Course Overview AP Calculus AB

Content Area: **Math**
Course(s):
Time Period:
Length:
Status: **Published**

School Mission Statement

The mission of Chartertech is to provide artists the opportunity to blend principles of artistic expression with cutting-edge technology, so artists will excel in academic, career, and civic pursuits and contribute to the harmony and productivity of the 21" century.

Artistic integration: Performing arts will be accessible to all artists as a skill and content area and will serve as a vehicle for imparting, enlivening, and motivating excellence in all academic topics, as well as providing a platform for learning multicultural appreciation and empathy, not just tolerance.

Technological integration: Technology will serve as the foundation for instructional delivery systems leading to knowledge acquisition, concept understanding, and skill mastery in all academic subjects. Technology will not be studied as a separate entity but infused into the very fabric of educational pursuits, exactly as it occurs in the business world. Artists will be prepared to compete in the modern workplace or post-secondary institution.

"Education has always been torn between vocational and utilitarian purposes on one hand and creative and holistic purposes on the other... We are rapidly entering a world that is hard to imagine. By developing the problem-solving skills, creativity, and discipline required in the arts, artists can prepare for life in the 21st century."

From Understanding How the Arts Contribute to Excellent Education

National Endowment for the Arts, 1991

School Goals

<u>Goals for Arts Education:</u>

Artists will learn the knowledge, skills, and abilities necessary to turn their passions and gifts in the arts into vocations or serious avocations.

Objective 1: Each year, each artist will take two semesters (10 credits) of career-oriented training (80 minutes per day) in their artistic major.

Objective 2: Each marking period, each artist will perform or produce frequently, in diverse settings and for diverse audiences.

Objective 3: Artistic instruction will be integrated into the study of all academic subjects.
Objective 4: Each year, each artist will complete at least twenty after-school “lab” hours in their artistic major. These will constitute career-oriented service to the school and/or community, and demonstrate accomplishment of the NJCCCS crosscutting workplace readiness standards.

Goal for Technology:
Chartertech will model the technology-intense workplace and artists will be able to compete successfully and perform well in a technology-intense workplace.

Objective 5: Each artist will routinely use technology in a workplace-like manner to acquire, analyze, communicate, and present information in every subject.
Objective 6: Each artist will have access to a computer every day, every class so that automated sources will be the main conduit for educational content.
Objective 7: All administrative and instructional functions of the school will be supported by the most modern technology available.

Goals for Academic Achievement:
Artists will apply themselves in the serious pursuit of knowledge and skills, especially skills in critical thinking, problem solving, decision making, and communication.

Objective 8: Each year, and to be promoted to the next grade each artist will pass five credits in English, Health, Social Studies, Science, Mathematics, and PE/Health. Between grades 9-12 artists will also complete 1year of Spanish.

Objective 9: In each academic subject, each year, each artist will complete a significant project that involves critical thinking, problem solving, decision making, and communication skills, and which demonstrates cross-content workplace readiness skills.

Objective 10: Each year artists will develop a artist resume to guide his/her academic and artistic studies and to document his/her academic and artistic accomplishments. This work will be done under the mentorship of the faculty in the artist’s artistic major.

Objective 11: Academic instruction in all subjects will be highly cross-curricular, in accordance with curricula design and continuously improved by teachers, in compliance with the New Jersey Artist Learning Standards.

Course Description

Course Title:	AP Calculus AB
Department:	Mathematics
Prerequisite:	Honors Precalculus
Number of Credits:	5
Grade Level(s):	12

