Unit 4: The Integral

Content Area: **Mathematics**
Course(s): **Calculus**
Time Period: **March**
Length: **12 weeks**
Status: **Published**

Unit Overview

The basic problem in intergral calculus is finding the area under a curve. We will explore the relationship between derivatives and integrals. We will find that one thing that connects these two concepts is they are both computed using limits. Farther exploration and the Fundamental Theorem of Caclulus will allow us to see that differentiation and integration are inverse relationships. This will play a truly fundamental role in nearly all applications of calculus, both theoretical and practical.

During this unit, students will.......

- study that the definition of a definite integral involves Riemann sums
- learn how to approximate a definite integral using different methods
- learn how to compute definite integrals using geometry
- will become familar with the basic techniques of integration and the properties of integrals
- learn that integrals are used in finding area, volume and they are used in motion applications
- also see that the definite integral is an accumulation function
- grasp the relationship between integration and differentation as expressed in the Fundamental Theorem of Calculus
- learn how to work with and anaylze functions defined by an integral

Transfer

Students will be able to independently use their learning to...

-What kinds of long term, independent accomplilshments are desired?

- recognize antiderivatives of basic functions
- determine antiderivatives
- integrate both definite and indefinite integrals
- approximate definite integrals in a variety of ways (ie. Riemann sums, using geometry, using rules of integrations,graphs, tables etc.,)
- explain the Fundamental Theorem of Calculus both Part I and Part II
- use the Fudamental Theorem of Calculus both Part I and Part II to integrate and find derivatives
- find total area vs. net change
- interpret the meaning of an integral within a problem
- apply definite integrals to problems involving average value of a function

- apply definite integrals in real life applications
- use the definite integral to solve problems in various context
- interpret, create, and solve differential equations from problems in context

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For more information, read the following article by Grant Wiggins.

http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60

Meaning

Understandings

Students will understand that...

-What specifically do you want students to understand?

-What inferences should they make/grasp/realize?

- integrals can be used to find antiderivatives
- the meaning of an antiderivative
- integrals can be used to find area under a curve
- integrals can be approximated using tables and graphs (ie. Riemann sums, trapezoids, geometry...)
- integrals can be represented graphically, numerically, algebraically, and verbally
- how to use the properties of definite integrals
- integrals represent an accumulation of a rate of change
- the definite integral of the rate of chane of a quantity over an interval gives the net change of that quantity over that interval
- the limit of an approximating Riemann sum can be interpreted as a definite integral
- antiderivatives can be used to find specific solutions to differential equations with given initial conditions, including applications to motion along a line, exponential growth and decay
- solutions to differential equations may be subject to domain restrictions
- integrals can be used to solve real life applications

Essential Questions

Students will keep considering...

- What is the relationship between a definite integral and area under a curve?
- What is the relationship between finding slopes of tangent lines (differential calculus) and finding areas under curves (integral calculus)?
- What is the Fundamental Theorem of Calculus and what is its significance?
- What is the importance of the indefinite integral in determing real life motion problems?
- What relationship exists between derivatives and integrals when working with the position, velocity, and accleration functions?
- How can one apply numerical techniques to compute an integral with knowing the associated antiderivative?
- How can you find the area under the curve using calculus as well as basic geometric area formulas?

Application of Knowledge and Skill

Students will know...

Students will know...

What facts and basic concepts should students know and be able to recall?

- how to recognize and interpret integral notaion
- determine the indefinite integral of a function using antiderivative formulas
- determine a value for constant of integration
- apply indefinite integrals in discovering initial value problems (ie. positon, velocity, and acceleration functions)
- how to interpret the definite integral as the limit of a Riemann sum
- how to calculuate definite integrals using areas and properties of definite integrals
- that definite integrals can be approximated for functions that are represented graphically, numerically, algebraically and verbally
- to approximate area under a curve using inscribed, circumscribed and midpoints of rectangles as well as approimating area using other geometric formulas
- how to apply the substitution method in order to solve problems involving integrals
- how to use different techniques for finding antiderivatives including algebraic manipulation, and substitution of variables

- how to apply and interpret the Fundamental Theorem of Calculus Part I and Part II
- how to interpret the meaning of a definite integral within a problem
- how to use integrals to solve real life problems
- how to use definite integrals to solve problems in various context
- how to anaylze differential equations to obtain general and specific solutions

Students will be skilled at...

