Geometry Unit 3: Similarity, Proof, and Polygons (Gr. 9 - 11)

Content Area: **Mathematics**
Course(s): **Geometry**
Time Period: **2nd Marking Period**
Length: **10 Weeks**
Status: **Published**

Unit Overview

This unit opens with a study of dilations which leads into the exploration of similar polygons and their properties. Following this, students will learn to identify perpendicular bisectors, angle bisectors, medians, altitudes, and midsegments of triangles, as well as use their properties to solve related problems. Students will then learn to write formal geometric proofs of theorems relating lines, angles, and triangles. Next, students will review polygon vocabulary and develop formulas used to find the interior and exterior angle measures of convex polygons. From there, students will move on to study quadrilaterals. They will investigate and prove the properties of parallelograms and special parallelograms, as well as use these to solve for unknowns in applied problems.

By the end of January, administer the Link IT CC Geometry Form B TEI

Transfer

Students will be able to independently use their learning to...

- Identify dilations and similar polygons as they occur in nature, art, and other applications.
- Sketch dilations and similar polygons to meet given criterion.
- Use proportional reasoning to solve applied problems as they arise.
- Identify segments related to triangles, as well as their unique properties.
- Determine the properties of polygons and quadrilaterals that are pertinent to solving various problems.
- Write valid geometric proofs to demonstrate the properties of lines, angles, triangles, and special quadrilaterals.

Meaning

Understandings

Students will understand that...

- Logical mathematical arguments, known as *proofs*, are used to prove that certain statements are true, and to establish rules in geometry.
- Geometric relationships and definitions can be used to solve real-world problems.
- When one figure is dilated to produce another, similar figures result.
- Similar polygons have congruent corresponding angles, and side lengths that are in proportion to one

another.
- There are shortcuts that can be used to show that two triangles are similar.
- The more sides that a convex polygon has, the greater the sum of its interior angles.
- The sum of the exterior angles of a convex polygon is consistent, regardless of the number of sides that the polygon has.
- The unique properties of parallelograms and other special quadrilaterals allow us to solve for unknowns involving these quadrilaterals.

Essential Questions

Students will keep considering...

- How can geometric figures and their properties be described by careful use of geometric language?
- Why is it useful to classify geometric figures?
- How can a desired image be produced through the manipulation of a given figure in a plane?
- How are transformations used in various careers and in the real world?
- How can available information and logical reasoning be used to develop and prove conjectures?

Application of Knowledge and Skill

Students will know...

- That formal geometric proofs are logical arguments which contain reasons to support each statement that is made.
- That transformations produce similar figures.
- The definition and properties of similar polygons.
- That the AA Postulate, SAS and SSS Similarity Theorems can be used to prove triangles similar.
- The Triangle Proportionality Theorems and the Triangle Angle Bisector Theorem.
- That ratios and proportions are useful in a variety of applications.
- The names of polygons with ten or fewer sides.
- The formulas used to determine angle measures related to polygons.
- The definitions and unique characteristics of parallelograms, rectangles, rhombi, squares, trapezoids and kites.

Students will be skilled at...

- Writing formal geometric proofs involving line and angle relationships, triangles, and quadrilaterals.

- Identifying the scale factor for similar polygons.
- Verifying that given polygons are similar.
- Solving for unknown measurements in similar polygons.
- Using proportional relationships to solve for unknowns.
- Calculating the interior and exterior angle sums of polygons, as well as individual angle measures in regular polygons.
- Classifying polygons based on given angle measurements.
- Identifying parallelograms, rectangles, rhombi, squares, trapezoids, kites, and similar figures.
- Using the properties of unique quadrilaterals to solve for unknowns.
- Using given information to prove that certain quadrilaterals are parallelograms, rectangles, rhombi, or squares.

Academic Vocabulary

- base angle of a trapezoid
- base of a trapezoid
- concave
- convex
- decagon
- deductive reasoning
- diagonal
- dilation
- directed line segment
- heptagon
- hexagon
- indirect measurement
- isosceles trapezoid
- justify
- kite
- leg of a trapezoid
- midsegment of a trapezoid
- n-gon
- nonagon
- octagon
- parallelogram
- pentagon
- proportion
- prove
- quadrilateral

- ratio
- rectangle
- regular polygon
- rhombus
- scale
- scale drawing
- scale factor
- side of a polygon
- similar
- similar polygons
- similarity ratio
- similarity transformations
- square
- trapezoid
- vertex of a polygon

Learning Goal 3.1

Students will verify the properties of dilations given by a center and a scale factor.

