

Geometry Honors Unit 4: Right Triangles, Area & Volume (Gr. 9 - 10)

Content Area: **Mathematics**
Course(s): **Geometry Honors**
Time Period: **3rd Marking Period**
Length: **8 Weeks**
Status: **Published**

Unit Overview

This unit opens with a review of simplifying radical expressions. From there, students will review, prove, and solve problems involving the Pythagorean Theorem as well as its converses. They will move on to examine the relationships within special right triangles and use these to solve for unknowns. Students will next study trigonometric functions and their application in solving real-world problems. During the second part of the unit, students will review known area, perimeter, and volume formulas, and build new ones. Emphasis will be placed on using these formulas in modeling situations.

Transfer

Students will be able to independently use their learning to...

- Solve for unknown measurements in right triangles.
- Solve applied problems involving right triangles, sketching diagrams as needed.
- Use perimeter, area, circumference, and volume formulas to solve for quantities of interest.

Meaning

Understandings

Students will understand that...

- Values reported using radical notation are more accurate than those given in rounded decimal form.
- The Pythagorean Theorem is used to solve for unknown side lengths in right triangles when two side lengths are given.
- The patterns that exist in Special Right Triangle allow us to solve easily for unknown measurements in these triangles.
- Right triangle trigonometry is useful in solving a variety of problems.
- Formulas can be applied to solve for any single unknown quantity that they contain.
- The areas and volumes of composite and irregular figures can be found by breaking them down into

simpler figures and adding/subtracting their individual areas/volumes.

- Different cross sections of solids may yield different two-dimensional shapes.
- Two- and three-dimensional geometric figures can be used to model real-world situations and figures.

Essential Questions

Students will keep considering...

- How can geometric concepts and figures be used to model real-world phenomena?
- What relationships exist between the side lengths and angle measures of right triangles?
- How can right triangle relationships be used to calculate inaccessible measurements?
- How are the formulas for area, perimeter, circumference, and volume of geometric figures applied in solving problems?
- How can I find the area, perimeter, or volume of figures composed of a combination of various geometric figures?
- How can geometric figures be used in solving real-world problems?

Application of Knowledge and Skill

Students will know...

- That when the altitude is drawn to the hypotenuse of a right triangle, the triangles formed are similar to each other, and to the original right triangle.
- The Pythagorean Theorem and its converses.
- The unique relationships that exist within 45-45-90 and 30-60-90 triangles.
- The definitions of the sine, cosine, and tangent ratios.
- That the sine and cosine of complementary angles are equal to one another.
- That right triangles are useful in solving a variety of real-world problems.
- The formulas used to find areas, perimeters, circumference, and volume of geometric figures.
- That pi represents the ratio between the circumference and diameter of a circle.

Students will be skilled at...

- Simplifying radical expressions.
- Using the Pythagorean Theorem to solve for unknown lengths in right triangles.
- Classifying triangles as acute, right, obtuse, or not possible when given their side lengths.
- Using Special Right Triangle relationships to solve for unknown measurements.

- Using trigonometric ratios to solve for unknown side lengths and angle measures.
- Choosing the most efficient method to solve various right triangles.
- Creating sketches appropriate to solving applied problems involving right triangles.
- Calculating the area, perimeter/circumference or volume of a given figure.
- Using given measurements and quantities to solve for unknown dimensions of geometric figures.
- Identifying various parts of solid figures.
- Applying geometric concepts in modeling situations.

Academic Vocabulary

- 30-60-90 Triangle
- 45-45-90 Triangle
- angle of depression
- angle of elevation
- apothem
- center of a circle
- center of a regular polygon
- center of a sphere
- central angle of a regular polygon
- circle
- composite figure
- cone
- cosine
- cross section
- cube
- cylinder
- edge
- face
- geometric mean
- great circle
- hemisphere
- net
- prism
- pyramid
- Pythagorean Theorem

- radius of a sphere
- sine
- sphere
- tangent
- trigonometric ratios
- vertex of a solid
- volume

Learning Goal 4.1

Students will use the Pythagorean Theorem and Special Right Triangle relationships to solve application problems involving right triangles.

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Target 4.1.1 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT simplify, add, subtract, multiply, and divide radical expressions.