Students will be skilled at...

What discrete skills and processes should students be able to use?

- defining both definite and indefinite integrals
- calculating both definite and indefinite integrals
- approximating definite integrals for functions that are represented graphcially, numericall, algebraically, and verbally
- using algebriac manipulation and substitution method in order to calculate antiderivatives
- find the area under a curve using definite integrals
- approximating definite integrals using Riemann sums, trapezoids and othe geometric area formulas
- solving differential equations to obtain both general and specific solutions
- determining position, velocity and acceleration formulas using integrals
- applying definite integrals to problems involving the average value of a function
- interpreting the meaning of a definite integral within a problem
- using the definite integral to solve problems in various contexts
- interpreting, creating, and solving differential equations from problems in context
- solving real life applications involving integrals

Academic Vocabulary

- Riemann Sums
- Antiderivatives
- Constant of Integration
- Initial Value Problems
- integrals
- definite integrals
- indefinite integrals
- summation notation
- signed area
- The Fundamental Theorem of Calculus PartI and Part II
- net change

- total area
- total
- marginal cost
- substitution method
- seperable integrals
- differential equations

Learning Goal 1

Students will be able to define antidiffernetiation as the inverse process of differentiation, and be able to use antiderivatives to solve initial value problems.

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

Target 1

- Students will be able to recognize antiderivatives of basic functions and use differentiation rules as the foundation for finding antiderivatives. (Level of Difficulty - Level 2 Comprehension)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

MPAC 5: Building notational fluency

Students can:

▸know and use a variety of notations (e.g.,);

MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Target 2

- Students will be able to solve initial value problems using antiderivatives and calculating constant of integration. (Level of Difficulty - 3 Analysis)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

MPAC 3: Implementing algebraic/computational processes

Students can:

▸complete algebraic/computational processes correctly;

▸connect the results of algebraic/computational processes to the question asked.

MPAC 5: Building notational fluency

Students can:

▸assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸use accurate and precise language and notation;

▸explain the meaning of expressions, notation, and results in terms of a context (including units);

▸explain the connections among concepts;

▸analyze, evaluate, and compare the reasoning of others

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Learning Goal 2

Students will be able to define the definite integral of a functions over an interval of time as the limit of a Riemann sum over that interval and be able to calculate the definite integral using a variety of strategies.

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

Target 1

- Students will be able to interpret the definite integral as the limit of a Riemann sum and express the limit of a Riemann sum in integral notation. (Level of Difficulty - 2 Comprehension)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸use definitions and theorems to build arguments, to justify conclusions or answers,

and to prove results;

▸apply definitions and theorems in the process of solving a problem;

▸develop conjectures based on exploration with technology; and

MPAC 2: Connecting concepts

Students can:

▸relate the concept of a limit to all aspects of calculus;

▸use the connection between concepts (e.g., rate of change and accumulation) or

processes (e.g., differentiation and its inverse process, antidifferentiation) to solve

problems;

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

MPAC 5: Building notational fluency

Students can:

▸know and use a variety of notations (e.g.,);

▸connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);

▸connect notation to different representations (graphical, numerical, analytical, and verbal); and

▸assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way

	(algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 2

- Students will be able to approimate definite integrals of a function that are represented graphically, numerically, algebraically and verbally (Level of Difficulty - 3 Analysis)
- Students will be able to approximate definite integrals using Riemann sums, trapezoids and other geometric formulas (Level of Difficulty - 3 Analysis)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸connect concepts to their visual representations with and without technology; and

▸identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▸select appropriate mathematical strategies;

▸complete algebraic/computational processes correctly;

▸apply technology strategically to solve problems;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸identify how mathematical characteristics of functions are related in different representations;

