# Unit 3: Behaviors of Function (Shapes of Graphs) 

Content Area: Mathematics
Course(s): Calculus
Time Period: 3rd Marking Period
Length:
6 weeks
Status:
Published

## Unit Overview

In this unit the derivative is put to work. We will use the first and second derivative to analyze functions and there behaviors.

During this unit, students will....

- use derivatives to anaylze the behavior of a graph
- apply derivatives to determine whether a graph is increasing or decreasing
- apply derivatives to determine local maximums and minimums
- apply derivatives to determine absolute maximums and mimums
- apply derivatives to determine concavity of a graph
- apply derivatives to determine inflection points on a graph
- apply derivatives to real world situations


## Transfer

Students will be able to independently use their learning to...
-What kinds of long term, independent accomplilshments are desired?

- determine behaviors of functions
- determine intervals on which functions increase and decrease
- determine values where local max and mins occurr
- determine intervals on which a function is concave up or down
- determine values where a function has points of inflection
- determine the completeness of a graph

For more information, read the following article by Grant Wiggins.
http://www.authenticeducation.org/ae bigideas/article.lasso?artid=60

## Meaning

## Understandings

Students will understand that...

- that derivatives can be used to analyze the properties of a function
- first and second derivatives of a function can provide information about the function and its graph including intervals of increase or decrease, local (relative) and global (absolute) extrema, intervals of upward and downward concavity, and points of inflection.
- key features of functions and their derivatives can be identified and related to their graphical, numerical, and analytical representations
- key features of the graphs of $f, f^{\prime}$, and $f^{\prime \prime}$ are related to one another


## Essential Questions

Students will keep considering...

- How do the graphs of the first and second derivatives relate to the original functions graph?
- What information does derivatives give us concerning the graphs and behavior of functions?


## Students will know...

Students will know...
What facts and basic concepts should students know and be able to recall?

- first derivative can be used to determine the intervals on which the original function increases or decreases
- first derivatives can be used to determine where the original function has local and absolute extremas
- second derivatives can be used to determine the intervals on which the original function is concave up or down
- second derivative can be use to determine where the original function has points of inflection
- how to determine where position, velocity and acceleration are increasing or decreasing
- how to determine where moving objects change direction
- how to detemine if speed of an object is increasing or decreasing


## Students will be skilled at...

Students will be skilled at...
What discrete skills and processes should students be able to use?

- finding critical points
- determining the behaviors of a given function (where it increases/decreases, has local/absolute max and mins, where it is concave up or down, where it has inflection points)
- discussing how the key features of the graphs of the original, the first derivative and the second derivative are related to one another
- solve problems involving position, velocity and acceleration
- determining where moving objects change direction
- determining where speed increase and decreases


## Academic Vocabulary

- Extreme Values
- Extremas
- Absolute/Global Maximum
- Absolute/Global Minimum
- Local/Relative Maximum
- Local/Relative Minimum
- Critical Points
- Cusp
- Rolle's Theorem
- Increasing/Decreasing Behavior
- The Mean Value Theorem
- First Derivative Test
- Concavity
- Inflection Points
- Second Derivative Test
- Summary Chart
- Average Rate of Change
- Instantaneous Rate of Change


## Learning Goal 1

Students will be able to use a functions first and second derivative ( which itself are functions) to determine the behavior of a function.
(ie: increase/decrease, max/min, concavity, inflection points, direction of movement etc.)

## Standards

## AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

## Collegeboard

## Target 1

- Students will be able to use first derivative to determine where a function has intervals on which it increases or decreases, and where it has relative (local) and global (absolute) extremas. (Level of Difficulty - 3 Analysis)


## Standards

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017
Collegeboard

MPAC 1: Reasoning with definitions and theorems
Students can:
-apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts
Students can:
-use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;
connect concepts to their visual representations with and without technology; and

MPAC 3: Implementing algebraic/computational processes
Students can:

- complete algebraic/computational processes correctly;
- apply technology strategically to solve problems;
attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations
Students can:
associate tables, graphs, and symbolic representations of functions;
develop concepts using graphical, symbolical, or numerical representations with and without technology;
identify how mathematical characteristics of functions are related in different representations;
extract and interpret mathematical content from any presentation of a function
(e.g., utilize information from a table of values);
construct one representational form from another (e.g., a table from a graph or a graph from given information); and
consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating
Students can:
clearly present methods, reasoning, justifications, and conclusions;
explain the connections among concepts;
critically interpret and accurately report information provided by technology; and
analyze, evaluate, and compare the reasoning of others

| MA.F-IF.B. 4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. |
| :---: | :---: |
| MA.F-IF.B. 5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. |
| MA.F-IF.B. 6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| MA.F-IF.C | Analyze functions using different representations |
| MA.F-IF.C. 7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| MA.F-IF.C. 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). |
| MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. |
| MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. |
| MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. |
| MA.F-IF.C.7e | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. |
| MA.F-IF.C.8a | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. |

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

MA.A-SSE.B. 3

MA.A-SSE.B.3a
MA.A-SSE.B.3b
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

Factor a quadratic expression to reveal the zeros of the function it defines.
Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

## Target 2

- Students will be able to use the second derivative to determine where a function has intervals on which it opens up or down, and where it has points of inflection. (Level of Difficulty - 3 Analysis)


## Standards

## AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

## Collegeboard

MPAC 1: Reasoning with definitions and theorems
Students can:
apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts
Students can:
use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;
connect concepts to their visual representations with and without technology; and

MPAC 3: Implementing algebraic/computational processes
Students can:
complete algebraic/computational processes correctly;
apply technology strategically to solve problems;
attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
-connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations
Students can:

- associate tables, graphs, and symbolic representations of functions;
-develop concepts using graphical, symbolical, or numerical representations with and without technology;
identify how mathematical characteristics of functions are related in different representations;
extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
- construct one representational form from another (e.g., a table from a graph or a graph from given information); and
- consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating
Students can:
-clearly present methods, reasoning, justifications, and conclusions;

- explain the connections among concepts;
critically interpret and accurately report information provided by technology; and
analyze, evaluate, and compare the reasoning of others

| MA.F-IF.B. 4 | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. |
| :---: | :---: |
| MA.F-IF.B. 5 | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. |
| MA.F-IF.B. 6 | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| MA.F-IF.C | Analyze functions using different representations |
| MA.F-IF.C. 7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. |
| MA.F-IF.C. 9 | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). |
| MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. |
| MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. |
| MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. |
| MA.A-APR.B. 3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. |
| MA.A-REI.B. 3 | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. |
| MA.A-REI.B. 4 | Solve quadratic equations in one variable. |
| MA.A-REI.B.4a | Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x-p)^{2}=q$ that has the same solutions. Derive the quadratic formula from this form. |
| MA.A-REI.B.4b | Solve quadratic equations by inspection (e.g., for $x^{2}=49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm b i$ for real numbers $a$ and $b$. |
| MA.A-REI.C | Solve systems of equations |
| MA.A-SSE.B. 3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. |
| MA.A-SSE.B.3a | Factor a quadratic expression to reveal the zeros of the function it defines. |
| MA.A-SSE.B.3b | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. |
| MA.A-SSE.B.3c | Use the properties of exponents to transform expressions for exponential functions. |

## Target 3

- Students will be able to determine that key features of functions and their derivatives can be identified and related to their graphical, numerical and anaytical representations. (Level of Difficulty - 3 Analysis)


## Standards

## AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

## Collegeboard

MPAC 4: Connecting multiple representations
Students can:

- associate tables, graphs, and symbolic representations of functions;
develop concepts using graphical, symbolical, or numerical representations with and without technology;
-identify how mathematical characteristics of functions are related in different representations;
extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
construct one representational form from another (e.g., a table from a graph or a graph from given information); and
consider multiple representations of a function to select or construct a useful representation for solving a problem.

| MA.F-IF.B. 4 | For a function that models a relationship between two quantities, interpret key features of <br> graphs and tables in terms of the quantities, and sketch graphs showing key features given <br> a verbal description of the relationship. |
| :--- | :--- |
| MA.F-IF.B. 5 | Relate the domain of a function to its graph and, where applicable, to the quantitative <br> relationship it describes. |
| MA.F-IF.B. 6 | Calculate and interpret the average rate of change of a function (presented symbolically or <br> as a table) over a specified interval. Estimate the rate of change from a graph. |
| MA.F-IF.C | Analyze functions using different representations <br> Graph functions expressed symbolically and show key features of the graph, by hand in <br> simple cases and using technology for more complicated cases. |
| MA.F-IF.C. 9 | Compare properties of two functions each represented in a different way (algebraically, <br> graphically, numerically in tables, or by verbal descriptions). |
| MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. |

