Geometry Unit 4: Trigonometric Ratios and Geometric Equations (Gr. 9 - 11)

Content Area: **Mathematics**
Course(s): **Geometry**
Time Period: **3rd Marking Period**
Length: **10 Weeks**
Status: **Published**

Unit Overview

This unit opens with the study of coordinate geometry as students are introduced to the use of coordinates to prove simple geometric theorems. Following this, students will examine the relationships within special right triangles and use these to solve for unknowns. They will then move into the study of right triangle trigonometric functions and their application in solving real-world problems. During the final portion of the unit, students will learn about circles—their equations, areas, related vocabulary, and properties.

Transfer

Students will be able to independently use their learning to...

- Use coordinates to prove geometric theorems.
- Solve for unknown measurements in right triangles.
- Solve applied problems involving right triangles, sketching diagrams as needed.
- Use the equation of a circle to identify its center and radius, or derive the equation when these values are given.
- Use the relationships between the angles, arcs, and segments of circles to solve applied problems.

Meaning

Understandings

Students will understand that...

- When working in the coordinate plane, the slopes of two lines can be used to determine whether the lines are parallel, perpendicular, or neither.
- The patterns that exist in Special Right Triangles allow us to solve quickly for unknown measurements in these triangles.
- Right triangle trigonometry is useful in solving a variety of problems.
- The equation of a circle is derived from the Pythagorean Theorem.
- Relationships exist between the angle and arc measures of circles.
- Angles can be measured in degrees or radians.
- Sectors and arcs are portions of circles.

Essential Questions

Students will keep considering...

- How can geometric concepts and figures be described by careful use of geometric language?
- How can various geometric properties be verified by using the coordinate plane?
- How can geometric concepts and figures be used to model real-world phenomena?
- What relationships exist between the side lengths and angle measures of right triangles?
- How can right triangle relationships be used to calculate inaccessible measurements?
- How are the relationships between the angles, arcs, and segments of circles applied in solving problems?
- How should I decide what method to use when solving problems involving circles?

Application of Knowledge and Skill

Students will know...

- The distance and midpoint formulas.
- The relationships between the slopes of parallel and perpendicular lines in the coordinate plane.
- The unique relationships that exist within 45-45-90 and 30-60-90 triangles.
- The definitions of the sine, cosine, and tangent ratios.
- That the sine and cosine of complementary angles are equal to one another.
- That right triangles are useful in solving a variety of real-world problems.
- That equations can be written to describe circles in the coordinate plane.
- The definitions of the terminology related to circles.
- That the relationships that exist between the angles, segments, and arcs associated with circles can be used to solve for unknown measures.

Students will be skilled at...

- Using coordinates to demonstrate properties of figures in the coordinate plane.
- Using varied given information to write the equation of a line.
- Simplifying radical expressions.
- Using Special Right Triangle relationships to solve for unknown measurements.
- Using trigonometric ratios to solve for unknown side lengths and angle measures.
- Choosing the most efficient method to solve various right triangles.
- Creating sketches appropriate to solving applied problems involving right triangles.
- Writing and graphing equations of circles in the coordinate plane.

- Using the relationships between angles, arcs and segments of circles to solve for unknown measures.
- Calculating arc lengths and areas of sectors of circles.

Academic Vocabulary

- 30-60-90 triangle
- 45-45-90 triangle
- angle of depression
- angle of elevation
- arc
- arc length
- central angle
- chord
- common tangent
- concentric circles
- congruent arcs
- congruent circles
- cosine
- distance formula
- equation of a circle
- exterior of a circle
- inscribed angle
- intercepted arc
- interior of a circle
- linear equation
- major arc
- midpoint formula
- minor arc
- point of tangency
- point-slope form
- secant
- sector of a circle
- segment of a circle
- semicircle
- sine
- slope

- slope-intercept form
- special right triangles
- tangent
- tangent circles
- tangent of a circle
- tangent segment
- trigonometric ratios

Learning Goal 4.1

Students will use coordinates to prove simple geometric theorems algebraically.

Daily Target 4.1.1 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT apply the distance and midpoint formulas to solve related problems.

MA.G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Daily Target 4.1.2 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

Slope Criterion for Perpendicular Lines: https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/5/tasks/1876

MA.G-GPE.B.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Daily Target 4.1.3 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT prove or disprove that a figure defined by four given points in the coordinate plane is a parallelogram, rectangle, rhombus, square, trapezoid, or kite.

A Midpoint Miracle: https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/4/tasks/605

MA.G-GPE.B.4	Use coordinates to prove simple geometric theorems algebraically.
MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Daily Target 4.1.4 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT find the perimeters and areas of polygons in the coordinate plane, applying the Pythagorean Theorem and/or distance formula as needed.

Triangle Perimeters: https://www.illustrativemathematics.org/content-standards/HSG/GPE/B/7/tasks/1816

MA.G-GPE.B.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Daily Target 4.1.5 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT locate a point on a directed line segment that creates two segments of a given ratio.

MA.G-GPE.B.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Learning Goal 4.2

Students will define trigonometric ratios and solve applied problems involving right triangles.

Daily Target 4.2.1 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT multiply, and divide radical expressions.

MA.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

Daily Target 4.2.2 (Level of Difficulty: Comprehension, DOK: 2 - Strategic Skill)

SWBAT identify and determine unknown side lengths in 45-45-90 and 30-60-90 triangles.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Objective 4.2.3 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT show and explain that definitions for trigonometric ratios derive from similarity of right triangles.

Defining Trigonometric Ratios: https://www.illustrativemathematics.org/content-standards/HSG/SRT/C/6/tasks/1635

MA.K-12.7	Look for and make use of structure.
LA.RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
MA.G-SRT.C.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Daily Target 4.2.4 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT use the sine, cosine, and tangent ratios and their inverses to solve for unknown side lengths and angle measures in right triangles.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Daily Target 4.2.5 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT explain and use the relationship between the sine and cosine of complementary angles.