▸extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building notational fluency

Students can:

▸connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);

▸connect notation to different representations (graphical, numerical, analytical, and verbal); and

▸assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸explain the meaning of expressions, notation, and results in terms of a context (including units);

▸explain the connections among concepts;

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others

MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 3

- Students will be able to calculate definite integrals using areas and properties of definite integrals. (Level of Difficulty - 3 Analysis)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸connect concepts to their visual representations with and without technology; and

MPAC 3: Implementing algebraic/computational processes

Students can:

▸select appropriate mathematical strategies;

▸sequence algebraic/computational procedures logically;

▸complete algebraic/computational processes correctly;

▸apply technology strategically to solve problems;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸identify how mathematical characteristics of functions are related in different representations;

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Learning Goal 3

Students will be able to explain how the Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

Target 1

- Student will be able to anaylze functions defined by an integral. (Level of Difficulty - 3 Analysis)

- Students will be able to use graphical, numerical, analytical, and verbal representations of a function ***f*** to provide information about the function ***g*** which is defined as the definite integral of **f(t)** from [a, x]. (Level of Difficulty - 4 Knowledge Utilization)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;

▸apply definitions and theorems in the process of solving a problem;

▸develop conjectures based on exploration with technology; and

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸connect concepts to their visual representations with and without technology; and

▸identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▸complete algebraic/computational processes correctly;

▸apply technology strategically to solve problems;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸identify how mathematical characteristics of functions are related in different representations;

▸extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building notational fluency

Students can:

▸connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);

▸connect notation to different representations (graphical, numerical, analytical, and verbal); and

MPAC 6: Communicating

Students can:

▸explain the meaning of expressions, notation, and results in terms of a context

(including units);

▸explain the connections among concepts;

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others

MA.N-Q	Quantities
MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.F-BF.B	Build new functions from existing functions
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are

	closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 2

- Students will be able to evaluate definite integrals using the Fundamental Theorem of Calculus Part I. (Level of Difficulty - 3 Analysis)
- Students will be able to take derivatives of integrals using the Fundamental Theorem of Calculus Part II. (Level of Difficulty - 2 Comprehension)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▸select appropriate mathematical strategies;

▸sequence algebraic/computational procedures logically;

▸complete algebraic/computational processes correctly;

▸apply technology strategically to solve problems;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating

Students can:

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others

MA.N-Q	Quantities
MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-BF	Building Functions
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.

MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.N-VM.A	Represent and model with vector quantities.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 3

- Students will be able to calculate antiderivatives using a variety of techniques that include algebraic manipulation and substitution method. (Level of Difficulty - 3 Analysis)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

MPAC 3: Implementing algebraic/computational processes

Students can:

▸select appropriate mathematical strategies;

▸sequence algebraic/computational procedures logically;

▸complete algebraic/computational processes correctly;

▸apply technology strategically to solve problems;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸explain the connections among concepts;

▸analyze, evaluate, and compare the reasoning of others

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step

	problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.A-SSE.A	Interpret the structure of expressions
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.B	Write expressions in equivalent forms to solve problems

Learning Goal 4

Students will be able to use the definite integral of a function over an interval as a mathematical tool to solve and interpret problems involving applications of accumulation and motion.

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

Target 1

Students will be able to interpret the meaning of definite integrals within a problem and use definite integrals to solve these problems.

- solve problems involving accumulation of a rate of change (Level of Difficulty - 4 Knowledge Utilization)
- solve problems involving net change vs total change (Level of Difficulty - 3 Analysis)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸connect concepts to their visual representations with and without technology; and

▸identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸use accurate and precise language and notation;

▸explain the meaning of expressions, notation, and results in terms of a context

(including units);

▸explain the connections among concepts;

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 2

- Students will be able to use definite integrals to solve problems involving the average value of a function. (Level of Difficulty- 3 Analysis)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 3: Implementing algebraic/computational processes

Students can:

▸complete algebraic/computational processes correctly;