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|-------------|--|
| MA.N-RN.A.2 | Rewrite expressions involving radicals and rational exponents using the properties of exponents. |
| MA.K-12.6 | Attend to precision. |
| MA.K-12.7 | Look for and make use of structure. |

Target 4.1.2 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT use the geometric mean to find segment lengths in right triangles.

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|--------------|---|
| MA.K-12.1 | Make sense of problems and persevere in solving them. |
| MA.K-12.6 | Attend to precision. |
| MA.K-12.7 | Look for and make use of structure. |
| MA.G-SRT.B.4 | Prove theorems about triangles. |

Target 4.1.3 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT apply the Pythagorean Theorem to solving for unknown segment lengths in right triangles and applied problems.

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|-----------|---|
| MA.K-12.1 | Make sense of problems and persevere in solving them. |
| MA.K-12.4 | Model with mathematics. |
| MA.K-12.7 | Look for and make use of structure. |

MA.G-SRT.C.8

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Target 4.1.4 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT prove the Pythagorean Theorem using triangle similarity.

MA.K-12.1

Make sense of problems and persevere in solving them.

MA.K-12.3

Construct viable arguments and critique the reasoning of others.

MA.G-SRT.B.4

Prove theorems about triangles.

Target 4.1.5 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

Given their side lengths, SWBAT use the Converses to The Pythagorean Theorem to classify triangles as acute, right, obtuse, or not possible.

MA.K-12.1

Make sense of problems and persevere in solving them.

MA.K-12.2

Reason abstractly and quantitatively.

MA.K-12.7

Look for and make use of structure.

MA.G-SRT.C.8

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Target 4.1.6 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT identify and determine unknown side lengths in 45-45-90 and 30-60-90 triangles.

MA.K-12.1

Make sense of problems and persevere in solving them.

MA.K-12.4

Model with mathematics.

MA.K-12.6

Attend to precision.

MA.K-12.8

Look for and express regularity in repeated reasoning.

MA.G-SRT.C.6

Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Learning Goal 4.2

Students will define and use trigonometric ratios to solve application problems involving right triangles.

- Students will use the Pythagorean Theorem and Special Right Triangle relationships to solve application problems involving right triangles.

Target 4.2.1 (Level of Difficulty: Retrieval, DOK: 1 - Recall)

SWBAT recognize that by similarity, side ratios in right triangles are properties of the angles in the triangle,

and define of trigonometric ratios for acute angles.

MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
LA.RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
MA.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Target 4.2.2 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT use the sine, cosine, and tangent ratios and their inverses to solve for unknown side lengths and angle measures in right triangles.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Target 4.2.3 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT explain and use the relationship between the sine and cosine of complementary angles.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.

Target 4.2.4 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT sketch right triangles and use trigonometric relationships to solve real-world problems.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied

problems.

Target 4.2.5 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT solve multi-step trigonometry problems involving attached and overlapping triangles.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Target 4.2.6 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT identify and use the most efficient method to solve right triangles in applied problems.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Learning Goal 4.3

Students will explain and apply formulas for finding the perimeter, circumference, and areas of figures.

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Target 4.3.1 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

Based on diagrams and/or written descriptions, SWBAT calculate the area and perimeter of each of the figures listed below. Given the area and one or more measurements of each of the figures listed, SWBAT calculate unknown measurements.

- Triangles
- Parallelograms
- Trapezoids
- Rhombi
- Kites
- Circles

- Regular polygons

Note: Problems should require frequent use of right triangle relationships.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Target 4.3.2 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT describe the ratio represented by pi. Based upon this, SWBAT give informal arguments for the area and circumference of circles.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.G-GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.

Target 4.3.3 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT find the areas of composite figures, and will be able to use composite figures to estimate the areas of irregular shapes.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Target 4.3.4 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT find the perimeters and areas of figures in the coordinate plane.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.

MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 4.3.5 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT use geometric shapes, their measures, and their properties to describe two-dimensional objects.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.7	Look for and make use of structure.
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Target 4.3.6 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT apply concepts of density based on area in modeling situations (e.g., persons per square mile).

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.K-12.7	Look for and make use of structure.
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Learning Goal 4.4

Students will relate two and three-dimensional figures, and will explain volume formulas and use them to solve problems.