\(\left.\left.$$
\begin{array}{ll}\text { MA.F-IF.C.7b } & \begin{array}{l}\text { Graph square root, cube root, and piecewise-defined functions, including step functions } \\
\text { and absolute value functions. }\end{array} \\
\text { MA.F-IF.C.7c } & \begin{array}{l}\text { Graph polynomial functions, identifying zeros when suitable factorizations are available, } \\
\text { and showing end behavior. }\end{array} \\
\text { MA.F-IF.C.7d } & \begin{array}{l}\text { Graph rational functions, identifying zeros and asymptotes when suitable factorizations } \\
\text { are available, and showing end behavior. }\end{array} \\
\text { Use the process of factoring and completing the square in a quadratic function to show } \\
\text { zeros, extreme values, and symmetry of the graph, and interpret these in terms of a } \\
\text { context. }\end{array}
$$\right\} \begin{array}{l}Identify zeros of polynomials when suitable factorizations are available, and use the zeros <br>

to construct a rough graph of the function defined by the polynomial.\end{array}\right\}\)| Solve linear equations and inequalities in one variable, including equations with |
| :--- |
| coefficients represented by letters. |

## Target 4

- Students will be able to solve problems involving postion, velocity and acceleration using the first and second derivatives of the function. (Level of Difficulty - 4 Knowledge Utilization)
- Students will be able to determine which direction an object is moving. (Level of Difficulty - 4 Knowledge Utilization)
- Students will be able to determine when an object changes direction and when its at rest. (Level of Difficulty - 4 Knowledge Utilization)
- Students will be able to determine whether the speed of a function is increasing or decreasing. (Level of Difficulty - 4 Knowledge Utilization)


## Standards

## AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

## Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:
-use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;

- apply definitions and theorems in the process of solving a problem;
develop conjectures based on exploration with technology; and

MPAC 2: Connecting concepts
Students can:
use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;
connect concepts to their visual representations with and without technology; and

MPAC 3: Implementing algebraic/computational processes
Students can:
select appropriate mathematical strategies;
sequence algebraic/computational procedures logically;
-complete algebraic/computational processes correctly;
apply technology strategically to solve problems;
attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations
Students can:
associate tables, graphs, and symbolic representations of functions;
develop concepts using graphical, symbolical, or numerical representations with
and without technology;
identify how mathematical characteristics of functions are related in different representations;

- extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
construct one representational form from another (e.g., a table from a graph or a graph from given information); and
- consider multiple representations of a function to select or construct a useful representation for solving a problem.


## MPAC 6: Communicating

Students can:
-clearly present methods, reasoning, justifications, and conclusions;

- explain the connections among concepts;
critically interpret and accurately report information provided by technology; and
analyze, evaluate, and compare the reasoning of others

| MA.N-Q.A | Reason quantitatively and use units to solve problems. |
| :--- | :--- |
| MA.N-Q.A. 1 | Use units as a way to understand problems and to guide the solution of multi-step <br> problems; choose and interpret units consistently in formulas; choose and interpret the <br> scale and the origin in graphs and data displays. |
| MA.N-Q.A. 2 | Define appropriate quantities for the purpose of descriptive modeling. <br> Choose a level of accuracy appropriate to limitations on measurement when reporting <br> quantities. |
| MA.N-Q.A. 3 | For a function that models a relationship between two quantities, interpret key features of <br> graphs and tables in terms of the quantities, and sketch graphs showing key features given <br> a verbal description of the relationship. |
| MA.F-IF.B.4 | Relate the domain of a function to its graph and, where applicable, to the quantitative <br> relationship it describes. |
| MA.F-IF.B. 6 | Calculate and interpret the average rate of change of a function (presented symbolically or <br> as a table) over a specified interval. Estimate the rate of change from a graph. |
| MA.F-IF.C | Analyze functions using different representations |
| MA.F-IF.C. 7 | Graph functions expressed symbolically and show key features of the graph, by hand in <br> simple cases and using technology for more complicated cases. |


| MA.F-IF.C.9 | Compare properties of two functions each represented in a different way (algebraically, <br> graphically, numerically in tables, or by verbal descriptions). |
| :--- | :--- |
| MA.F-IF.C.7a | Graph linear and quadratic functions and show intercepts, maxima, and minima. |
| MA.F-IF.C.7b | Graph square root, cube root, and piecewise-defined functions, including step functions <br> and absolute value functions. |
| MA.F-IF.C.7c | Graph polynomial functions, identifying zeros when suitable factorizations are available, <br> and showing end behavior. |
| MA.F-IF.C.7d | Graph rational functions, identifying zeros and asymptotes when suitable factorizations <br> are available, and showing end behavior. |
| Mse the process of factoring and completing the square in a quadratic function to show |  |
| zeros, extreme values, and symmetry of the graph, and interpret these in terms of a |  |
| context. |  |