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 3

- Students will be able to use definite integrals to solve problems involving motion. (acceleration- veloctiy- position). (Level of Difficulty - 3 Analysis)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸connect concepts to their visual representations with and without technology; and

▸identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▸complete algebraic/computational processes correctly;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸identify how mathematical characteristics of functions are related in different representations;

▸extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating

Students can:

▸explain the meaning of expressions, notation, and results in terms of a context (including units);

▸explain the connections among concepts;

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 4

- Students will be able to use definite integrals to find the area under a curve. (Level of Difficulty - 3 Analysis)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or

processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸connect concepts to their visual representations with and without technology; and

MPAC 3: Implementing algebraic/computational processes

Students can:

▸complete algebraic/computational processes correctly;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

MPAC 4: Connecting multiple representations

Students can:

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸analyze, evaluate, and compare the reasoning of others

MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Target 5

- Students will be able to use the definite integral to solve problems in various context (problems involving area, infromation about accumulation and net change in applied context, motion problems, etc.) (Level of Difficulty - 4 Knowledge Utilization)

MPAC 1: Reasoning with definitions and theorems

Students can:

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸connect concepts to their visual representations with and without technology; and

▸identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▸complete algebraic/computational processes correctly;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸identify how mathematical characteristics of functions are related in different representations;

▸extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating

Students can:

▸explain the meaning of expressions, notation, and results in terms of a context (including units);

▸explain the connections among concepts;

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-CED	Creating Equations
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.

MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Learning Goal 5

Students wil be able to explain how antidifferentiation is an underlying concept involved in solving seperable equations and use antiderivatives in order to solve these types of differential equations.

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

Target 1

- Students will be able anaylze and solve differential equations to obtain both general and specific solutions. (Level of Difficulty - 3 Analysis)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 2: Connecting concepts

Students can:

▶use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▸select appropriate mathematical strategies;

▸sequence algebraic/computational procedures logically;

▸complete algebraic/computational processes correctly;

▸connect the results of algebraic/computational processes to the question asked.

MPAC 5: Building notational fluency

Students can:

▸assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸explain the connections among concepts;

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others

MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a

	rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-SSE	Seeing Structure in Expressions
MA.A-SSE.A	Interpret the structure of expressions
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Target 2

- Students will be able to interpret, create and solve differential equations from problems in context. (Level of Difficulty - 4 Knowledge of Utilization)

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸use definitions and theorems to build arguments, to justify conclusions or answers,

and to prove results;

▸apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸connect concepts to their visual representations with and without technology; and

▸identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▸select appropriate mathematical strategies;

▸sequence algebraic/computational procedures logically;

▸complete algebraic/computational processes correctly;

▸apply technology strategically to solve problems;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸identify how mathematical characteristics of functions are related in different representations;

▸extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸explain the meaning of expressions, notation, and results in terms of a context (including units);

▸explain the connections among concepts;

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.G-MG	Modeling with Geometry
MA.G-MG.A	Apply geometric concepts in modeling situations
MA.G-MG.A.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

MA.G-MG.A.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MA.G-MG.A.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Summative Assessment

- Quizzes
- Tests
- Unit Exams
- Packets
- Projects
- Writing Assignments
- Labs

21st Century Life and Careers

Select all applicable standards from the applicable standards

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.
CAEP.9.2.12.C.4	Analyze how economic conditions and societal changes influence employment trends and future education.

Formative Assessment and Performance Opportunities

- Interactive Learning Activities
- Academic Games
- class discussions
- class work
- homework
- warm ups
- Active Learning Activites
- Teacher Observation
- Cooperative Groups
- Student Tracking- Proficiency Scales

Accommodations and Modifications

- 504 Accomadations
- IEP Modifications
- Extension Activites
- Extra Practice Activities
- Technology
- Stations
- Collaborative Corner
- Projects
- Small Group Instruction
- Scaffolding of Questions

Unit Resources

- Textbook

- Online Textbook
- Collegeboard Website
- Practice Workbooks