Unit 5: Probability, Statistics, Sequences & Series

Content Area: **Mathematics**
Course(s): **Algebra II Honors**
Time Period: **May**
Length: **7 Weeks**
Status: **Published**

Unit Overview

During this unit, students will…

- Understand and evaluate random processes underlying statistical experiments.
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
- Understand independence and conditional probability and use them to interpret data.
- Use the rules of probability to compute probabilities of compound events in a uniform probability model.
- Determine patterns of numbers (sequences) and their sums (series).
- Write and evaluate sequences and series.

CHAPTER 7, 8, 9

By the end of the year, administer the Link IT CC Algebra 2 Form C TEI

Transfer

Students will be able to independently use their learning to...

- Look at a report and determine the validity of conclusions reached from data.
- Reach conclusions based on given data.
- Arrange and read data from statistical charts.
- Determine whether events are independent or dependent.

For more information, read the following article by Grant Wiggins.

http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60

Meaning

Understandings

Students will understand that...

- Probability and statistics can be used to make predictions in real-life situations.
- Statistical data can be collected using a variety of methods.
- The way data is collected may yield misleading results.
- Specific formulas can be helpful in calculating various probabilities.
- Formulas can be used to predict numbers in a sequence or series.

Essential Questions

Students will keep considering...

- Can data sets be added together to obtain a larger sample size and hence more meaningful conclusion?
- How can we establish and quantify a cause and effect relationship between two variables?
- How can mathematical models be used as tools to describe and help explain real-life situations?
- Can bad data be corrected with good statistical analysis?
- How to write and evaluate sequences and series?

Application of Knowledge and Skill

Students will know...

Students will know...

- How sample size reflects the reliability of the data.
- When calculators, spreadsheets, or charts are best used to organize data.
- Mathematical models can be used to describe physical relationships.
- Strategies for predicting a number in a sequence or series.

Students will be skilled at...

Students will be skilled at...

- Choosing and using appropriate formulas to calculate probabilities.
- Making reasonable predictions based on data and sample sets.
- Determining how two variables can affect each other
- Determining whether sequences are arithmetic or geometric.

- Writing and evaluating sequences and series.

Academic Vocabulary

Biased sample

Binomial experiment

Census

Combination

Conditional probability

Control group

Controlled experiment

Convenience sample

Converge

Dependent events

Diverge

Experiment

Experimental probability

Explicit Formula

Factorial

Finite Sequence

Independent events

Infinite Sequence

Iteration

Limit

Margin of error

Null hypothesis

Observational study

Outcome

Parameter

Permutation

Population

Random sample

Recursive Formula

Sample

Sequence

Series

Statistic

Term of a sequence

Theoretical probability

Treatment group

Learning Goal 5.1

SWBAT determine the relationship between experimental and theoretical probability and use probabilities to analyze outcomes.

Target 5.1.1 (+)

SWBAT:

- Solve problems involving the Fundamental Counting Principle. **(DOK 2)**
- Solve problems involving permutations and combinations. **(DOK 2)**

MA.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.
	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to

bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Target 5.1.2 (+)

SWBAT:

- Find the theoretical probability of an event. **(DOK 3)**
- Find the experimental probability of an event. **(DOK 3)**

MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.
MA.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

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Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Target 5.1.3

SWBAT...

- Determine whether events are independent or dependent. **(DOK 3)**
- Find the probability of independent or dependent events. **(DOK 3)**

MA.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
MA.S-CP.A.3	Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as

	the probability of B.
MA.S-CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
MA.S-CP.B.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
MA.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, $P(A\ and\ B)$ = $[P(A)]$ × $[P(B\mid A)]$ = $[P(B)]$ × $[P(A\mid B)]$, and interpret the answer in terms of the model.
MA.S-IC.A.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.
MA.S-ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

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Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Target 5.1.4

SWBAT Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. **(DOK 3)**

MA.S-CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
MA.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.S-ID.B.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

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Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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Target 5.1.5 (+)

SWBAT...

- Find the probability of mutually exclusive events. **(DOK 4)**
- Find the probabilty of inclusive events. **(DOK 4)**

MA.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
MA.S-CP.B.7	Apply the Addition Rule, $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$, and interpret the answer in terms of the model.
MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.
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Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

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Learning Goal 5.2

SWBAT analyze data and use binomial and normal distributions to analyze outcomes.

LGBTQ Inclusive: "As teachers teach about data collection and relevance, they should include whether it is beneficial to include gender or biological sex, being sure to reinforce the difference between those two terms. When students are creating their own surveys, if they want to include data for biological sex, teachers need to be sure they include both *intersex* and *other* as choices, and if the students want to include data for gender, a variety of choices need to be included, such as *agender, genderfluid, female, male, nonbinary, transman, transwoman*, and *other*. These additional categories will create more work for the analyses of the data sets, give more representative results, and deepen our students’ understanding of both statistical analyses and the diversity of the human population." Resource: https://www.glsen.org/blog/how-do-we-make-math-class-more-inclusive-trans-and-non-binary-identities

Target 5.2.1

SWBAT...