- Students will relate two and three-dimensional figures, and will explain volume formulas and use them to solve problems.

Target 4.4.1 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT classify prisms, cylinders, cones, pyramids, and spheres, and will identify:

- the shapes of two-dimensional cross-sections of these figures
- the three-dimensional figures that can be formed by given nets

MA.K-12.1	Make sense of problems and persevere in solving them.
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MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
LA.RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
MA.G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Target 4.4.2 (Level of Difficulty: Retrieval/executing, DOK: 3 - Strategic Thinking)

SWBAT explain and apply the formulas used to find the volumes of prisms, cylinders, cones, and pyramids.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Target 4.4.3 (Level of Difficulty: Retrieval/executing, DOK: 3 - Strategic Thinking)

SWBAT apply the formulas used to find the area and volume of a sphere.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.G-GMD.A.2	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
MA.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Target 4.4.4 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.

MA.K-12.4	Model with mathematics.
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Target 4.4.5 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT apply concepts of density based on volume in modeling situations (ex. BTUs per cubic foot).

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

21st Century Life and Careers

WORK.9-12.9.1.12.1	The ability to recognize a problem and apply critical thinking and problem-solving skills to solve the problem is a lifelong skill that develops over time.
WORK.9-12.9.1.12.1	Collaboration and teamwork enable individuals or groups to achieve common goals with greater efficiency.
WORK.9-12.9.1.12.2	Critical thinking and problem solving in the 21st century are enhanced by the ability to work in cross-cultural teams in face-to-face and virtual environments.
WORK.9-12.9.1.12.2	Leadership abilities develop over time through participation in groups and/or teams that are engaged in challenging or competitive activities.
WORK.9-12.9.1.12.A.1	Apply critical thinking and problem-solving strategies during structured learning experiences.
WORK.9-12.9.1.12.F.2	Demonstrate a positive work ethic in various settings, including the classroom and during structured learning experiences.
WORK.9-12.9.3.12.C.6	Develop job readiness skills by participating in structured learning experiences and employment seeking opportunities.

Summative Assessment

- Projects
- Quizzes
- Student Portfolios
- Tests
- Unit 1 Assessment (Common Assessment)

Formative Assessment and Performance Opportunities

- "I have...Who has..." Review Activities
- Academic Games
- Carousel Activities
- Class Discussions
- Classwork
- Closure Activities
- Concept Sorting Activities
- Do Nows
- Exit Tickets
- Four Corners Activities
- Graphic Organizers
- Homework
- Placemat Activities
- Question-All-Writes
- Quiz-Quiz-Trade Activities
- Station Activities
- Student Interviews
- Student Response Systems
- Student Self-Ratings
- Teacher Observation
- Teacher Questioning
- Think, Pair, Share Discussions
- Thumbs Up/Down
- Whip Around
- Whiteboard Use

Differentiation/Enrichment

- 504 Accommodations
- Challenge Problems
- IEP Modifications
- Learning Centers/Stations
- Leveled Practice Opportunities
- Scaffolding Questions
- Small Group Instruction
- Student Companion Website Resources
- Technology
- Use of Manipulatives (Paper Strips, Exploragons, etc.)

Unit Resources

- Textbook: Geometry, Common Core Ed. (Holt McDougal, 2012)
- Textbook Resource Kit & Companion Website: <https://my.hrw.com/>
- Geometer's Sketchpad
- Kuta Software

Additional Websites:

- Dan Meyer's 3-Act Math Tasks:
<https://docs.google.com/spreadsheets/pub?key=0AjIqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM1UWowTEE&output=htmlG>
- Engage NY: Geometry Lesson Notes & Handouts: <https://www.engageny.org/resource/high-school-geometry>
- Geometry Teacher Mike Patterson's Common Core Teaching Notes:
<http://www.geometrycommoncore.com/>
- Khan Academy: <https://www.khanacademy.org/>
- NCTM Illuminations Website: Resources for Teaching Math:
<http://illuminations.nctm.org/Default.aspx>
- PARCC Educator Resources: <http://www.parcconline.org/for-educators>
- The Geometer's Sketchpad Resource Center: <http://www.dynamicgeometry.com/>