Unit 1: Limits and Continuity

Content Area: **Mathematics**
Course(s): **Calculus**
Time Period: **1st Marking Period**
Length: **6 weeks**
Status: **Published**

Unit Overview

The idea of limits is essential in calculus because it helps us develop definitions, formulas and theorems. This unit will set us up for the study of derivatives.

During this unit, students will....

- compute various limits (regular limits, one-sided limits, limits as you approach infinity....)
- be able to determine limits graphically, algebraically, numerically, and through tables
- be able to apply limits to explain the behavior of functions near a given point.
- be able to use limits to understand the concept of continuity.
- use limits to talk about the asymptotic and unbounded behaviros of a function
- use limits to talk about the types of discontinuity in given functions

Transfer

Students will be able to independently use their learning to...

- determine limits algebraically, graphically, anayltically, through tables, and technology (graphing calculator)
- determine which method is most appropriate for finding a given limit
- to read tables, graphs and symbolic representations of a function
- discuss how limits are used to define continuity and how continuity is a key property of a function
- use concepts of limits to understand the behavior of a function (such as asypmptotic and unbounded behavior, and types of discontinuity)
- use concepts of limits to model real world situations

For more information, read the following article by Grant Wiggins.

http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60

Meaning

Understandings

Students will understand that..

- limits are useful in describing real life situations
- limits can be solved in using a various methods
- the concept of limit can be extended to include one-sided limits, limits to infinity, and infinite limits
- limits may not exist for some functions at a given value of x
- numerical and graphical information can be used to estimate limits
- limits of various functions (limits of sums, differences, products, quotients, etc.) can be found using basic theorems of limits and algebraic rules
- limit of a function can be found using algebraic manipulation, alternate forms of trig functions, and squeeze (sandwich theorem)
- asymptotic and unbounded behavior of functions can be explained and described using limits
- limits are used to define continuity
- there are different types of discontinuity (jump, removable, infinite discontinutiy)
- continuity is an essential conditon for theorems such as Intermediate Value Theorem, Extreme Value Theorem, and Mean Value Theorem

Essential Questions

Students will keep considering...

- How do limits serve as a means to better understand functions and their behavior?
- Why are limits essential to the concept of continuity?
- Why are limits essential to the underlying meaning of calculus?
- How can the concept of a limit be used as a analogy to real-world situations?

Application of Knowledge and Skill

Students will know...

Students will know...

What facts and basic concepts should students know and be able to recall?

- how to determine limits in a variety of ways (anayltically, algebraically, graphically, through tables, etc)
- how to use limits to determine behaviors of a function (determine if function is continuous, if so the type of discontinuity, determine if a function has asymptotoes, etc.)
- that limits are needed for developing various theorems (Intermediate Value Theorem, Extreme Value Theorem, and Mean Value Theorem)

Students will be skilled at...

Students will be skilled at...

What discrete skills and processes should students be able to use?

- students need to be skilled at algebraically manipulating functions
- students need to be skilled at reading functions graphically and through tables
- students need to be skilled at use graphing calculators to determine limts
- students need to be skilled at using alternate forms of trigonometric functions

Academic Vocabulary

- limit
- rates of change
- tangent lines
- interpret
- zeros
- secant lines

- one-sided limits
- right handed limits
- left handed limits
- infinite limits
- continuity
- discontinuity
- removable discontinuity
- infinite discontinuity
- jump discontinuity
- end behavior model
- piece wise functions
- composite functions
- indeterminite
- conjugate
- asymptotic behavior
- horizontal asymptote
- vertical asymptote
- oblique asymptote

Learning Goal 1

Students will be able to explain how and use limits to understand functions and their behaviors and explain why limits are central to the underlying meaning of calculus.

Standards

AP Curriculum Framework – AP Calculus AB and AP Calculus BC 2016-2017

Collegeboard

Target 1

- Students will be able to express limits symbolically using correct notation. (Level of Difficulty - 1 Retrival)

- Students will be able to interpret limits expressed symbolically. (Level of Difficulty - 1 Retrival)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

By Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸use definitions and theorems to build arguments, to justify conclusions or answers,

and to prove results;

MPAC 5: Building notational fluency

Students can:

▸know and use a variety of notations (e.g.,);

▸connect notation to definitions (e.g., relating the notation for the definite integral to

that of the limit of a Riemann sum);

▸connect notation to different representations (graphical, numerical, analytical, and

verbal); and

▸assign meaning to notation, accurately interpreting the notation in a given problem

and across different contexts.

