Geometry Unit 2: Transformations, Congruence & Constructions (Gr. 9 - 11)

Content Area: **Mathematics**
Course(s): **Geometry**
Time Period: **November**
Length: **8 Weeks**
Status: **Published**

Unit Overview

Through this unit, students will investigate transformations, with a focus on isometries. They will define, sketch, and compare various transformations, as well as describe the transformations that can be used to map one figure onto another. Building upon this work, students will define and identify the properties of congruent figures. The study of congruent figures will focus on triangles, as students explore and justify different methods that can be used to demonstrate triangle congruence. At the conclusion of the unit, students will study and make formal geometric constructions, recognizing the foundation that congruence provides for these.

Transfer

Students will be able to independently use their learning to...

- Transform figures in a plane.
- Determine when figures are congruent.
- Use the properties of congruence to reach accurate conclusions.
- Use geometric relationships as a basis to solve real-life and mathematical problems using numerical and algebraic expressions and equations
- Create formal geometric constructions.

Meaning

Understandings

Students will understand that...

- Geometric relationships and definitions can be used to construct geometric figures and solve real-world problems.
- Isometries are transformations that do not change the size or shape of a figure.
- An object in a plane can be oriented infinitely many different ways while maintaining its size, shape, and original properties.
- Specific instructions can be used to produce a desired image through the manipulation of a given image.
- Knowing the properties of triangles enables us to reach conclusions and solve for unknown measures related to triangles.

- Geometric constructions are precise drawings made with only a compass and a straightedge.

Essential Questions

Students will keep considering...

- How can geometric figures and their properties be described by careful use of geometric language?
- How can a desired image be produced through through the manipulation of a given figure in a plane?
- When is it possible to form a triangle with given constraints?
- How can we verify that two figures are congruent to one another?
- How can formal geometric constructions be created using a variety of tools?

Application of Knowledge and Skill

Students will know...

- That transformations are functions that take points in the plane as inputs and give other points as outputs.
- The definitions of reflections, rotations, translations, and symmetry.
- The properties of isometries.
- The five basic methods used to prove triangles congruent.
- That all corresponding parts of congruent figures are congruent.
- How to make basic formal geometric constructions.

Students will be skilled at...

- Drawing specified transformations of figures.
- Specifying a sequence of transformations that will carry a given figure onto another.
- Identifying symmetry in figures.
- Identifying congruent figures.
- Using the SSS, SAS & ASA postulates, as well as the AAS & HL theorems to demonstrate triangle congruence.
- Creating basic formal geometric constructions and explaining why they work.

Academic Vocabulary

- acute triangle
- base
- base angle
- circumscribed
- compass
- compositions of transformations
- congruent polygons
- construct
- construction
- corresponding angles
- corresponding sides
- CPCTC
- demonstrate
- equidistant
- glide reflection
- glide reflection symmetry
- included angle
- included side
- inscribed
- isometry
- justify
- line of symmetry
- line symmetry
- reflection
- rigid motion
- rotation
- rotational symmetry
- straightedge
- symmetry
- transformation
- translation
- translation symmetry
- triangle rigidity

Learning Goal 2.1

Students will identify the properties of, sketch, and describe transformations in the plane.

Daily Target 2.1.1 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

Students will draw transformed figures using graph paper, tracing paper, and/or geometry software.

MA.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
MA.K-12.5	Use appropriate tools strategically.
MA.G-CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

Daily Target 2.1.2 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT describe a transformation F as a function that assigns each point P in the plane to a point F(P) of the plane, and will develop formal mathematical definitions of reflection, rotation, and translation.

Defining Rotations: https://www.illustrativemathematics.org/content-standards/HSG/CO/A/4/tasks/1509

MA.G-CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
MA.G-CO.A.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
MA.K-12.7	Look for and make use of structure.
LA.RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

Daily Target 2.1.3 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT compare rigid motions to horizontal and vertical stretches, or dilations, distinguishing preserved distances and angles from those that are not preserved.

Horizontal Stretch of the Plane: https://www.illustrativemathematics.org/content-

standards/HSG/CO/A/2/tasks/1924

MA.G-CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

Daily Target 2.1.4 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT draw compositions of transformations.

MA.G-CO.A.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
MA.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
MA.K-12.5	Use appropriate tools strategically.
MA.G-CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

Daily Target 2.1.5 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT specify a sequence of transformations that map one figure onto another.

A particular case of showing triangle congruence: https://www.illustrativemathematics.org/content-standards/HSG/CO/A/5/tasks/1547

MA.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

Daily Target 2.1.6 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT identify lines of symmetry and describe the rotations and/or reflections that carry rectangles,

parallelograms, trapezoids, and regular polygons onto themselves.

Symmetries of Rectangles: https://www.illustrativemathematics.org/content-standards/HSG/CO/A/3/tasks/1469

Seven Circles II: https://www.illustrativemathematics.org/content-standards/HSG/CO/A/3/tasks/708

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.G-CO.A.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
MA.G-CO.A.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
MA.K-12.7	Look for and make use of structure.

Daily Target 2.1.7 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT identify three-dimensional objects generated by rotations of two-dimensional objects.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.7	Look for and make use of structure.
MA.G-GMD.B.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Learning Goal 2.2

Students will be able to understand congruence in terms of rigid motions.

Daily Target 2.2.1 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

Given two figures, SWBAT decide if they are congruent by applying rigid motions.

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.G-CO.B.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
LA.RST.9-10.4	Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to grades 9-10 texts and topics.