Standards:	Aligned to College Board Standards
Description of Course	Students do best when they have an understanding of the underpinnings of calculus. Rather than making the course a long laundry list of skills that students have to memorize, we stress the "why" behind the major ideas. If students can grasp the reasons for an idea of theorem, they can usual figure out how to apply it to the problem at hand. We explain to them that they will study four major ideas during the year: limits, derivatives, indefinite integrals, and definite integrals. As we develop the concepts, we explain how the mechanics go along with the topics. During the first few weeks, we spend extra time familiarizing students with their graphing calculators. Students are taught the rule of three: ideas can be investigated analytically, graphically, numerically, and verbally. Students are expected to relate the various representations to each other. **[SC4, SC5, SC6 & SC7]** It is important for them to understand that graphs and tables are not sufficient to prove an idea. Verification always requires an analytic argument. Each chapter exam includes one or two questions that involve only graphs or numerical data. I believe it is important to maintain a high level of student expectation. I have found that students will rise to the level that I expect of them. A teacher needs to have more confidence in the students than they have in themselves. We also stress communication as a major goal of the course. Students are expected to explain problems using proper vocabulary and terms. Students are also expected to work and communicate professionally on group projects and assignments. Like many teachers, I have students explain solutions orally on the board to their classmates. This lets me know which students need extra help and which topics need additional reinforcement. Also, I have students explain and/or justify their solutions to problems in well-written sentences. **[SC8 & SC9]** Much of calculus depends on an understanding of a concept taught in a previous lesson. During introductory activities connecting previously taught material to new concepts in the AP Calculus course, students will form groups to tutor themselves.

AP® Calculus AB: Syllabus

Scoring Components		Page(s)
SC1	The course teaches all topics associated with Functions, Graphs, and Limits as delineated in the Calculus AB Topic Outline in the AP Calculus course description.	4
SC2	The course teaches all topics associated with Derivatives as delineated in the Calculus AB Topic Outline in the AP Calculus course description.	5, 6
SC3	The course teaches all topics associated with Integrals as delineated in the Calculus AB Topic Outline in the AP Calculus course description.	6, 7
SC4	The course provides students the opportunity to work with functions represented graphically.	2, 3
SC5	The course provides students with the opportunity to work with functions represented numerically.	2, 3
SC6	The course provides students with the opportunity to work with functions represented analytically.	2
SC7	The course provides students with the opportunity to work with functions represented verbally.	2
SC8	The course teaches students how to explain solutions to problems orally.	2
SC9	The course teaches students how to explain solutions to problems in written sentences.	2
SC10	The course teaches students how to use graphing calculators to help solve problems.	3
SC11	The course teaches students how to use graphing calculators to experiment.	3
SC12	The course teaches students how to use graphing calculators to interpret results and support conclusions.	3

Calculator Ideas

The graphing calculator is used to help students develop an intuitive feel for concepts before they are approached through typical algebraic techniques. We use the calculator as a tool to illustrate ideas and make discoveries about functions in calculus. The four required functionalities of graphing technology are:

1. Finding a root
2. Sketching a function in a specified window
3. Approximating the derivative at a point using numerical methods
4. Approximating the value of a definite integral using numerical methods

Students are also required to make connections between the graphs of functions and their analysis, and draw conclusions about the behavior of functions when using a graphing calculator. **[SC10 & SC12]**

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Activities

The following sample activities ways to help students gain an increased understanding of calculus.

Limits

If your calculator has a "table" feature, it can be used to zoom in on a limit numerically. **[SC5]**

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For example, to find

$$\lim_{x \to 2} \frac{x-2}{x^2-4}$$

we view the values of the function from x-values from 1.5 to 2.5 with an increment step of 0.1. At $x = 2$, the table records "error" or "not defined." Students should see that the y-values seem to follow a pattern. Redo the process beginning at 1.9 with a step size of 0.01, and observe that the y-values are converging to 0.25. The process can be repeated with smaller and smaller steps.

The limit can also be shown visually by graphing the function in a window that has a pixel step of 0.1. Trace the function beginning at $x = 1$. Each step shows the corresponding x- and y-coordinates, but at $x = 2$, the y-coordinate disappears. It "reappears" when the tracing continues at $x = 2.1$. Students can see graphically that the y-coordinates cluster at about 0.25 as x is near 2. For comparison, do the same exploration with

$$\lim_{x \to 2} \frac{x^2+4}{x-4}$$

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This function is also undefined at $x = 2$, but the y-values do not converge as x approaches 2. Instead, the values explode, giving students a numerical look at asymptotic behavior.

