Unit 5: Applications Of The Integral

Content Area: **Mathematics**
Course(s): **Calculus**
Time Period: **May**
Length: **3 weeks**
Status: **Published**

Unit Overview

In the last unit we used the integral to compute areas under curves and net change. In this unit students will discuss some of the other quantites that are represented by integrals.

In this unit students will.....

- use integrals to calculate area between curves
- use the disk method and washer methods to calculate volumes of revolution using integrals
- find volume as the integral of cross-sectional area

Transfer

Students will be able to independently use their learning to...

-What kinds of long term, independent accomplilshments are desired?

- apply integrals in solving real life problems
- use integrals in calculating area under a curve and between curves
- use integrals in finding volumes of revolutions using disk and washer methods
- find volume as the integral of cross-sectional areas
- understand that integrals can be used in finding areas and volumes of irregularly shaped objects that have no geometric formulas for finding area and volume

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For more information, read the following article by Grant Wiggins.

http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60

Meaning

Understandings

Students will understand that...

-What specifically do you want students to understand?

-What inferences should they make/grasp/realize?

- integrals have many real life applications
- that integrals can be used to find the area and volume of irregularly shaped objects
- that integrals are used in finding area between curves which can translate into real life applications
- that integrals are used in finding volumes of revolutions
- that integrals can be used to find volumes of shapes with given cross-sectional areas

Essential Questions

Students will keep considering...

- How can we expand area under a curve to include the area between curves?
- What are the similarities and differences among the disk and washer methods of integration?
- How can cross-sectional areas and integrals be used to find volumes of irregularly shaped objects?
- What are practical applications of finding such volumes?

Application of Knowledge and Skill

Students will know...

Students will know...

What facts and basic concepts should students know and be able to recall?

- how to calculate definite integrals using Fundamental Theorem of Calculs
- know that definite integrals can be approximated for functions that are represented graphically, numerically, algebraically, and verbally
- in some cases that definite integrals can be evaluated using geometry and the connections between definite integral and area
- techniques for evaluating definite integrals include algebraic manipulatiion and substitution of variables
- areas of certain regions in the plane can be calculated with definite integrals
- area between curves can be calculated using definite integrals
- volume of solids with known cross sections can be calculated with definite integrals
- volume of revolutions can be calculated using disk and wahser methods which rely on the use of definite integrals

Students will be skilled at...

Students will be skilled at...

What discrete skills and processes should students be able to use?

- calculating definite integrals using properties of integrals, using rules of integration, using algebraic manipulation, and substitution method

- calculating and approximating definite integrals whose functions are represented graphically, numerically, algebraically, and verbally
- choosing the most appropriate method for solving a definite integral
- finding the area under a curve using definite integrals
- finding the area between curves using definite integrals
- finding volume of revolutions using disk and washer methods which rely on the use of definite integrals
- finding volume of solids with known cross sections which uses definite integrals
- applying definite integrals to solve real world problems especially those involving area and volumes of irregularly shaped objects

Academic Vocabulary

- definite integrals

- area between curves
- volume of revolutions
- disk method
- washer method
- cross-sections
- cross-sectional area
- irregular shapes

Learning Goal 1

Students will be able to use the definite integral of a function over an interval of time as a mathematical tool to solve applications of derivatives including area and volume of irregularly shaped objects.

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

Target 1

- Students will be able to calculate area between curves using definite integrals. (Level of Difficulty - 3 Analysis)

Standards

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Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▶use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;

▶apply definitions and theorems in the process of solving a problem;

▶develop conjectures based on exploration with technology; and

MPAC 2: Connecting concepts

Students can:

▶use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▶connect concepts to their visual representations with and without technology; and

▶identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▶select appropriate mathematical strategies;

▶sequence algebraic/computational procedures logically;

▶complete algebraic/computational processes correctly;

▶apply technology strategically to solve problems;

▶attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▶connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▶associate tables, graphs, and symbolic representations of functions;

▶develop concepts using graphical, symbolical, or numerical representations with and without technology;

▶identify how mathematical characteristics of functions are related in different representations;

▶extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▶construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▶consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building notational fluency

Students can:

▶connect notation to different representations (graphical, numerical, analytical, and verbal); and

▶assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating

Students can:

▶clearly present methods, reasoning, justifications, and conclusions;

▶explain the meaning of expressions, notation, and results in terms of a context (including units);

▶explain the connections among concepts;

▶critically interpret and accurately report information provided by technology; and

▶analyze, evaluate, and compare the reasoning of others

MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.A-APR	Arithmetic with Polynomials and Rational Expressions
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 2

- Students will be able to calculate volumes of solids with known cross-sections using definite integrals. (Level of Difficulty - 3 Analysis)

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Collegeboard

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MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.G-GMD.A	Explain volume formulas and use them to solve problems
MA.G-GMD.A.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
MA.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
MA.G-GMD.B	Visualize relationships between two-dimensional and three-dimensional objects
MA.G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 3

- Students will be able to find volumes of revolutions using the Disk and Washer Method which relys on definite integrals. (Level of Difficulty - 3 Analysis)

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Summative Assessment

- Quizzes
- Tests
- Unit Exams
- Packets
- Projects
- Writing Assignments
- Labs

21st Century Life and Careers

Select all applicable standards from the applicable standards

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.

Formative Assessment and Performance Opportunities

- Interactive Learning Activities
- Academic Games
- class discussions
- class work
- homework
- warm ups
- Active Learning Activites
- Teacher Observation
- Cooperative Groups

- Student Tracking- Proficiency Scales

Accommodations and Modifications

- 504 Accomadations
- IEP Modifications
- Extension Activites
- Extra Practice Activities
- Technology
- Stations
- Collaborative Corner
- Projects
- Small Group Instruction
- Scaffolding of Questions

Unit Resources

- Textbook
- Online Textbook
- Collegeboard Website
- Practice Workbooks