Unit 2: Differentiation

Content Area: **Mathematics**
Course(s): **Calculus**
Time Period: **November**
Length: **12 weeks**
Status: **Published**

Unit Overview

Differential Calulus is the study of the derivative , and differentiation is the process of computing derivatives. There are three equally important answers to what a derivative is: A derivative is a rate of change, it is the slope of a tangent line, and (more formally) it is the limit of a difference quotient. In this unit all three of these definitions will be explored and we will develop the basic rules of differentiation.

During this unit, students will....

- build the concept of derivative using concepts of limits
- use limits to compute rates of change
- apply derivatives to finding tangent lines
- be able to use the definition of derivatives to estimate derivatives from graphs and tables
- apply various derivative rules and properties
- apply derivatives to real world situations

Transfer

Students will be able to independently use their learning to...

-What kinds of long term, independent accomplilshments are desired?

- calculate the derivatives of various problems
- use derivatives in real-world situations
- explain the concept of derivative and how it applies to everyday situations
- determine higher order derivatives
- use derivatives to anaylze properties of functions
- interpret derivatives within a problem
- solve problems involving rates of change, related rates and optimization

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For more information, read the following article by Grant Wiggins.

http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60

Meaning

Understandings

Students will understand that...

-What specifically do you want students to understand?

-What inferences should they make/grasp/realize?

- derivatives are useful in describing real-life situations
- derivatives can be found in a variety of ways
- derivatives can be represented in a variety of contexts (graphically, algebraically, through tables, etc.)
- derivatives are the essential building block of calculus
- there is connection between differentiability and continuity

Essential Questions

Students will keep considering...

- How are slopes of the tangent line and limits used to discover and describe the concept of a derivative?
- What is the relationship between continuity, differentiability and the graph of a function?
- How can a function be transformed prior to differentiation in order to apply a simpler differentiation rule?
- In what cases is it necessary to use implicit differentiation?

Application of Knowledge and Skill

Students will know...

Students will know...

- how to estimate the rate of change at a given point given the equation of a function
- how to use the difference quotient
- find derivatives using the defintion (Fermat's Method)
- the different notations for the derivative
- derivatives can be represented graphically, numerically, analytically, and verbally
- derivatives can be estimated from tables and graphs
- to calculate derivatives of various types of functions
- specific rules can be used to calculate derivatives of different types of functions
- sums, differences, products and quotients of functions can be differentiated using derivative rules
- use the chain rule
- how to determine higher order derivatives
- continuity and differentiablitly are connected
- how to interpret the meaning of a derivative is the instantaneous rate of change of the function
- how to solve problems involving tangent lines, related rates, and optimization
- how to take derivatives implicitly

Students will be skilled at...

Students will be skilled at...

- approximate rates of change at x=c using calculator
- use algebra to find derivatives using Fermat's Method
- find the derivatives of various functions using the derivative rules
- using derivatives to solve problems involiving velocity and acceleration
- using derivative rules to differentiate sum, differences, products and quotients
- using the chain rule to differentiate composite functions
- implicit differentiation
- finding the instantaneous rate of change for given functions
- interpreting derivatives given tables and graphs
- solving problems involving tangent lines, rates of change, related rates and optimization

Academic Vocabulary

- Difference Quotient
- derivative
- differentiate
- tangent line
- secant line
- power rule
- product rule
- quotient rule
- sum and difference rules
- rate of change
- slope
- instantaneous rate of change
- velocity
- acceleration
- position
- higher order derivatives
- chain rule
- inverse functions
- implicit differentation
- explicit differentation
- related rates
- how fast?
- rate
- optimization

Learning Goal 1

Students will be able to define the derivative of a function as the limit of the difference quotient and use a variety of strategies to determine derivatives of various types of functions. Students will also begin using derivatives in context.

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

By Collegeboard

Target 1

Students will be able to

- identify the derivative of a function as the limit of a difference quotient (Fermat's Method) (Level of Difficulty - 2 Comprehension)
- determine that the difference quotient expresses the average rate of change of a function over an interval (Level of Difficulty - 2 Comprehension)
- define derivative as the instantaneous rate of a function by applying Fermat's Method (Level of Difficulty - 2 Comprehension)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

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MPAC 1: Reasoning with definitions and theorems

Students can:

▶use definitions and theorems to build arguments, to justify conclusions or answers,

and to prove results;

▶apply definitions and theorems in the process of solving a problem;

MPAC 2: Connecting concepts

Students can:

▶relate the concept of a limit to all aspects of calculus;

MPAC 3: Implementing algebraic/computational processes

Students can:

▶select appropriate mathematical strategies;

▶sequence algebraic/computational procedures logically;

MPAC 6: Communicating

Students can:

▶explain the meaning of expressions, notation, and results in terms of a context

(including units);

MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.

Target 2

Students will be able to...

- identify the different notations for derivative (Level of Difficutly - 1 Retrieval)
- represent derivatives graphically, numerically, analytically, and verbally (Level of Difficulty - 3 Analysis)
- calculate derivatives graphically, numerically, analytically and verbally (Level of Difficulty - 3 Analysis)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

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MPAC 2: Connecting concepts

Students can:

▶connect concepts to their visual representations with and without technology; and

MPAC 3: Implementing algebraic/computational processes

Students can:

▶attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

MPAC 4: Connecting multiple representations

Students can:

▶associate tables, graphs, and symbolic representations of functions;

▶develop concepts using graphical, symbolical, or numerical representations with and without technology;

▶identify how mathematical characteristics of functions are related in different representations;

▶extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▶construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▶consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building notational fluency

Students can:

▶know and use a variety of notations (e.g.,);

▶connect notation to different representations (graphical, numerical, analytical, and verbal); and

▶assign meaning to notation, accurately interpreting the notation in a given problem

and across different contexts.

MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model

Target 3

Students will be able to ...

- calculate derivatives of sums, differences, products and quotients using differentiation rules (Level of Difficulty - 2 Comprehension)
- calculate derivatives using chain rule (Level of Difficulty - 2 Comprehension)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

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MPAC 1: Reasoning with definitions and theorems

Students can:

▶apply definitions and theorems in the process of solving a problem;

▶relate the concept of a limit to all aspects of calculus;

▶use the connection between concepts (e.g., rate of change and accumulation) or

processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▶connect concepts to their visual representations with and without technology; and

▶identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▶select appropriate mathematical strategies;

▶sequence algebraic/computational procedures logically;

▶complete algebraic/computational processes correctly;

▶connect the results of algebraic/computational processes to the question asked.

MPAC 6: Communicating

Students can:

▶clearly present methods, reasoning, justifications, and conclusions;

MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.N-RN.B.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.

Target 4

Students will be able to .....

- solve rate problems (Level of Difficulty - 4 Knowledge Utilization)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

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MPAC 1: Reasoning with definitions and theorems

Students can:

▶apply definitions and theorems in the process of solving a problem;

▶interpret quantifiers in definitions and theorems (e.g., "for all," "there exists");

MPAC 2: Connecting concepts

Students can:

▶use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

MPAC 6: Communicating

Students can:

▶clearly present methods, reasoning, justifications, and conclusions;

▶use accurate and precise language and notation;

▶explain the meaning of expressions, notation, and results in terms of a context (including units);

▶explain the connections among concepts;

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.N-Q.A.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
MA.N-VM.A	Represent and model with vector quantities.
MA.N-VM.A.1	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $\|\boldsymbol{v}\|$, $\|\|\boldsymbol{v}\|\|$, $\boldsymbol{v}$).
MA.N-VM.A.3	Solve problems involving velocity and other quantities that can be represented by vectors.

Target 5

Students will be able to....

- calculate derivatives of trigonometric functions (Level of Difficulty - 2 Comprehension)
- calculate derivatives of inverse trigonometric functions (Level of Difficulty - 2 Comprehension)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

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MPAC 1: Reasoning with definitions and theorems

Students can:

▶apply definitions and theorems in the process of solving a problem;

MPAC 3: Implementing algebraic/computational processes

Students can:

▶select appropriate mathematical strategies;

MPAC 4: Connecting multiple representations

Students can:

▶associate tables, graphs, and symbolic representations of functions;

▶develop concepts using graphical, symbolical, or numerical representations with and without technology;

▶construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▶consider multiple representations of a function to select or construct a useful representation for solving a problem.

MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x)$ + $r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.G-SRT.C	Define trigonometric ratios and solve problems involving right triangles

Target 6

Students will be able to ....

- find derivatives of implicit functions (Level of Difficulty - 3 Analysis)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

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MPAC 1: Reasoning with definitions and theorems

Students can:

▶apply definitions and theorems in the process of solving a problem;

MPAC 3: Implementing algebraic/computational processes

Students can:

▶select appropriate mathematical strategies;

MPAC 5: Building notational fluency

Students can:

▶know and use a variety of notations (e.g.,);

▶assign meaning to notation, accurately interpreting the notation in a given problem

and across different contexts.

Learning Goal 2

Students will be able to discuss the multiple interpretations of the derivative as well as solve appliction problems of derivatives including those that involve instantaneous rates of change, optimization, and related rates.

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

Target 1

Students will be able to....

- Solve related rate problems (Level of Difficulty - 4 Knowledge Utilization)
- Solve optimization problems (Level of Difficulty - 4 Knowledge Utilization)

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Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

By Collegeboard

MPAC 2: Connecting concepts

Students can:

▶use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▶identify a common underlying structure in problems involving different contextual situations.

MPAC 3: Implementing algebraic/computational processes

Students can:

▶select appropriate mathematical strategies;

▶apply technology strategically to solve problems;

▶connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▶construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▶consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating

Students can:

▶clearly present methods, reasoning, justifications, and conclusions;

▶explain the meaning of expressions, notation, and results in terms of a context

(including units);

▶explain the connections among concepts;

▶analyze, evaluate, and compare the reasoning of others.

MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.A-CED.A	Create equations that describe numbers or relationships
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.A-REI.B	Solve equations and inequalities in one variable
MA.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MA.A-REI.B.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
MA.G-GMD.A.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

Summative Assessment

- Quizzes
- Tests
- Unit Exams
- Packets
- Projects
- Writing Assignments
- Labs

21st Century Life and Careers

Select all applicable standards from the applicable standards

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.

Formative Assessment and Performance Opportunities

- Interactive Learning Activities
- Academic Games
- class discussions
- class work
- homework
- warm ups
- Active Learning Activites
- Teacher Observation
- Cooperative Groups
- Student Tracking- Proficiency Scales

Accommodations and Modifications

- 504 Accomadations
- IEP Modifications
- Extension Activites
- Extra Practice Activities
- Technology
- Stations
- Collaborative Corner
- Projects
- Small Group Instruction
- Scaffolding of Questions

Unit Resources

- Textbook
- Online Textbook
- Collegeboard Website
- Practice Workbooks