Lindenwold Public Schools

Calculus Unit 2: Derivatives and Applications of Derivatives

Mid-November - January (45 instructional days)

Targeted Standards

#	STUDENT LEARNING OBJECTIVES	CORRESPONDING NJSLS-Math
1	Identify the derivative of a functions as the limit of a difference quotient.	HSF.LE.A.1 ID.A.4 F.IF.7 F.IF.8 F.IF.9 ID.A.1 F.IF.4
2	Estimate and evaluate derivatives using the power rule, sum and difference rules, product and quotient rules, and chain rule.	
3	Estimate and evaluate derivatives of trigonometric functions.	
4	Determine higher order derivatives.	
5	Use derivatives to analyze properties of a function.	
6	Recognize the connection between differentiability and continuity.	
7	Interpret the meaning of a derivative within a problem.	
8	Solve problems involving the slope of a tangent line.	
9	Solve problems involving related rates, optimization, rectilinear motion in applied contexts.	

10	Determine the applicability of important calculus theorems using continuity.	

Rationale and Transfer Goals:

Using derivatives to describe the rate of change of one variable with respect to another variable allows students to understand change in a variety of contexts. Students build the derivative using the concept of limits and use the derivative primarily to compute the instantaneous rate of change of a function. Applications of the derivative include finding the slope of a tangent line to a graph at a point, analyzing the graph of a function (for example, determining whether a function is increasing or decreasing and finding concavity and extreme values), and solving problems involving rectilinear motion. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. In addition, students should be able to solve separable differential equations, understand and be able to apply the Mean Value Theorem, and be familiar with a variety of real-world applications including related rates, optimization, and growth and decay models. (College Board)

Enduring Understandings:

Derivatives allow us to determine rates of change at an instant by applying limits to knowledge about rates of change over intervals.

Recognizing that a functions derivative may also be a function allows us to develop knowledge about the related behaviors of both.

Recognizing opportunities to apply derivative rules can simplify differentiation.

Reasoning with definitions, theorems, and properties can be used to determine a limit.

Derivatives allow us to solve real-world problems involving rates of change.

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Essential Questions:

How can a state determine the rate of change in high school graduates at a particular level of public investment in education (in graduates per dollar) based on a model for the number of graduates as a function of the state's education budget?

Why do mathematical properties and rules for simplifying and evaluating limits apply to differentiation?

If you knew that the rate of change in high school graduates at a particular level of public investment in education (in graduates per dollar) was a positive number, what might that tell you about the number of graduates at that level of investment?

Content/Objectives		Instructional Actions	
Content ***What students will know***	**Skills** ***What students will be able to do***	**Activities/Strategies** ***How we teach content and skills***	**Evidence (Assessments)** ***How we know students have learned***
➔ The difference quotients express the average rate of change of a function over an interval. ➔ The instantaneous rate of change of a function at $x = a$ can be expressed by $\lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$ or $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$, provided the limit exists. There are	➔ Determine average rates of change using difference quotients. ➔ Represent the derivative of a function as the limit of a difference quotient. ➔ Represent the derivative of a function as the limit of a difference quotient. ➔ Determine the equation of a line tangent to a curve at a given point.	Math practice individually, whole group, and small group. Peer group leadership Student presentations of concepts and demonstration of skills Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability)	**Formative/Summative:** ● Written section assessments ● Review Games ● Practice exercises and assignments ● White board demonstrations ● Desmos Activities ● Written Topic Assessments ● Technology Assessments ● Benchmark 2 Assessment

equivalent forms of the definition of the derivative and are denoted f'(a). ➔ The derivative of f is the function whose value at x is $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, provided this limit exists. ➔ For y=f(x), notations for the derivative include dy/dx, f'(x), and y'. ➔ The derivative can be represented graphically, numerically, analytically, and verbally. ➔ The derivative of a function at a point is the slope of the line tangent to a graph of the function at that point. ➔ The derivative at a point can be estimated from information given in tables or graphs. ➔ Technology can be used to calculate or estimate the value of a derivative of a function at a point. ➔ If a function is differentiable at a point then it is	➔ Estimate derivatives. ➔ Explain the relationship between differentiability and continuity. ➔ Calculate derivatives of familiar functions. ➔ Interpret a limit as a definition of a derivative. ➔ Calculate derivatives of products and quotients of differentiable functions. ➔ Calculate derivatives of compositions of differentiable functions. ➔ Calculate derivatives of implicitly defined functions. ➔ Calculate derivatives of inverse and inverse trigonometric functions. ➔ Determine higher order derivatives of a function. ➔ Interpret the meaning of a derivative in context. ➔ Calculate rates of change in applied contexts. ➔ Approximate a value on a curve using the equation of a tangent line.	Open Source activities below from Illustrative Math, Desmos, Geogebra: ● Sketchy Derivatives ● Average Value of a Function ● Derivative Match ● Functions and their derivatives ● Mean Value Theorem ● Related Rates Activities ● Optimization Problems ● Sketch the Derivative	

continuous at that point. In particular, if a point is not in the domain of f, then it is not in the domain of f'. ➔ A continuous function may fail to be differentiable at a point in its domain. ➔ Direct application of the definition of the derivative and specific rules can be used to calculate the derivative four functions of the form $f(x) = x'$. ➔ Sums, differences, and constant multiples of functions can be differentiated using derivative rules. ➔ The power rule combined with sum, difference, and constant multiple properties can be used to find the derivatives for polynomial functions. ➔ Specific rules can be used to find the derivatives for sine, cosine, exponential, and logarithmic functions.	➔ Determine limits of functions that result in indeterminate forms. ➔ Justify conclusions about functions by applying the Mean Value Theorem over an interval. ➔ Calculate minimum and maximum values in applied contexts or analysis of functions.		

➔ The chain rule provides a way to differentiate composite functions. ➔ The derivative of a function can be interpreted as the instantaneous rate of change with respect to its independent variable. ➔ The derivative can be used to express information about rates of change in applied contexts. ➔ The derivative can be used to solve optimization problems; that is, finding a minimum or maximum value of a function on a given interval.			

Spiraling for Mastery

Content or Skill for this Unit	Spiral Focus from Previous Unit	Instructional Activity
➔ Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations. ➔ Identify an appropriate mathematical rule or procedure based on the relationship between concepts or processes to solve problems.	● Find Limits using graphs ● Find one-sided limits using graphs ● determine if a limit exists ● Find limits using limit laws ● Find limits of polynomials and rational functions ● Find limits at vertical asymptotes using graphs	Students given handouts of powerpoint notes Students given access to online textbook - Derivatives, Applications of Derivatives Partners or group work (groups formed heterogeneously according to ability) iXL Review Sections:

➔ Use appropriate mathematical symbols and notation. ➔ Apply appropriate mathematical rules or procedures, with and without technology. ➔ Provide reasons or rationales for solutions and conclusions.	• Determine end behavior using graphs	• Average Rate of Change • Find instantaneous rates of change • Velocity as a rate of change • Find values of derivatives using limits • Find the slope of a tangent line using limits • Sum and Difference Rules • Product Rule • Quotient Rule • Power Rule • Chain Rule • Derivatives of Polynomials • Derivatives using Implicit Differentiation

21st Century Skills:

CRP2. Apply appropriate academic and technical skills.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11. Use technology to enhance productivity.

Career and Technical Education

9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth

Key resources:

Calculus, by Larson, 9e
TI-84Plus Graphing Calculators
www.khanacademy.org
Test Prep materials from the College Board and other publishers
Teacher created worksheets and activities

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Interdisciplinary Connections

NJSLS ELA

NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

NJSLA Science

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.