The Derivative of the Sine Function (This activity works on a smart-board and online graphing utili

Graph the function $y = \sin x$ in a standard trigonometric viewing window. Estimate the slope of the tangent line at various x-values and plot the slope values as a function of x on the smart-board. **[SC4 & SC11]** (The slope values are clearly 0 at the turning points and can be estimated to be +1 or –1 at the x-intercepts. A few more estimates will enable students to guess the curve.) Students should see that the slope curve follows the path of the cosine function. To test this conjecture, graph the numerical derivative of the sine in the same window. Then graph the cosine function and note that the two graphs are superimposed. Tracing gives the same values on both curves. From this point, it is easy to proceed to an analytic proof of

$$\frac{d}{dx}(\sin x) = \cos x$$

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AP Calculus AB Course Outline

Unit 1: Precalculus Review (2–3 weeks)

A. Lines

1. Parallel and perpendicular lines
2. Slope as rate of change
3. Equations of lines

Overview & Pacing

Unit #	Major Content	Expected Time

<table>
<tr>
<td>Unit 1: Precalculus Review</td>
<td>A. Lines

1. Parallel and perpendicular lines
2. Slope as rate of change
3. Equations of lines

B. Functions and graphs

1. Functions
2. Domain and range
3. Families of functions
4. Piecewise functions
5. Composition of functions

C. Exponential and logarithmic functions

1. Exponential growth and decay
2. Inverse functions
3. Logarithmic functions
4. Properties of logarithms

D. Trigonometric functions

1. Graphs of basic trigonometric functions
a. Domain and range
b. Transformations
c. Inverse trigonometric functions

2. Applications</td>
<td>2-3 Weeks</td>
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Unit 2: Limits and Continuity	A. Rates of change B. Limits at a point 1. Properties of limits 2. Two-sided 3. One-sided C. Limits involving infinity 1. Asymptotic behavior 2. End behavior 3. Properties of limits 4. Visualizing limits D. Continuity 1. Continuous functions 2. Discontinuous functions a. Removable discontinuity b. Jump discontinuity c. Infinite discontinuity E. Instantaneous rates of change	3 Weeks
Unit 3: The Derivative	A. Definition of the derivative B. Differentiability 1. Local linearity 2. Numeric derivatives using the calculator 3. Differentiability and continuity C. Derivatives of algebraic functions D. Derivative rules of combining functions E. Applications to velocity and acceleration F. Derivatives of trigonometric functions G. The chain rule H. Implicit derivatives 1. Differential method	5 Weeks

	2. y' method I. Derivatives of inverse trigonometric functions J. Derivatives of logarithmic and exponential functions	
Unit 4: Applications of the Derivative	A. Extreme values 1. Local (relative) extrema 2. Global (absolute) extrema B. Using the derivative 1. Mean value theorem 2. Rolle's theorem 3. Increasing and decreasing functions C. Analysis of graphs using the first and second derivatives 1. Critical values 2. First derivative test for extrema 3. Concavity and points of inflection 4. Second derivative test for extrema D. Optimization problems E. Linearization models F. Related rates	4 Weeks
Unit 5: The Definite Integral	A. Approximating areas 1. Riemann sums 2. Trapezoidal rule 3. Definite integrals B. The fundamental theorem of calculus (part 1) C. Definite integrals and antiderivatives 1. The average value theorem D. The fundamental theorem of calculus (part 2)	3 Weeks

Unit 6: Differential Equations and Mathematical Modeling	A. Antiderivatives B. Integration using *u*-substitution C. Separable differential equations 1. Growth and decay 2. Slope fields 3. General differential equations	3-4 Weeks
Unit 7: Applications of Definite Integrals	A. Summing rates of change B. Particle motion C. Areas in the plane D. Volumes 1. Volumes of solids with known cross sections 2. Volumes of solids of revolution a. Disk method b. Shell method	3 Weeks

- This schedule leaves 4-6 Weeks of flexibility with teaching and learning time management.