Unit 1: Trigonometry Functions, Graphs, and Identities

Content Area: **Mathematics**
Course(s):   **Pre-Calculus Honors**
Time Period: **2 marking periods**
Length: **15 Weeks**
Status: **Published**

Unit Overview

During this unit, students will…

- Define trigonometry vocabulary as well as put these terms into practice.
- Use angles of rotation and finding arc lengths of circles.
- Use trigonometric functions and their inverses.
- Solve problems involving trigonometric functions
- Factor to solve trigonometric functions.
- Use trigonometric functions to model real-world problems.
- Solve trigonometric equations by using algebra and graphs.
- Extend the domain of trigonometric functions using the unit circle.
- Model periodic phenomena with trigonometric functions.
- Prove and apply trigonometric identities.

Blitzer: CHAPTER 4, 5, 6

Transfer

Students will be able to independently use their learning to...

- Evaluate trigonometric values of both radians and degrees.
- Model real-world situations using trigonometric functions.
- Solve for all missing pieces of triangles.
- Solve and apply trigonometric equations.

For more information, read the following article by Grant Wiggins.

http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60

Meaning

Understandings

Students will understand that...

- The unit circle can be used to evaluate trigonometric functions.
- The graphs of trigonometric functions are periodic.
- Radian measure is a different unit system for measuring angles.
- The Law of Sines and Law of Cosines can be used to solve for missing pieces of right and oblique triangles.
- Real world phenomena can be modeled by periodic functions.

Essential Questions

Students will keep considering...

- How are trigonometric functions and their graphs used to model real-world data?
- How is the unit circle used to determine trigonometric values?
- How are trigonometric identities used to verify statements and solve trigonometric equations?
- What does a trig function look like and how can I use it to model real-life applications?
- How can the Law of Sines and Law of Cosinces help to solve triangles?

Application of Knowledge and Skill

Students will know...

Students will know...

- The relationship between radians and degrees.
- How to build the unit circle from right triangle trigonometry.
- The benefits of using the unit circle.
- Real-life applications of right triangles.
- What the graphs of trigonometric functions look like.
- The Fundamental Trigonometric identities.
- The Law of Sines and Law of Cosines.
- Real-Life applications of oblique triangles.

Students will be skilled at...

Students will be skilled at...

- Converting between radians and degrees.
- Drawing and rotating angles in standard position.
- Working with special right triangles in order to build the unit circle.
- Use the unit circle to create the graphs of sine, cosine, and tangent functions.
- Identifying period, amplitude, maximum and minimums.
- Using the unit circle and right triangles to evaluate trigonometric values of both radians and degrees.
- Using Fundamental Trig Identities to solve trigonometric equations.
- Applying the Law of Sines and the Law of Cosines.

Academic Vocabulary

Angle of rotation

Coterminal Angle

Initial Side

Radian

Reference Angle

Standard Position

Terminal Side

Trigonometric Function

Unit Circle

Amplitude

Cycle

Frequency

Period

Periodic Function

Phase shift

Quadrant angle

Angles of Depression

Learning Goal 1.1

SWBAT solve, graph, and model real-life applications using algebraic and trigonometric properties.

Target 1.1.1 Trigonometric Vocabulary

SWBAT:

- Define, recognize, and draw trigonometric vocabulary such as angles (initial side, terminal side, vertex). **(DOK 1)**
- Recall radian-degree conversion by memory for special angles (30, 45, 60, 90, 180, 270, 360) **(DOK 1)**
- Draw angles in standard position. **(DOK 1)**
- Convert angle measures between degrees and radians. **(DOK 2)**
- Draw and compute coterminal angles in degrees and in radians**. (DOK 2)**
- Determine arc length given radius and radian measure. **(DOK 2)**
- Define a radian. **(DOK 1)**

Section 4.1

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Target 1.1.2 (Right Triangle Trigonometry)

SWBAT:

- Utilize SOH CAH TOA ratios to determine the values of the trigonometric functions for an angle in standard position using right triangle trigonometry. **(DOK 2)**
- Compute trigonmetric expressions and trigonmetric equations using graphing calculator technology. **(DOK 2)**
- Recall special right triangles in degrees and radians in order to evaluate trig expressions and solve trig equations in first quadrant. **(DOK 2)**
- Define and draw angles of evaluation and angles of depression in order to solve word problems. **(DOK 3)**
- Create and solve real-life application problems utilizing right triangle trigonometry **(DOK 4)**

Section 4.3

MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT	Similarity, Right Triangles, and Trigonometry
MA.G-SRT.C	Define trigonometric ratios and solve problems involving right triangles
MA.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
MA.G-SRT.D	Apply trigonometry to general triangles
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

Target 1.1.3 (Unit Circle)

SWBAT:

- Construct the unit circle using 30, 45, 60 special right triangles **(DOK 4)**
- Evaluate simple and complex trig expressions using unit circle **(DOK 2)**

Section 4.2

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.G-SRT	Similarity, Right Triangles, and Trigonometry
MA.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in

	the triangle, leading to definitions of trigonometric ratios for acute angles.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

Target 1.1.4 (Trigonometric Graphs)

SWBAT

- Create sine and cosine graphs using unit circle **(DOK 4)**
- Determine domain, range, amplitude, and period of sine and cosine graphs **(DOK 1)**
- Determine amplitude and range of sine and cosine equations and its effect on the original sine and cosine graphs **(DOK 2)**
- Write sine and cosine equations in y=asin(bx) and/or y=acos(bx) form by looking at a graph. **(DOK 3)**
- Create and recogonize tangent graph using unit circle and SOH CAH TOA ratios. **(DOK 4)**

use definitions of trig functions and reference angles to evaluate trigonometric functions.

