| ➔ Use appropriate units of measure.<br>➔ Explain how an approximated value relates to the actual value. | ● Area of compound figures<br>● Find Limits using graphs<br>● Find one-sided limits using graphs<br>● determine if a limit exists | Students given handouts of powerpoint notes<br><br>Students given access to online textbook: Integration and Applications of Integration |

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| ➔ Identify a re-expression of mathematical information presented in a given representation.<br>➔ Identify an appropriate mathematical rule or procedure based on the relationship between Concepts or processes to solve problems.<br>➔ Identify how mathematical characteristics or properties of functions are related in different representations.<br>➔ Apply an appropriate mathematical definition, theorem, or test.<br>➔ Apply appropriate mathematical rules or procedures, with and without technology. | • Find limits using limit laws<br>• Find limits of polynomials and rational functions<br>• Find limits at vertical asymptotes using graphs<br>• Determine end behavior using graphs | Partners or group work (groups formed heterogeneously according to ability) |

**21st Century Skills:**

CRP2. Apply appropriate academic and technical skills.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP11. Use technology to enhance productivity.

**Career and Technical Education**

9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth

**Key resources:**

*Calculus*, by Larson, 9e

TI-84Plus Graphing Calculators

www.khanacademy.org

Test Prep materials from the College Board and other publishers

Teacher created worksheets and activities

| **Interdisciplinary Connections** |
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| **NJSLS ELA**<br>NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.<br><br>**NJSLA Science**<br>HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.<br>HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known. |