Target 2

- Determine the existence of a limit. (Level of Difficulty - 2 Comprehension)
- Calculate both one and two-sided limits using several methods including but not limited to algebraic manipulation, alternate forms of trigonometric functions, squeeze theorem, numerically, and graphically. (Level of Difficulty - 2 Comprehension)

- Use properties of limits to aide in problem solving. (Level of Difficulty - 3 Analysis)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

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MPAC 1: Reasoning with definitions and theorems

Students can:

▸use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;

▸apply definitions and theorems in the process of solving a problem;

▸develop conjectures based on exploration with technology; and

MPAC 2: Connecting concepts

Students can:

▸connect concepts to their visual representations with and without technology; and

MPAC 3: Implementing algebraic/computational processes

Students can:

▸select appropriate mathematical strategies;

▸sequence algebraic/computational procedures logically;

▸complete algebraic/computational processes correctly;

▸apply technology strategically to solve problems;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸identify how mathematical characteristics of functions are related in different representations;

▸extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸use accurate and precise language and notation;

▸critically interpret and accurately report information provided by technology; and

▸analyze, evaluate, and compare the reasoning of others.

MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.

Target 3

Students will be able to:

- Deduce and interpret behavior of functions using limits (ie. determine asymptotic and unbounded behavior of functions, comparing relative magnitude and rates of change of functions.) (Level of Difficulty - 3 Analysis)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

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MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or

processes (e.g., differentiation and its inverse process, antidifferentiation) to solve

problems;

▸connect concepts to their visual representations with and without technology; and

MPAC 3: Implementing algebraic/computational processes

Students can:

▸select appropriate mathematical strategies;

▸sequence algebraic/computational procedures logically;

▸complete algebraic/computational processes correctly;

▸apply technology strategically to solve problems;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸identify how mathematical characteristics of functions are related in different representations;

▸extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

Target 4

Students will be able to

- Analyze functions for intervals of continuity or points of discontinuity (Level of Difficulty - 3

Analysis)
- Identify types of discontinuity (removable, jump, infinite, etc.) (Level of Difficulty - 2 Comprehension)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

By Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;

▸apply definitions and theorems in the process of solving a problem;

▸develop conjectures based on exploration with technology; and

▸produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

MPAC 2: Connecting concepts

Students can:

▸use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, antidifferentiation) to solve problems;

▸connect concepts to their visual representations with and without technology; and

MPAC 3: Implementing algebraic/computational processes

Students can:

▸select appropriate mathematical strategies;

▸sequence algebraic/computational procedures logically;

▸complete algebraic/computational processes correctly;

▸apply technology strategically to solve problems;

▸attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and

▸connect the results of algebraic/computational processes to the question asked.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

▸develop concepts using graphical, symbolical, or numerical representations with and without technology;

▸identify how mathematical characteristics of functions are related in different representations;

▸extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);

▸construct one representational form from another (e.g., a table from a graph or a graph from given information); and

▸consider multiple representations of a function to select or construct a useful representation for solving a problem.

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸use accurate and precise language and notation;

▸explain the meaning of expressions, notation, and results in terms of a context (including units);

▸explain the connections among concepts;

MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.

Target 5

Students will be able to

- determine the applicability of important calculus theorems (Intermediate Value Theorem, Extreme Value Theorem, Mean Value Theorem) (Level of Difficulty - 4 Knowledge Utilization)
- describe how continuity is an essential condition for the above theorems (Level of Difficulty 2 - Comprehension)

Standards:

AP Curriculum Framework - AP Calculus AB and AP Calculus BC 2016-2017

By Collegeboard

MPAC 1: Reasoning with definitions and theorems

Students can:

▸use definitions and theorems to build arguments, to justify conclusions or answers,

and to prove results;

▸confirm that hypotheses have been satisfied in order to apply the conclusion of a

theorem;

▸apply definitions and theorems in the process of solving a problem;

▸produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

MPAC 4: Connecting multiple representations

Students can:

▸associate tables, graphs, and symbolic representations of functions;

MPAC 6: Communicating

Students can:

▸clearly present methods, reasoning, justifications, and conclusions;

▸explain the connections among concepts;

Summative Assessment

- Quizzes
- Tests
- Packets
- Unit Exams
- Projects
- Writing Assignments
- Labs

21st Century Life and Careers

Select all applicable standards from the applicable standards

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.

Formative Assessment and Performance Opportunities

- Cooperatvie Groups
- Class Discussions
- Warm Ups
- Exit Tickets
- Active Learning Activities
- Teacher Observation
- Student Tracking- Proficiency Scales
- Academic Games
- Interactive Activities
- Homework
- Classwork

Accommodations and Modifications

- 504 Accomadations
- IEP Modifications
- Extension Activities
- Extra Practice Activites
- stations
- Collaborative Corner
- projects
- small group instruction
- technology
- scaffolding of quesitons

Unit Resources

- Textbook
- Online Textbook
- Collegeboard Website
- Practice Workbooks