Sine and Cosine of Complementary Angles: https://www.illustrativemathematics.org/content-standards/HSG/SRT/C/7/tasks/1443

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.C.7	Explain and use the relationship between the sine and cosine of complementary angles.

Daily Target 4.2.6 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT solve right triangles (determine all side lengths and all angle measures) using trigonometric ratios, special right triangle relationships, and the Pythagorean Theorem.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Daily Target 4.2.7 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT sketch and use right triangles to solve applied problems efficiently.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.C.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Learning Goal 4.3

Students will translate between the geometric description and the equation for a circle.

Daily Target 4.3.1 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

Given the center and radius, SWBAT use the Pythagorean Theorem to derive the equation of a circle.

Explaining the equation for a circle: https://www.illustrativemathematics.org/content-standards/HSG/GPE/A/1/tasks/1425

MA.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Daily Target 4.3.2 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

Given an equation of a circle in any form, SWBAT complete the square to find the center and radius of the circle.

MA.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Daily Target 4.3.4 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT prove or disprove that a given point lies on a circle of a given center and radius or point on the circle.

MA.G-GPE.A.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Learning Goal 4.4

Students will understand and apply theorems and formulas relating circles and their measures.

Daily Target 4.4.1 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT prove that all circles are similar.

Similar Circles: https://www.illustrativemathematics.org/content-standards/HSG/C/A/1/tasks/1368

MA.G-C.A.1	Prove that all circles are similar.

Daily Target 4.4.2 (Level of Difficulty: Retrieval, DOK: 1 - Recall)

SWBAT identify and define each of the following terms associated with circles, as well as their symbolic notations:

- tangents
- secants
- chords

- arcs
- minor arcs
- major arcs
- semicircles
- central angles
- inscribed angles
- sector of a circle

LA.RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

Daily Target 4.4.3 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT identify, describe, and solve problems using relationships among angles, radii, and chords of circles, including:

- the radius of a circle is perpendicular to the tangent at the point of tangency
- the relationships between central, inscribed, and circumscribed angles and their intercepted arcs
- angles inscribed in a semi-circle are right angles
- the measures of angles and arcs defined by secants, chords and tangents of circles

MA.G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords.

Daily Target 4.4.4 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT prove that opposite angles of a quadrilateral inscribed in a circle are supplementary.

MA.G-C.A.2	Identify and describe relationships among inscribed angles, radii, and chords.

Daily Target 4.4.5 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT use similarity to recognize that the length of the arc intercepted by an angle is proportional to the radius of a circle, and will define the radian measure of an angle as the constant of proportionality.

MA.G-C.B.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Daily Target 4.4.6 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT find arc lengths and areas of sectors of circles.

Cosmos - Eratosthenes calculates Earth's circumference: https://vimeo.com/78787366

MA.G-C.B.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

21st Century Life and Careers

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.

Technology

TECH.8.1.12.A.1	Create a personal digital portfolio which reflects personal and academic interests, achievements, and career aspirations by using a variety of digital tools and resources.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.B.CS2	Create original works as a means of personal or group expression.
TECH.8.1.12.C.CS1	Interact, collaborate, and publish with peers, experts, or others by employing a variety of digital environments and media.
TECH.8.1.12.C.CS2	Communicate information and ideas to multiple audiences using a variety of media and formats.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS2	Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media.
TECH.8.1.12.E.CS3	Evaluate and select information sources and digital tools based on the appropriateness for specific tasks.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.

Summative Assessment

- Projects

- Quizzes
- Student Portfolios
- Tests
- Unit #4 Assessment (Common Assessment)

Formative Assessment and Performance Opportunities

- "I have...Who has..." Review Activities
- Academic Games
- Carousel Activities
- Class Discussions
- Classwork
- Closure Activities
- Concept Sorting Activities
- Desmos Activities
- Do Nows
- Edulastic
- Exit Tickets
- Four Corners Activities
- Graphic Organizers
- Homework
- Kahoot! Activities
- Placemat Activities
- Question-All-Writes
- Quiz-Quiz-Trade Activities
- Station Activities
- Student Interviews
- Student Response Systems
- Student Self-Ratings
- Teacher Observation
- Teacher Questioning
- Think, Pair, Share Discussions
- Thumbs Up/Down
- Whip Around
- Whiteboard Use

Accommodations and Modifications

- 504 Accomodations

- Challenge Problems
- IEP Modifications
- Learning Centers/Stations
- Leveled Practice Opportunities
- Scaffolding Questions
- Small Group Instruction
- Student Companion Website Resources
- Technology
- Use of Manipulatives (Paper Strips, Exploragons, etc.)

Unit Resources

- Textbook: Geometry, Common Core Ed. (Holt McDougal, 2012)
- Textbook Resource Kit & Companion Website: https://my.hrw.com/
- Geometer's Sketchpad
- Kuta Software

Additional Websites:

- Dan Meyer's 3-Act Math Tasks: https://docs.google.com/spreadsheet/pub?key=0AjIqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM1UWowTEE&output=htmlG
- Engage NY: Geometry Lesson Notes & Handouts: https://www.engageny.org/resource/high-school-geometry
- Geometry Teacher Mike Patterson's Common Core Teaching Notes: http://www.geometrycommoncore.com/
- Khan Academy: https://www.khanacademy.org/
- NCTM Illuminations Website: Resources for Teaching Math: http://illuminations.nctm.org/Default.aspx
- PARCC Educator Resources: http://www.parcconline.org/for-educators
- The Geometer's Sketchpad Resource Center: http://www.dynamicgeometry.com/