Section 4.5

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and

appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

MA.F-BF	Building Functions
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-BF.A.1c	Compose functions.
MA.F-TF	Trigonometric Functions
MA.F-TF.A	Extend the domain of trigonometric functions using the unit circle
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B	Model periodic phenomena with trigonometric functions
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

Target 1.1.5 (Evaluating Trigonometric Functions)

SWBAT

- Define and sketch reference angles and their properties **(DOK 1)**
- Determine reference angles in all 4 quadrants (in degrees and in radians) **(DOK 2)**
- Evaluate trig functions in all 4 quadrants **(DOK 2)**
- Determine missing angle of trig equations using special right triangles and/or unit circle without a

calculator **(DOK 2)**

Section 4.4

MA.G-SRT	Similarity, Right Triangles, and Trigonometry
MA.G-SRT.C	Define trigonometric ratios and solve problems involving right triangles
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

Target 1.1.6 (Fundamental Trigonometric Identities)
SWBAT

- Develop all fundamental identities and categorize as reciprocal, quotient, Pythagorean, even, or odd identities. **(DOK 1)**
- Use fundamental trigonometric identities to simplify and rewrite expressions. **(DOK 2)**
- Use fundamental trigonometric identities and algebraic properties to verify other identities. **(DOK 3)**

Section 5.1

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.6	Attend to precision.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-TF.C.8	Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Target 1.1.7 (Solving Trigonometric Equations)
SWBAT

- Solve equations involving trigonometric functions by applying fundamental trigonometric identities and algebraic properties. **(DOK 2)**
- Utilize special right triangles to solve trigonometric functions in all 4 quadrants. **(DOK 2)**

Section 5.3

MA.K-12.1	Make sense of problems and persevere in solving them.

MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-TF	Trigonometric Functions
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.
MA.F-TF.C.8	Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Target 1.1.8 (Law of Sines & Law of Cosines)

SWBAT

- Use the Law of Sines to find the side lengths and angle measures of a triangle. **(DOK 2)**
- Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case. **(DOK 2)**
- Use the Law of Cosines to find the side lengths and angle measures of a triangle. **(DOK 2)**
- Use Heron's Formula to find the area of a triangle as well as the Law of Sines area formula. **(DOK 2)**
- Use the Law of Sines and the Law of Cosines to sketch triangles that model real-life applications as well as use these formulas to solve multiple step real-life applications. **(DOK 3)**
- Create and solve real-life applications problems involving both right triangle trigonometry as well as LOS and/or LOC. **(DOK 4)**

Section 6.1 & 6.2

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.G-SRT.D.10	Prove the Laws of Sines and Cosines and use them to solve problems.
MA.G-SRT.D.11	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
MA.F-TF.C	Prove and apply trigonometric identities

Formative Assessment and Performance Opportunities

- academic games
- Class discussions

- Classwork
- Do nows
- Exit tickets
- Homework
- Problem based learning
- student interviews
- Teacher observation
- whiteboard/communicator opportunities

Summative Assessment

- Link It Exams
- Projects
- Quizzes
- student interviews
- Tests
- Unit Exam

21st Century Life and Careers

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP2.1	Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.
CAEP.9.2.12.C	Career Preparation
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.

Technology

TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.C	Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
TECH.8.1.12.D	Digital Citizenship: Students understand human, cultural, and societal issues related to

	technology and practice legal and ethical behavior.
TECH.8.1.12.D.CS1	Advocate and practice safe, legal, and responsible use of information and technology.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS4	Process data and report results.
TECH.8.2.12.E.1	Demonstrate an understanding of the problem-solving capacity of computers in our world.

Accommodations and Modifications

- 504 Accommodations
- Academic Games of Review Packet for each section
- centers/stations
- challenge questions
- IEP Modifications
- Individual vs. Large Group Wipeboard Q & A
- manipulatives (highlighting, underlining, starring critical information)
- Provide YouTube videos (special right triangles, create unit circle demonstration, create sine & cosine graph with linguine from unit circle, SOH CAH TOA anticipatory set
- scaffolding questions
- small group instruction (opportunity to work with teacher 1-on-1)
- use of technology such as google classroom to provide answer keys

Unit Resources

- Google Classroom
- Kuta software
- NCTM website
- online textbook materials
- Text
- YouTube & Internet Videos

Interdisciplinary Connections

Real life application problems involving the Law of Sines, Law of Cosines, angle of depression and elevation can be used to predict the height of architectural structures and the distance between objects as well as surverying land and pilot navigation. (MA.9-12.G-SRT.D.10)

9-12.HS-ETS1-4.5	Using Mathematics and Computational Thinking