Unit 2: Polynomials & Rational Equations

Content Area: **Mathematics**
Course(s): **Algebra II Honors**
Time Period: **December**
Length: **8 Weeks**
Status: **Published**

Unit Overview

During this unit, students will…

- Interpret the structure of expressions.
- Understand the relationship between zeros and factors of polynomials.
- Use polynomial identities to solve problems.
- Rewrite rational expressions.
- Analyze functions using different representations.
- Solve problems with variation functions.
- Simplifying rational expressions.
- Graphing rational funcions.
- Solving rational equations and inequalities.

CHAPTER 3, SUPPLEMENTAL (old unit 1a/1b), CHAPTER 6, & SOME OF CHAPTER 5

By the end of January, administer the Algebra II Link It! NJSLS Form B.

Transfer

Students will be able to independently use their learning to...

- Estimate the number of zeros a polynomial will have based on its graph.
- Identify the number of solutions of a polynomial based on its degree.
- Rewrite polynomial expressions in various forms.
- Apply arithmetic and geometric models to real-life situations.
- Write algebraic models and functions to describe real-world situations.
- Describe the relationship between rational expressions.
- Write rational functions & solve rational equations.
- Recognize real-life situations that can be modeled by rational functions and write functions to model such situations.

For more information, read the following article by Grant Wiggins.

http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60

Meaning

Understandings

Students will understand that...

- The zeros of a polynomial and the graphs of a polynomial are related.
- Algebraic expressions can be simplified and rewritten in different equivalent forms.
- Geometric and arithmetic sequences can be used to model real-life data.
- Algebraic models are useful in describing real-life situations.
- Rational expressions can be simplified using factoring.
- Extraneous solutions may arise during the process of solving rational equations.
- Rational expressions are often useful in modeling real-life situations.

Essential Questions

Students will keep considering...

- How can mathematical models be used as tools to describe and help explain real-life situations?
- How can various mathematical expressions be simplified both effectively and efficiently?
- How do you apply known formulas to problems?
- How is graphing useful in both factoring expressions and solving equations?
- How can algebraic concepts and their properties be described by careful use of mathematical language?
- How do mathematical ideas interconnect and build on one another to produce a coherent whole?
- How are rational expressions translated from one form to another? Why is it often useful to rewrite them?
- How can a given equation be solved both effectively and efficiently?

Application of Knowledge and Skill

Students will know...

Students will know...

- Different representations of functions and how they relate to each other.

- The rules and procedures used to manipulate, divide, and factor polynomial expressions.
- The Fundamental Theorem of Algebra can be used to verify the number of complex/real solutions of a polynomial.
- The Remainder Theorem can be used to verify factors of polynomials.
- The rules of adding, subtracting, multiplying & dividing rational expressions.
- How to solve rational equations.
- How to graph rational equations.

Students will be skilled at...

Students will be skilled at...

- Rewriting polynomials including simple, rational, and radical expressions.
- Manipulation of polynomials using arithmetic.
- Identifying the number of solutions of a polynomial equation.
- Factoring and performing operations on polynomial expressions.
- Finding terms of geometric and arithmetic sequences.
- Writing and using algebraic expressions and functions to model real-world situations.
- Analyzing the relationships among functions represented as tables of values, algebraic formulas, written statements, and graphs.
- Simplifying rational expressions.
- Multiplying and dividing rational expressions.
- Adding and subtracting rational expressions.
- Graphing rational functions.
- Solving rational equations and inequalities.

Academic Vocabulary

Asymptotes

Binomial

Combined Variation

Common Denominator

Complex Fraction

Correlation

Cubic Function

Data display

Defined

Degree of a polynomial

Domain

Even/Odd function

End behavior

Extraneous Solution

Factoring

Formula

Function composition

Graph

Greatest Common Factor

Hole (in a graph)

Interpret

Inverse functions

Leading coefficient

Least Common Denominator

Linear Function

Local Maximum

Local Minimum

Level of Accuracy

Limitation

Measurement

Modeling

Monomial

Multiplicity

One-to-one function

Origin

Parent function

Polynomial

Polynomial Function

Proportion

Quadratic Function

Quantity

Range

Rational Equation

Rational Exponent

Rational Expression

Rational Function

Rational Inequality

Reflection

Simplest Form

Simplify

Solution

Stretch

Synthetic Division

Terms

Transformation

Trinomial

Turning Point

Undefined

Unit

Zero of a Function

Learning Goal 2.1

SWBAT apply operations with polynomials and apply polynomial functions in context.

Note: Students should have completed polynomial operations in Algebra 1. However, division of polynomials still needs to be covered.

Target 2.1.1

SWBAT use long division and synthetic division to divide polynomials. **(DOK 3)**

	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
	Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
MA.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Target 2.1.2

SWBAT:

- Use the Factor Theorem to determine factors of a polynomial. **(DOK 3)** *(how to factor when a is greater than 1-factoring should've been covered in Algebra 1 so should only be a quick review)*
- Factor the sum and difference of two cubes. **(DOK 3)**

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MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
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MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.A-APR.C.4	Prove polynomial identities and use them to describe numerical relationships.