Daily Target 3.1.1 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT dilate figures in the coordinate plane. Given a figure and its dilated image, SWBAT identify the center of dilation and the scale factor.

MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.G-SRT.A.1	Verify experimentally the properties of dilations given by a center and a scale factor:

Daily Target 3.1.2 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT describe the effect of given dilations on lines and line segments:

- A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Dilating a Line: https://www.illustrativemathematics.org/content-standards/HSG/SRT/A/1/tasks/602

MA.K-12.1	Make sense of problems and persevere in solving them.

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.G-SRT.A.1	Verify experimentally the properties of dilations given by a center and a scale factor:
MA.G-SRT.A.1a	A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
MA.G-SRT.A.1b	The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Learning Goal 3.2

Students will understand similarity in terms of similarity transformations, and will solve problems involving similar polygons.

Daily Target 3.2.1 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT define similarity in terms of similarity transformations. Based on this definition, students will determine when two polygons are similar.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
LA.RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.
MA.G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Daily Target 3.2.3 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT identify properties of similar polygons, and apply these properties in solving for unknowns.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.A.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of

	all corresponding pairs of sides.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Daily Target 3.2.3 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT use ratios and scale drawings to make indirect measurements and solve problems.

MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Daily Target 3.2.4 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT use the properties of similarity transformations to establish the AA, SSS, and SAS criterion for two triangles to be similar.

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.G-SRT.A.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Daily Target 3.2.5 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT state and apply the Triangle Proportionality Theorem, its converse, and the Triangle Angle Bisector Theorems in solving for unknown lengths.

Tangent Line to Two Circles: https://www.illustrativemathematics.org/content-standards/HSG/SRT/B/5/tasks/916

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.7	Look for and make use of structure.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Learning Goal 3.3

Students will be able to state and apply properties of perpendicular and angle bisectors, medians, altitudes, and midsegments of triangles.

Daily Target 3.3.1 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT state and apply theorems about perpendicular and angle bisectors of triangles.

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.G-CO.C.10	Prove theorems about triangles.
MA.G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Daily Target 3.3.2 (Level of Difficulty: Analysis, DOK: 4 - Extended Thinking)

SWBAT construct the inscribed and circumscribed circles of a triangle.

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.G-C.A.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Daily Target 3.3.3 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT state and apply theorems about medians of triangles.

MA.K-12.6	Attend to precision.
MA.G-CO.C.10	Prove theorems about triangles.

Daily Target 3.3.4 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT state and apply theorems about altitudes of triangles.

MA.K-12.6	Attend to precision.
MA.G-CO.C.10	Prove theorems about triangles.

Daily Target 3.3.5 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT state and apply theorems about midsegments of triangles.

MA.K-12.6	Attend to precision.
MA.G-CO.C.10	Prove theorems about triangles.

Learning Goal 3.4

Students will construct and explain formal proofs of theorems involving lines, angles, and triangles.

Daily Target 3.4.1 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT construct and explain proofs of theorems about lines and angles, including:

- Vertical angles are congruent
- Theorems describing the angle relationships formed when parallel lines are intersected by transversals (For a transformation-based approach to this, see https://www.illustrativemathematics.org/content-standards/HSG/CO/C/9/tasks/1922)
- Points on a perpendicular bisector of a segment are exactly those equidistant from the endpoints of the segment: https://www.illustrativemathematics.org/content-standards/HSG/CO/C/9/tasks/967

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.6	Attend to precision.
MA.G-CO.C.9	Prove theorems about lines and angles.

Daily Target 3.4.2 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT construct and explain proofs of theorems about triangles, including:

- The sum of the interior angles of a triangle is 180˚
- Base angles of an isosceles triangle are congruent
- The segment joining the midpoints of two sides of a triangle is parallel to the third side of the triangle, and half of its length
- A line parallel to one side of a triangle divides the other two sides proportionally
- The medians of a triangle meet at one point
- The Pythagorean Theorem (using triangle similarity): https://www.illustrativemathematics.org/content-standards/HSG/SRT/B/4/tasks/1568

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.6	Attend to precision.
MA.G-CO.C.10	Prove theorems about triangles.

Learning Goal 3.5

Students will prove relationships that exist in unique quadrilaterals, and will solve problems involving polygons and quadrilaterals.

Daily Target 3.5.1 (Level of Difficulty: Retrieval, DOK: 1 - Recall)

SWBAT classify polygons and quadrilaterals according to their sides, angles, and unique properties.