- Find measures of central tendency and measures of variation for statitstical data. **(DOK 3)**
- Examine the effects of outliers on statistical data. **(DOK 3)**

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.S-ID.A.1	Represent data with plots on the real number line (dot plots, histograms, and box plots).
MA.S-ID.A.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
MA.S-ID.A.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
MA.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
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Target 5.2.2

SWBAT...

- Examine how random samples can be used to make inferences about a population. **(DOK 3)**
- Use probability to analyze decisions and strategies. **(DOK 3)**

MA.N-Q.A	Reason quantitatively and use units to solve problems.
MA.N-Q.A.2	Define appropriate quantities for the purpose of descriptive modeling.
MA.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MA.S-MD.B.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
MA.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

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Target 5.2.3

SWBAT focus on the commonalities and differences between surveys, experiments, and observational studies. **(DOK 4)**

MA.S-IC.B.3

Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

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Target 5.2.4

SWBAT use simulations and hypothesis testing to compare treatments from a randomized experiment. **(DOK 4)**

MA.S-IC.B.5

Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

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Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

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Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Target 5.2.5

SWBAT...

- Estimate population means and proportions and develop margin of error from simulations involving random sampling. **(DOK 4)**
- Analyze surveys, experiments, and observational studies to judge the validity of the conclusion. **(DOK 4)**

MA.S-IC.B.4

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

MA.S-IC.B.6

Evaluate reports based on data.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Target 5.2.6

SWBAT...

- Use the Binomial Theorem to expand a binomial raised to a power. **(DOK 3)**
- Find binomial probabilities and test hypotheses. **(DOK 3)**

MA.S-MD.A.4

Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

MA.A-APR.D.6

Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

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Target 5.2.7

SWBAT...

- Use tables to estimate areas under normal curves. **(DOK 4)**
- Recognize data sets that are not normal. **(DOK 4)**

MA.K-12.4	Model with mathematics.
MA.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a

procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

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Target 5.2.8

SWBAT explain that probability can be used to help determine if good decisions are made. Use probabilites to analyze decisions and strategies. **(DOK 3)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
MA.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Learning Goal 5.3

SWBAT use arithmetic and geometric sequences and series to analyze and predict outcomes.

Target 5.3.1

SWBAT:

- Find the nth term of a sequence. **(DOK 2)**
- Write rules for sequences. **(DOK 2)**

MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.7	Look for and make use of structure.

Target 5.3.2

SWBAT evaluate the sum of a series expressed in sigma notation. **(DOK 3)**

MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.

Target 5.3.3

SWBAT:

- Find the indicated terms of an arithmetic sequence. **(DOK 2)**
- Find the sums of arithmetic series. **(DOK 2)**

MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

Target 5.3.4

SWBAT:

- Find terms of a geometric sequence, including geometric means. **(DOK 2)**
- Find the sums of geometric series. **(DOK 2)**

MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.A-SSE.B.4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Target 5.3.5

SWBAT:

- Find sums of infinite geometric series. **(DOK 2)**
- Use mathematical induction to prove statements. **(DOK 3)**

MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.7	Look for and make use of structure.

Target 5.3.6 (+) Extension

SWBAT: approximate area under a curve by using rectangles. **(DOK 4)**

MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.

Formative Assessment and Performance Opportunities

- academic games
- Albert
- Class discussions
- Classwork
- Do nows
- Exit tickets
- Google Forms
- Homework
- Kahoot!
- Khan Academy
- Problem based learning
- Quizizz
- student interviews
- Teacher observation
- whiteboard/communicator opportunities

Summative Assessment

- Link-It Exams
- Projects
- Quizzes
- student interviews
- Tests
- Unit Exam

21st Century Life and Careers

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.

Technology

TECH.8.1.12.A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
TECH.8.1.12.B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge

	and develop innovative products and process using technology.
TECH.8.1.12.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

Accommodations and Modifications

- 504 Accommodations
- centers/stations
- challenge questions
- Common Core Workbook Reference chapter 7 probability (permutations & combinations, theoretical and experimental probability, independent and dependent events, two-way tables, compund events)
- Common Core Workbook Reference Chapter 8 Data Analysis and Statistics (Measures of central tendency, data gathering, surveys/experiments/observational studies, significance of experimental results, sampling distributions, binomial distributions, normal distribution, analyzing decisions))
- Common Core Workbook Reference Chapter 9 Sequences and Series (Introduction to sequences, series and summation notation, arithmetic sequences and series, geometric sequences and series, mathematical induction and infinite geometric series)
- Graphing Calculator Simulation Activity
- Honors Extra Credit Unit 5 Activity
- IEP Modifications
- manipulatives
- projects
- scaffolding questions
- small group instruction
- Unit 5 Review for exam
- use of technology

Unit Resources

- Albert
- Algebra 2 Teachers Collaboration Google Classroom - Digital Resources & Videos
- Desmos
- Explorations in Core Math for Common Core: Algebra 2 (Holt McDougal)
- Geometer sketchpad
- Google Classroom
- Kahoot
- Khan Academy
- Kuta software
- Loom

- NCTM website
- online textbook materials
- PARCC/NJSLA Released Questions
- Quizizz
- SJMAP Resources
- Text
- YouTube