Target 2.1.3

SWBAT...

- Identify the multiplicity of roots. **(DOK 4)**
- Use the Rational Root Theorem and the Irrational Root Theorem to solve polynomial equations. **(DOK 4)**

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Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.

MA.A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Target 2.1.4

SWBAT:

- Use the Fundamental Theorem of Algebra and its corollary to write a polynomial equation of least degree with given roots. **(DOK 3)**
- Identify all of the roots of a polynomial equation. **(DOK 3)**

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MA.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
MA.N-CN.C.8	Extend polynomial identities to the complex numbers.
MA.N-CN.C.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Target 2.1.5

SWBAT:

- Use properties of end behavior to analyze, describe, and graph polynomial functions. **(DOK 3)**
- Identify and use maxima and minima of polynomial functions to solve problems. **(DOK 3)**

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MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Target 2.1.6

SWBAT...

- Transform polynomial functions. **(DOK 2)**

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	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
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MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Target 2.1.7

SWBAT...

- Use finite differences to determine the degree of a polynomial that will fit a given set of data. **(DOK 4)**
- Use technology to find polynomial models for a given set of data. **(DOK 4)**

Possibly only Honors

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and

	conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Learning Goal 2.2

SWBAT apply algebraic reasoning to solve problems with rational expressions.

Target 2.2.1

SWBAT solve problems involving direct, inverse, joint, and combined variation. **(DOK 4)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Target 2.2.2

SWBAT:

- Simplify rational expressions. **(DOK 2)**
- Multiply and divide rational expressions. **(DOK 2)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.A-APR.D	Rewrite rational expressions

Target 2.2.3

SWBAT:

- Add and subtract rational expressions **(DOK 2)**
- Simplify Complex Fractions **(DOK 3)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Target 2.2.4

SWBAT understand under which operations rational expressions are closed. **(DOK 4)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Target 2.2.5

SWBAT:

- Graph rational functions. **(DOK 3)**
- Transform rational functions by changing parameters. **(DOK 3)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Target 2.2.6

SWBAT solve rational equations and inequalities. **(DOK 3)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.5	Use appropriate tools strategically.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Formative Assessment and Performance Opportunities

- academic games
- Albert
- Class discussions
- Classwork
- Do nows
- Exit tickets
- Google Forms
- Homework
- Kahoot!
- Khan Academy
- Problem based learning
- Quizizz
- student interviews
- Teacher observation
- whiteboard/communicator opportunities

Summative Assessment

- Link-it Exams
- Projects
- Quizzes
- student interviews
- Tests
- Unit Exams

21st Century Life and Careers

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.

CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.

Technology

TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
TECH.8.1.12.B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
TECH.8.1.12.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

Accommodations and Modifications

- 504 Accommodations
- centers/stations
- challenge questions
- Common Core Workbook Reference 3-1 (Polynomials)
- Common Core Workbook Reference 3-31 (Dividing Polynomials)
- Common Core Workbook Reference 3-4 (Factoring Polynomials)
- Common Core Workbook Reference 3-5 (Finding Real Roots of Polynomial Equations)
- Common Core Workbook Reference 3-6 (Fundamental Theorem of Algebra)
- Common Core Workbook Reference 3-7 (Investigating Graphs of Polynomial Functions)
- Common Core Workbook Reference 3-8 (Transforming Polynomial Functions)
- Common Core Workbook Reference 5-2 (Multiplying and Dividing Rational Expressions)
- Common Core Workbook Reference 5-3 (Adding and Subtracting Rational Expressions)
- Common Core Workbook Reference 5-4 (Rational Functions)
- Common Core Workbook Reference 5-5 (Solving Rational Equations and Inequalities))
- Graphic organizer for graphing rational functions
- IEP Modifications
- manipulatives - highlighting, underlying and starring critical information
- Relating operations on fractions to rational functions
- scaffolding questions
- small group instruction
- Unit 2 Exam Review
- use of technology

- Using prior knowledge of elementary division to relate to long division of polynomials

Unit Resources

- Albert
- Algebra 2 Teachers Collaboration Google Classroom - Digital Resources & Videos
- Desmos
- Explorations in Core Math for Common Core: Algebra 2 (Holt McDougal)
- Geometer sketchpad
- Google Classroom
- Kahoot
- Khan Academy
- Kuta software
- Loom
- NCTM website
- online textbook materials
- PARCC/NJSLA Released Questions
- Quizizz
- SJMAP Resources
- Text
- You Tube

Interdisciplinary Connections

Real World Applications involving multiplying and dividing polynomials to find a missing length or area helps students analyze engineering decisions. (MA.9-12.A.APR.D.6)

9-12.HS-ETS1-1.1.1	Analyze complex real-world problems by specifying criteria and constraints for successful solutions.