## Learning Goal 2

Students will be able to use The Mean Value Theorem to connect the behavior of a differentiable function over an interval to the behavior of the derivative of that function at a particular point in the interval. Students will be able to use The Mean Value Theorem to solve real world problems.

## Standards

## AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

## Collegeboard

## Target 1

Students will be able to ...

- apply The Mean Value Theorem (Level of Difficulty - 3 Analysis)
- use the The Mean Value Theorem to solve real life applications (Level of Difficulty - 4 Knowledge Utilization)


## Standards

## AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

## Collegeboard

MPAC 1: Reasoning with definitions and theorems
Students can:
-use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;
confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem;
apply definitions and theorems in the process of solving a problem;
interpret quantifiers in definitions and theorems (e.g., "for all," "there exists");

MPAC 2: Connecting concepts
Students can:
relate the concept of a limit to all aspects of calculus;
use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

## MPAC 3: Implementing algebraic/computational processes

Students can:

- complete algebraic/computational processes correctly;
- apply technology strategically to solve problems;
attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
connect the results of algebraic/computational processes to the question asked.


## MPAC 6: Communicating

Students can:
clearly present methods, reasoning, justifications, and conclusions;
critically interpret and accurately report information provided by technology; and
analyze, evaluate, and compare the reasoning of others

MA.F-IF.B. 4

MA.F-IF.B. 5

MA.F-IF.B. 6

MA.F-IF.C
MA.F-IF.C. 7

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

| MA.A-APR.A | Perform arithmetic operations on polynomials |
| :---: | :---: |
| MA.A-APR.D. 6 | Rewrite simple rational expressions in different forms; write $a(x) / b(x)$ in the form $q(x)+r(x) / b(x)$, where $a(x), b(x), q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. |
| MA.A-APR.D. 7 | Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. |
| MA.A-CED.A. 4 | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. |
| MA.A-REI.A. 1 | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. |
| MA.A-SSE.A | Interpret the structure of expressions |
| MA.A-SSE.A. 2 | Use the structure of an expression to identify ways to rewrite it. For example, see $x^{4}-y^{4}$ as $\left(x^{2}\right)^{2}-\left(y^{2}\right)^{2}$, thus recognizing it as a difference of squares that can be factored as $\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$. |
| MA.A-SSE.A.1a | Interpret parts of an expression, such as terms, factors, and coefficients. |
| MA.A-SSE.B | Write expressions in equivalent forms to solve problems |
| MA.A-SSE.B. 3 | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. |

## Summative Assessment

- Quizzes
- Tests
- Unit Exams
- Packets
- Projects
- Writing Assignments
- Labs


## 21st Century Life and Careers

Select all applicable standards from the applicable standards

CRP.K-12.CRP1
CRP.K-12.CRP2
CRP.K-12.CRP4
CRP.K-12.CRP6
CRP.K-12.CRP8
CRP.K-12.CRP10

Act as a responsible and contributing citizen and employee.
Apply appropriate academic and technical skills.
Communicate clearly and effectively and with reason.
Demonstrate creativity and innovation.
Utilize critical thinking to make sense of problems and persevere in solving them.
Plan education and career paths aligned to personal goals.

Use technology to enhance productivity.
Review career goals and determine steps necessary for attainment.
Modify Personalized Student Learning Plans to support declared career goals.
Identify transferable career skills and design alternate career plans.

## Formative Assessment and Performance Opportunities

- Interactive Learning Activities
- Academic Games
- class discussions
- class work
- homework
- warm ups
- Active Learning Activites
- Teacher Observation
- Cooperative Groups
- Student Tracking- Proficiency Scales


## Accommodations and Modifications

- 504 Accomadations
- IEP Modifications
- Extension Activites
- Extra Practice Activities
- Technology
- Stations
- Collaborative Corner
- Projects
- Small Group Instruction
- Scaffolding of Questions


## Unit Resources

- Textbook
- Online Textbook
- Collegeboard Website
- Practice Workbooks

