

Unit #3: Linear vs. Exponential Models

Content Area: **Math**
Course(s):
Time Period: **December**
Length: **7 weeks**
Status: **Published**

State Mandated Topics Addressed in this Unit

<u>State Mandated Topics Addressed in this Unit</u>	
N/A	N/A

Linear vs. Exponential Models

Learning Objectives

- Objective 1 - Understand that a function has one element of the domain assigned to exactly one element of the range.
- Objective 10 - Relate the domain of a function to its graph.
- Objective 11 - Relate the domain of a function to the quantitative relationship that it describes.
- Objective 12 - Calculate the average rate of change of a function from a graph or a function on an interval.
- Objective 13 - Interpret the average rate of change.
- Objective 14 - Estimate the average rate of change from a graph.
- Objective 15 - Graph simple functions by hand showing key features of the graph.
- Objective 16 - Graph complicated functions using technology.
- Objective 17 - Graph exponential & logarithmic functions showing intercepts and end behavior
- Objective 18 - Graph linear and quadratic functions showing key features including intercepts, maxima and minima.
- Objective 19- Write a function shown as an expression in equivalent forms.
- Objective 2 - $F(x)$ denotes the output of f corresponding to the input of x .
- Objective 20 - Reveal different properties of a function by expressing it in different forms.
- Objective 21 - Use factoring and completing the square on quadratic functions to show zeroes, extreme, values, and symmetry.
- Objective 22 - Interpret zeroes, extreme values, and symmetry of a quadratic in context.
- Objective 23 - Write a function shown as an expression in equivalent forms.
- Objective 24 - Reveal different properties of a function by expressing it in different forms.
- Objective 25 - Use the properties of exponents to interpret exponential functions.
- Objective 26 - Identify the percent rate of change in an exponential function.

- Objective 27 - Classify exponential functions as exponential growth or decay.
- Objective 28 - Use factoring and completing the square on quadratic functions to show zeroes, extreme, values, and symmetry.
- Objective 29 - Interpret zeroes, extreme values, and symmetry of a quadratic in context.
- Objective 3 - The graph of f is the graph of $y=f(x)$.
- Objective 30 - Use the properties of exponents to interpret exponential functions.
- Objective 31 - Identify the percent rate of change in an exponential function.
- Objective 32 - Classify exponential functions as exponential growth or decay.
- Objective 33 - Compare properties of two functions represented differently (algebraically, graphically, numerically, verbally)
- Objective 34 - Write a function shown as an expression in equivalent forms.
- Objective 35 - Reveal different properties of a function by expressing it in different forms.
- Objective 36 - Use the properties of exponents to interpret exponential functions.
- Objective 37 - Identify the percent rate of change in an exponential function.
- Objective 38 - Classify exponential functions as exponential growth or decay.
- Objective 39 - Write a function that describes a relationship between two quantities. *Determine an explicit expression, recursive process, or steps for calculations from a given context*
- Objective 4 - Use function notation to evaluate functions for inputs in their domain.
- Objective 40 - Combine functions using arithmetic operations
- Objective 41 - Build a function by combining two functions and relate the resulting functions to a model
- Objective 42 - Write arithmetic & geometric sequences recursively
- Objective 43 - Write arithmetic & geometric sequences with an explicit formula.
- Objective 44 - Model situations using arithmetic & geometric formulas.
- Objective 45 - Translate between arithmetic & geometric formulas.
- Objective 46 - Distinguish between situations that can be modeled with linear and exponential functions.
- Objective 47 - Prove that linear functions grow by equal differences over equal intervals.
- Objective 48 - Prove that exponential functions grow by equal factors over equal intervals.
- Objective 49 - Recognize situations where one quantity changes at a constant rate relative to another.
- Objective 5 - Interpret statements that use function notations in terms of context.
- Objective 50 - Recognize situations where a quantity grows or decays by a constant percent rate.
- Objective 51 - Construct linear and exponential functions (with a focus on arithmetic and geometric sequences) from a given graph, description of relationship or two input-output pairs
- Objective 52 - Observe using graphs that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or a polynomial.
- Objective 53 - Observe using tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or a polynomial.
- Objective 54 - Interpret the parameters of a linear or exponential functions in context.
- Objective 55 -
- Objective 6 - Recognize sequences are functions whose domain is a subset of the integers.
- Objective 7 - Sketch a graph using the key features of a function.

