Lindenwold Public Schools

Calculus Unit 1: Limits and Continuity

September - mid-November (45 instructional days)

Targeted Standards

#	STUDENT LEARNING OBJECTIVES	CORRESPONDING NJSLS-Math
1	Express limits symbolically using correct notation. and interpret limits expressed symbolically.	N.Q.1 F.IF.4 F.IF.5 F.IF.6 F.IF.9 G.CO.9, G.CO.10
2	Estimate limits of functions.	
3	Evaluate limits of functions.	
4	Deduce and interpret behavior of functions using limits.	
5	Analyze functions for intervals of continuity or points of discontinuity.	
6	Determine the applicability of important calculus theorems using continuity.	

Lindenwold Public Schools

Rationale and Transfer Goals :

The limit of a function is the primary concept that distinguishes calculus from algebra and analytic geometry. The notion of a limit is fundamental to the study of calculus. Thus, it is important to acquire a good working knowledge of limits before moving on to other topics in calculus. (Larson Textbook)

Many calculus concepts are developed by first considering a discrete model and then the consequences of a limiting case. Therefore, the idea of limits is essential for discovering and developing important ideas, definitions, formulas, and theorems in calculus. Students must have a solid, intuitive understanding of limits and be able to compute various limits, including one-sided limits, limits at infinity, the limit of a sequence, and infinite limits. They should be able to work with tables and graphs in order to estimate the limit of a function at a point. Students should know the algebraic properties of limits and techniques for finding limits of indeterminate forms, and they should be able to apply limits to understand the behavior of a function near a point. Students must also understand how limits are used to determine continuity, a fundamental property of functions. (College Board)

Enduring Understandings:

- Calculus allows us to generalize knowledge about motion to diverse problems involving change.
- Reasoning with definitions, theorems, and properties can be used to justify claims about limits.
- Existence theorems allow us to draw conclusions about a function's behavior on an interval without precisely locating that behavior.

Lindenwold Public Schools

Essential Questions:

- Can change occur at an instant?
- How does knowing the value of a limit, or that a limit does not exist, help you to make sense of interesting features of functions and their graphs?
- How do we close loopholes so that a conclusion about a function is always true?

Content/Objectives		Instructional Actions	
Content ***What students will know***	**Skills** ***What students will be able to do***	**Activities/Strategies** ***How we teach content and skills***	**Evidence (Assessments)** ***How we know students have learned***
• Calculus uses limits to understand and model dynamic change. • Because an average rate of change divides the change in one variable by the change in another, the average rate of change is undefined at a point where the change in the independent variable would be zero. • The limit concept allows us to define instantaneous rate of change in terms of average rates of change. • Given a function f, the limit of f(x) as x approaches c is a real	• Interpret the rate of change at an instant in terms of average rates of change over intervals containing an instant. • Represent limits analytically using correct notion. • Interpret limits expressing in analytic notation. • Estimate limits of functions. • Determine the limits of functions using limit theorems. • Justify conclusions about continuity at a point using the definition. • Determine intervals over which a function is continuous.	Math practice individually, whole group, and small group. Peer group leadership Student presentations of concepts and demonstration of skills Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) Open Source activities below from Illustrative Math, Desmos, Geogebra: • Runner's World	**Formative/Summative:** • Written section assessments • Review Games • Practice exercises and assignments • White board demonstrations • Desmos Activities • Written Topic Assessments • Technology Assessments • Benchmark 1 Assessment • AP Benchmark 1

number R if f (x) can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is lim f (x)= R . ● A limit can be expressed in multiple ways, including graphically, numerically, and analytically. ● The concept of a limit includes one sided limits. ● Graphical information about a function can be used to estimate limits. ● Because of issues of scale, graphical representation of functions may miss important function behavior. ● A limit might not exist for some functions at particular values of x. ● Some ways that the limit might not exist are if the function is unbounded, if the function is oscillating near this value, or if the limit from	● Interpret the behavior of functions using limits involving infinity. ● Explain the behavior of a function on an interval using the Intermediate Value Theorem.	● Average Cost ● The Intermediate Value Theorem ● Limits and Continuity ● Average Value of a Function ● Introduction to Limits ● One-Sided Limits ● Graphical Limits ● Infinite Limits and Limits at Infinity ● The slope of a secant line	

the left does not equal the limit from the right. ● Numerical information can be used to estimate limits. ● One-sided limits can be determined analytically or graphically. ● Limits of sums, differences, products, quotients, and composite functions can be found using limit theorems. ● Types of discontinuities include removable discontinuities, jump discontinuities, and discontinuities due to vertical asymptotes. ● A function *f* is continuous at *x = c* provided that f(c) exists, lim x-> c f(X) exists, and lim x->c f(x) = f(c). ● A function is continuous on an interval if the function is continuous at each point in the interval. ● Polynomial, rational, power, exponential, logarithmic, and trigonometric functions are			

continuous on all points in their domains. ● If f is a continuous function on the closed interval [a, b] and d is a number between f (a) and f (b), then the Intermediate Value Theorem guarantees that there is at least one number c between a and b, such that f(c) = d.			

Spiraling for Mastery

Content or Skill for this Unit	Spiral Focus from Previous Unit	Instructional Activity
● Identify mathematical information from graphical, symbolic, numerical, and/or verbal representations. ● Apply appropriate mathematical rules of procedures with and without technology. ● Identify an appropriate mathematical rule or procedure based on the classification of a given expression. ● Confirm whether hypotheses or conditions of selected definition, theorem, or test have been satisfied.	● Factoring rational expressions ● Graphing equations ● When is a graph continuous? ● Vertical asymptotes	Students given handouts of powerpoint notes Students given access to online textbook Partners or group work (groups formed heterogeneously according to ability) iXL Review Sections: ● Find Limits using graphs ● Find one-sided limits using graphs ● determine if a limit exists ● Find limits using limit laws ● Find limits of polynomials and rational functions ● Find limits at vertical asymptotes using graphs

• Identify a re-expression of mathematical information presented in a given representation. • Apply an appropriate definition, theorem, or test. • Identify how mathematical characteristics or properties of functions are related in different representations. • Provide reasons or rationales for solutions or conclusions.		• Determine end behavior using graphs • Identify graphs of continuous functions • determine continuity using graphs • Intermediate Value Theorem

21st Century Skills:

CRP2. Apply appropriate academic and technical skills.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP11. Use technology to enhance productivity.

Career and Technical Education

9.2.12.CAP.2: Develop college and career readiness skills by participating in opportunities such as structured learning experiences, apprenticeships, and dual enrollment programs.

9.2.12.CAP.3: Investigate how continuing education contributes to one's career and personal growth

Key resources:

Calculus, by Larson, 9e
Calculus Online Textbook - Openstax
TI-84Plus Graphing Calculators
www.khanacademy.org
Test Prep materials from the College Board and other publishers
Teacher created worksheets and activities
www.calculus-help.com

Interdisciplinary Connections

NJSLS ELA

NJSLSA.R7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.

NJSLA Science

HS-PS4-1. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.

HS-PS3-1. Create a computational model to calculate the change in the energy of one component in a system when the change in energy of the other component(s) and energy flows in and out of the system are known.