Daily Target 2.2.2 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

Properties of Congruent Triangles: https://www.illustrativemathematics.org/content-standards/HSG/CO/B/7/tasks/1637

MA.K-12.2	Reason abstractly and quantitatively.
MA.G-CO.B.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

Daily Target 2.2.3 (Level of Difficulty: Knowledge Utilization, DOK: 4 - Extended Thinking)

SWBAT show and explain how each of the following criteria for triangle congruence extend from the definition of congruence in terms of rigid motions:

- SSS Postulate
- SAS Postulate
- ASA Postulate
- AAS Theorem
- HL Theorem

Why does SAS work? https://www.illustrativemathematics.org/content-standards/HSG/CO/B/8/tasks/109

Why does SSS work? https://www.illustrativemathematics.org/content-standards/HSG/CO/B/8/tasks/110

Why does ASA work? https://www.illustrativemathematics.org/content-standards/HSG/CO/B/8/tasks/339

A quick verification that **SSA** does not guarantee congruence: https://www.illustrativemathematics.org/content-standards/7/G/A/2/tasks/2150

MA.K-12.2	Reason abstractly and quantitatively.
MA.G-CO.B.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

Daily Target 2.2.4 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT solve problems using triangle congruence criteria.

MA.K-12.7	Look for and make use of structure.
MA.G-SRT.B.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Learning Goal 2.3

Students will make formal constructions using a variety of tools and methods.

(Tools may include compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.)

Daily Target 2.3.1 (Level of Difficulty: Comprehension, DOK: 2 - Skill)

SWBAT perform formal constructions using a variety of tools and methods including:

- copying a segment
- copying an angle
- bisecting a segment
- bisecting an angle

Instructions for bisecting an angle: https://www.illustrativemathematics.org/content-standards/HSG/CO/D/12/tasks/1083

Angle bisection and midpoints of line segments: https://www.illustrativemathematics.org/content-standards/HSG/CO/D/12/tasks/1320

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.G-CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Daily Target 2.3.2 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT formally construct a line parallel to a given line through a point not on the line.

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.

MA.K-12.6	Attend to precision.
MA.G-CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Daily Target 2.3.3 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT formally construct:

- perpendicular lines
- perpendicular bisectors of given line segments

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.G-CO.D.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).

Daily Target 2.3.4 (Level of Difficulty: Analysis, DOK: 3 - Strategic Thinking)

SWBAT construct each of the following:

- an equilateral triangle
- a square
- a regular hexagon inscribed in a circle

(Note: See activity on pages 392 - 393 of the textbook.)

Inscribing an equilateral triangle in a circle: https://www.illustrativemathematics.org/content-standards/HSG/CO/D/13/tasks/1557

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.G-CO.D.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

21st Century Life and Careers

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP11	Use technology to enhance productivity.
CAEP.9.2.12.C.1	Review career goals and determine steps necessary for attainment.

Technology

TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.B.CS2	Create original works as a means of personal or group expression.
TECH.8.1.12.C.CS1	Interact, collaborate, and publish with peers, experts, or others by employing a variety of digital environments and media.
TECH.8.1.12.C.CS2	Communicate information and ideas to multiple audiences using a variety of media and formats.
TECH.8.1.12.D.CS2	Demonstrate personal responsibility for lifelong learning.
TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS2	Locate, organize, analyze, evaluate, synthesize, and ethically use information from a variety of sources and media.
TECH.8.1.12.E.CS3	Evaluate and select information sources and digital tools based on the appropriateness for specific tasks.
TECH.8.2.12.D.CS2	Use and maintain technological products and systems.

Summative Assessment

- Projects
- Quizzes
- Student Portfolios
- Tests
- Unit #2 Assessment (Common Assessment)

Formative Assessment and Performance Opportunities

- "I have...Who has..." Review Activities
- Academic Games
- Carousel Activities
- Class Discussions
- Classwork
- Closure Activities
- Concept Sorting Activities
- Desmos Activities
- Do Nows
- Edulastic
- Exit Tickets

- Four Corners Activities
- Graphic Organizers
- Homework
- Kahoot! Games
- Placemat Activities
- Question-All-Writes
- Quiz-Quiz-Trade Activities
- Station Activities
- Student Interviews
- Student Response Systems
- Student Self-Ratings
- Teacher Observation
- Teacher Questioning
- Think, Pair, Share Discussions
- Thumbs Up/Down
- Whip Around
- Whiteboard Use

Accommodations and Modifications

- 504 Accomodations
- Challenge Problems
- IEP Modifications
- Learning Centers/Stations
- Leveled Practice Opportunities
- Scaffolding Questions
- Small Group Instruction
- Stundent Companion Website Resources
- Technology
- Use of Manipulatives (Paper Strips, Exploragons, etc.)

Unit Resources

- Textbook: Geometry, Common Core Ed. (Holt McDougal, 2012)
- Textbook Resource Kit & Companion Website: https://my.hrw.com/
- Geometer's Sketchpad
- Kuta Software

Additional Websites:

- Dan Meyer's 3-Act Math Tasks: https://docs.google.com/spreadsheet/pub?key=0AjIqyKM9d7ZYdEhtR3BJMmdBWnM2YWxWYVM

1UWowTEE&output=htmlG

- Engage NY: Geometry Lesson Notes & Handouts: https://www.engageny.org/resource/high-school-geometry
- Geometry Teacher Mike Patterson's Common Core Teaching Notes: http://www.geometrycommoncore.com/
- Khan Academy: https://www.khanacademy.org/
- NCTM Illuminations Website: Resources for Teaching Math: http://illuminations.nctm.org/Default.aspx
- PARCC Educator Resources: http://www.parcconline.org/for-educators
- The Geometer's Sketchpad Resource Center: http://www.dynamicgeometry.com/