- Objective 8 - Interpret key features from a graph or a table of values. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Objective 9 - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Essential Skills

- Essential Skill 1 - Artists will be able to understand that a function has one member of the domain assigned to exactly one element of the range. $F(x)$ denotes the output of f corresponding to the input of x . The graph of f is the graph of $y=f(x)$
- Essential Skill 10 - Artists will be able to calculate the average rate of change of a function from a graph or a function on an interval.
- Essential Skill 11 - Artists will be able to interpret the average rate of change.
- Essential Skill 12 - Artists will be able to estimate the average rate of change from a graph.
- Essential Skill 13 - Artists will be able to graph linear and quadratic functions showing key features including intercepts, maxima and minima.
- Essential Skill 14 - Artists will be able to graph simple functions by hand showing key features of the graph.
- Essential Skill 15 - Artists will be able to graph complicated functions using technology.
- Essential Skill 16 - Artists will be able to graph exponential & logarithmic functions showing intercepts and end behavior
- Essential Skill 17 - Artists will be able to write a function shown as an expression in equivalent forms.
- Essential Skill 18 - Artists will be able to reveal different properties of a function by expressing it in different forms.
- Essential Skill 19 - Artists will be able to use factoring and completing the square on quadratic functions to show zeroes, extreme, values, and symmetry.
- Essential Skill 2 - Artists will be able to use function notation to evaluation functions for inputs in their domain.
- Essential Skill 20 - Artists will be able to interpret zeroes, extreme values, and symmetry of a quadratic in context.
- Essential Skill 21 - Artists will be able to write a function shown as an expression in equivalent forms.
- Essential Skill 22 - Artists will be able to reveal different properties of a function by expressing it in different forms.
- Essential Skill 23 - Artists will be able to use the properties of exponents to interpret exponential functions.
- Essential Skill 24 - Artists will be able to identify the percent rate of change in an exponential function.
- Essential Skill 25 - Artists will be able to classify exponential functions as exponential growth or decay.
- Essential Skill 26 - Artists will be able to compare properties of two functions represented differently (algebraically, graphically, numerically, verbally) Example: given a graph of a quadratic and algebraic expression, say which has the larger maximum.
- Essential Skill 27 - Artists will be able to write a function that describes a relationship between two quantities.

- Essential Skill 28 - Artists will be able to determine an explicit expression, recursive process, or steps for calculations from a given context.
- Essential Skill 29 - Artists will be able to write a function that describes a relationship between two quantities.
- Essential Skill 3 - Artists will be able to interpret statements that use function notations in terms of context.
- Essential Skill 30 - Artists will be able to combine functions using arithmetic operations.
- Essential Skill 31 - Artists will be able to build a function by combining two functions and relate the resulting functions to a model.
- Essential Skill 32 - Artists will be able to write arithmetic & geometric sequences recursively.
- Essential Skill 33 - Artists will be able to write arithmetic & geometric sequences with an explicit formula.
- Essential Skill 34 - Artists will be able to model situations using arithmetic & geometric formulas.
- Essential Skill 35 - Artists will be able to translate between the two forms.
- Essential Skill 36 - Artists will be able to distinguish between situations that can be modeled with linear and exponential functions.
- Essential Skill 37 - Artists will be able to prove that linear functions grow by equal differences over equal intervals.
- Essential Skill 38 - Artists will be able to prove that exponential functions grow by equal factors over equal intervals.
- Essential Skill 39 - Artists will be able to distinguish between situations that can be modeled with linear functions and with exponential functions.
- Essential Skill 4 - Artists will be able to recognize sequences are functions whose domain is a subset of the integers.
- Essential Skill 40 - Artists will be able to recognize situations where one quantity changes at a constant rate relative to another.
- Essential Skill 41 - Artists will be able to recognize situations where a quantity grows or decays by a constant percent rate.
- Essential Skill 42 - Artists will be able to construct linear and exponential functions (with a focus on arithmetic and geometric sequences) from a given graph, description of relationship or two input-output pairs.
- Essential Skill 43 - Artists will be able to observe using graphs that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or a polynomial)
- Essential Skill 44 - Artists will be able to observe using tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or a polynomial.
- Essential Skill 45 - Artists will be able to interpret the parameters of a linear or exponential functions in context.
- Essential Skill 46 - Artists will be able to understand that the graph of an equation in two variables is the set of all of its solutions plotted in the coordinate plane.
- Essential Skill 47 - Artists will be able to understand that the solution set to an equation in two variables often forms a curve (which could be a line).
- Essential Skill 48 - Artists will be able to explain why the solution of $f(x)=g(x)$ is the x coordinate of their intersection.
- Essential Skill 49 - Artists will be able to find the solutions approximately using technology to graph the function or create a table of values. Include $f(x)$ & $g(x)$ that are linear, polynomial, rational, absolute value, exponential, and logarithmic
- Essential Skill 5 - Artists will be able to sketch a graph using the key features of a function.

- Essential Skill 50 - Artists will be able to create an equation that represents relationships between quantities.
- Essential Skill 51 - Artists will be able to graph equations on axes with labels and scales.
- Essential Skill 52 - Artists will be able to represent solutions of equations, inequalities, and systems to real-world applications.
- Essential Skill 53 - Artists will be able to interpret solutions as viable based on the constraints of the application. Example: cannot have negative time
- Essential Skill 54 - Artists will be able to explain the steps to solving an equation.
- Essential Skill 55 - Artists will be able to construct a viable argument to justify a solution method.
- Essential Skill 6 - Artists will be able to interpret key features from a graph or a table of values.
- Essential Skill 7 - Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- Essential Skill 8 - Artists will be able to relate the domain of a function its graph.
- Essential Skill 9 - Artists will be able to relate the domain of a function to the quantitative relationship that it describes.