MA.K-12.6	Attend to precision.
LA.RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

Daily Target 3.5.2 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT develop and use formulas to determine the following measures in given convex polygons. Given any of these unique measures, SWBAT determine the associated polygon.

- The sum of the measures of the interior angles
- The sum of the measures of the exterior angles (360°)
- The measure of one interior angle of a regular polygon
- The measure of one exterior angle of a regular polygon

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.6	Attend to precision.
MA.K-12.8	Look for and express regularity in repeated reasoning.

Daily Target 3.5.3 (Level of Difficulty: Knowledge Utilization, DOK: 4 – Extended Thinking)

Students will use congruence criteria for triangles (CPCTC) to construct and explain formal proofs of theorems involving parallelograms, including:

- Opposite sides are congruent
- Opposite angles are congruent
- The diagonals of a parallelogram bisect each other

Is this a parallelogram? https://www.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/1321

Midpoints of the Sides of a Parallelogram: https://www.illustrativemathematics.org/content-standards/HSG/CO/C/11/tasks/35

MA.K-12.7	Look for and make use of structure.
MA.G-CO.C.11	Prove theorems about parallelograms.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Daily Target 3.5.4 (Level of Difficulty: Analysis, DOK: 4 – Extended Thinking)

Students will construct and explain formal proofs of theorems involving special parallelograms (for example, SWBAT prove that rectangles are parallelograms with congruent diagonals).

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.6	Attend to precision.
MA.G-CO.C.11	Prove theorems about parallelograms.

Daily Target 3.5.5 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT apply the properties of special quadrilaterals in solving for unknowns.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.G-CO.C.11	Prove theorems about parallelograms.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

21st Century Life and Careers

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP11	Use technology to enhance productivity.

Technology

TECH.8.1.12.A.1	Create a personal digital portfolio which reflects personal and academic interests, achievements, and career aspirations by using a variety of digital tools and resources.

TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.B.CS2	Create original works as a means of personal or group expression.
TECH.8.1.12.C.CS1	Interact, collaborate, and publish with peers, experts, or others by employing a variety of digital environments and media.
TECH.8.1.12.C.CS2	Communicate information and ideas to multiple audiences using a variety of media and formats.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS2	Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media.
TECH.8.1.12.E.CS3	Evaluate and select information sources and digital tools based on the appropriateness for specific tasks.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.

Summative Assessment

- Projects
- Quizzes
- Student Portfolios
- Tests
- Unit #3 Assessment (Common Assessment)

Formative Assessment and Performance Opportunities

- "I have...Who has..." Review Activities
- Academic Games
- Carousel Activities
- Class Discussions
- Classwork
- Closure Activities
- Concept Sorting Activities
- Do Nows
- Exit Tickets
- Four Corners Activities
- Graphic Organizers
- Homework
- Placemat Activities
- Question-All-Writes
- Quiz-Quiz-Trade Activities

- Station Activities
- Student Interviews
- Student Response Systems
- Student Self-Ratings
- Teacher Observation
- Teacher Questioning
- Think, Pair, Share Discussions
- Thumbs Up/Down
- Whip Around
- Whiteboard Use

Accommodations and Modifications

- 504 Accomodations
- Challenge Problems
- IEP Modifications
- Learning Centers/Stations
- Leveled Practice Opportunities
- Scaffolding Questions
- Small Group Instruction
- Student Companion Website Resources
- Technology
- Use of Manipulatives (Paper Strips, Exploragons, etc.)

Unit Resources

- Textbook: Geometry, Common Core Ed. (Holt McDougal, 2012)
- Textbook Resource Kit & Companion Website: https://my.hrw.com/
- Geometer's Sketchpad
- Kuta Software

Additional Websites:

- Dan Meyer's 3-Act Math Tasks: https://docs.google.com/spreadsheet/pub?key=0AjIqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM1UWowTEE&output=htmlG
- Engage NY: Geometry Lesson Notes & Handouts: https://www.engageny.org/resource/high-school-geometry
- Geometry Teacher Mike Patterson's Common Core Teaching Notes: http://www.geometrycommoncore.com/
- Khan Academy: https://www.khanacademy.org/
- NCTM Illuminations Website: Resources for Teaching Math: http://illuminations.nctm.org/Default.aspx
- PARCC Educator Resources: http://www.parcconline.org/for-educators

- The Geometer's Sketchpad Resource Center: http://www.dynamicgeometry.com/