Standards

MATH.9-12.F.BF.A.1.a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MATH.9-12.F.BF.A.1.b	Combine standard function types using arithmetic operations.
MATH.9-12.F.BF.A.1.c	Compose functions.
MATH.9-12.F.BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MATH.9-12.F.IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
MATH.9-12.F.IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MATH.9-12.F.IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MATH.9-12.F.IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MATH.9-12.F.IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MATH.9-12.F.IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MATH.9-12.F.IF.C.7.a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MATH.9-12.F.IF.C.7.b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MATH.9-12.F.IF.C.7.c	Graph polynomial functions, identifying zeros when suitable factorizations are available,

	and showing end behavior.
MATH.9-12.F.IF.C.7.d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MATH.9-12.F.IF.C.7.e	Graph exponential and logarithmic functions, showing intercepts and end behavior.
MATH.9-12.F.IF.C.7.f	Graph trigonometric functions, showing period, midline, and amplitude.
MATH.9-12.F.IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MATH.9-12.F.IF.C.8.a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MATH.9-12.F.IF.C.8.b	Use the properties of exponents to interpret expressions for exponential functions.
MATH.9-12.F.IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MATH.9-12.F.LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MATH.9-12.F.LE.A.1.a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MATH.9-12.F.LE.A.1.b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MATH.9-12.F.LE.A.1.c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MATH.9-12.F.LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MATH.9-12.F.LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.

Instructional Tasks/Activities

- Activity - Match the graph with the equation.
- Activity (QUIZZZ)
- Activity 1 - Ladder Activity
- Activity 2 - Academic games & Competitions
- Activity 3 - Worksheets
- Activity 4 - Formative Assessments
- Activity 5 - Arts inspired projects
- Activity 6 - Project: Zombie growth-comparing linear and exponential growth of a population
- Activity in QUIZZZ (slope- intercept)
- Converting linear graphs into standard form
- Converting standard form into slope-intercept form
- Intro intro to linear functions
- Intro to standard form (linear equations)
- Linear equations Review
- Linear equations Review (standard form)

- Linear equations word problems
- Make-up day on missing assignments, Quizizz, Quiz.
- Practice slope-intercept, standard form and linear graphs
- Quiz
- Solve Linear functions using slope-intercept
- Solve Linear functions using slope-intercept in graphs
- Worksheet on Average rate of change (Review)
- Worksheet on Average rate of change between two points
- Worksheet on Average rate of change in a graph
- Worksheet on Average rate of change of linear functions
- Worksheet on Average rate of change of non-linear functions

Assessment Procedure

- Classroom Total Participation Technique
- Classwork
- DBQ
- Essay
- Exit Ticket/Entrance Ticket/Do Now
- Journal / Student Reflection
- Kahoot
- Other named in lesson
- Peer Review
- Performance
- Problem Correction
- Project
- Quiz
- Rubric
- Teacher Collected Data
- Test
- Worksheet

Recommended Technology Activities

- Appropriate Content Specific Online Resource
- Chromebook
- Gimkit
- GoGuardian
- Google Classroom

- Google Docs
- Google Forms
- Google Slides
- Kahoot
- MagicSchool AI
- MATHia
- Other- Specified in Lesson
- PowerPoint
- Quizizz
- Screencastify

Accommodations & Modifications & Differentiation

Accommodations and Modifications should be used to meet individual needs. Their IEP and 504 plans should be used in addition to the following suggestions.

Gifted and Talented

- Compare & Contrast
- Conferencing
- Debates
- Jigsaw
- Peer Partner Learning
- Problem Solving
- Structured Controversy
- Think, Pair, Share
- Tutorial Groups

Instruction/Materials

- alter format of materials (type/highlight, etc.)
- color code materials
- eliminate answers
- extended time
- extended time
- large print
- modified quiz
- modified test

- Modify Assignments as Needed
- Modify/Repeat/Model directions
- necessary assignments only
- Other (specify in plans)
- other- named in lesson
- provide assistance and cues for transitions
- provide daily assignment list
- read class materials orally
- reduce work load
- shorten assignments
- study guide/outline
- utilize multi-sensory modes to reinforce instruction

Environment

- alter physical room environment
- assign peer tutors/work buddies/note takers
- assign preferential seating
- individualized instruction/small group
- modify student schedule (Describe)
- other- please specify in plans
- provide desktop list/formula

Honors Modifications

Resources

- Carnegie Learning MathBook
- Resource 1 - www.Khanacademy.com