Unit 3: Exponents, Logarithms & Radical Functions

Content Area: **Mathematics**
Course(s): **Algebra II Honors, Algebra II**
Time Period: **February**
Length: **6 Weeks**
Status: **Published**

Unit Overview

During this unit, students will…

- Write expressions in equivalent forms to solve problems.
- Extend the properties of exponents to rational exponents.
- Understand solving equations as a process of reasoning and explain the reasoning.
- Analyze functions using different representations.
- Construct exponential models and use them to solve problems.
- Interpret expressions for functions in terms of the situation they model.
- Simplifying radical expressions.
- Graphing radical functions.
- Solving radical equations and inequalities

CHAPTER 4 & PART OF CHAPTER 5

By the end of May, administer the Algebra II Link It! NJSLS Form C.

Transfer

Students will be able to independently use their learning to...

- Describe the relationship between exponential and logarithmic expressions.
- Describe the relationship between exponential, rational, and radical expressions.
- Simplify and evaluate exponential and logarithmic expressions.
- Explain key features of exponential models.
- Recognize real-life situations that can be modeled by exponential functions, and write functions to model such situations.
- Write radical functions and solve radical equations.
- Apply radical functions and equations to model a real-world situation.

For more information, read the following article by Grant Wiggins.

http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60

Meaning

Understandings

Students will understand that...

- Rational exponential expressions can be simplified using the properties of exponents.
- Rational exponential notation and radical notation are equivalent forms of writing expressions.
- Exponential and logarithmic expressions are inverses of one another.
- Extraneous solutions may arise during the process of solving rational exponential and/or radical equations.
- Exponential expressions are often useful in modeling real-life situations.
- Radical expressions can be simplified using the properties of exponents.
- Radical expressions are often useful in modeling real-life situations.

Essential Questions

Students will keep considering...

- How can various mathematical expressions be simplified both effectively and efficiently?
- How are radical and exponential expressions translated from one form to the other? Why is it often useful to rewrite them?
- How can a given equation be solved both effectively and efficiently?
- What is the relationship between exponential and logarithmic expressions?
- How can mathematical models be used as tools to describe and help explain real-life situations?
- How do mathematical ideas interconnect and build on one another to produce a coherent whole?

Application of Knowledge and Skill

Students will know...

Students will know...

- Rational, exponential and radical notations are different forms of expressing equivalent values.

- Simple exponential equations can be written in logarithmic form, as simple logarithmic equations can be written in exponential form.
- Extraneous solutions may arise during the process of solving rational and radical equations.
- How various aspects of exponential models affect the models.
- The rules of adding, subtracting, multiplying and dividing radical expressions.
- How to solve radical equations.
- How to graph radical equations.

Students will be skilled at...

Students will be skilled at...

- Converting expressions between radical and rational exponential notation.
- Converting expressions between exponential and logarithmic forms.
- Evaluating exponential and logarithmic expressions.
- Solving rational and radical equations.
- Writing exponential functions to model real-world situations.
- Graphing exponential and logarithmic equations.
- Simplifying radical expressions.
- Rewriting radical expressions by using rational exponents.
- Graphing radical functions and inequalities.
- Solving radical equations and inequalities.

Academic Vocabulary

Exponent

Base

Rational Exponent

Exponential Equation

Logarithm

Common logarithm

Natural logarithm

Radical Equation

Rational Equation

Extraneous Solution

Radical Function

Rational Function

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Radical Inequalitiy

square-root function

Learning Goal 3.1

SWBAT use and apply properties of, communicate the relationship between and solve problems using exponential and logarithmic functions.

Target 3.1.1

SWBAT:

- Write and evaluate exponential expressions to model growth and decay situations. **(DOK 3)**

MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the

meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Target 3.1.2

SWBAT:

- Graph and recognize inverses of relations and functions. **(DOK 2)**
- Find inverses of functions.**(DOK 3)**

MA.F-BF.B.4c	Read values of an inverse function from a graph or a table, given that the function has an inverse.

MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

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Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

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Target 3.1.3

SWBAT...

- Write equivalent forms for exponential and logarithmic functions. **(DOK 2)**
- Write, evaluate, and graph logarithmic functions. **(DOK 3)**

MA.F-BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

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Target 3.1.4

SWBAT:

- Use properties to simplify logarithmic expressions. **(DOK 3)**
- Translate between logarithms in any base. **(DOK 2)**

MA.F-BF.B.4	Find inverse functions.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

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Target 3.1.5

SWBAT...

- Solve exponential and logarithmic equations and inequalities. **(DOK 2)**
- Solve problems involving exponential and logarithmic equations. **(DOK 3)**

MA.F-LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab to the ct power = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
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Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

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Target 3.1.7

SWBAT...

- Transform exponential and logarithmic functions by changing parameters. **(DOK 3)**
- Describe the effects of changes in the coefficients of exponential and logarithmic functions.**(DOK 3)**

MA.F-BF.B.3

Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using

	technology.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
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Target 3.1.8

SWBAT...

- Model data by using exponential and logarithmic functions. **(DOK 4)**
- Use exponential and logarithmic models to analyze and predict. **(DOK 4)**

MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7

× 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Learning Goal 3.2

SWBAT apply algebraic reasoning to solve problems with radical expressions.

Target 3.2.1

SWBAT:

- Rewrite radical expressions by using rational exponents. **(DOK 2)**
- Simplify & evaluate radical expressions and expressions containing rational exponents. **(DOK 2)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.A-REI.D.12	Graph the solutions to a linear inequality in two variables as a half plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Target 3.2.2

SWBAT:

- Graph radical functions and inequalities. **(DOK 2)**
- Transform radical functions by changing parameters. **(DOK 3)**

MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Target 3.2.3

SWBAT solve radical equations and inequalities. **(DOK 2)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.6	Attend to precision.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.

Target 3.2.4

SWBAT solve equations graphically. **(DOK 3)**

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.A-REI.D.11	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Formative Assessment and Performance Opportunities

- academic games
- Class discussions
- Classwork
- Do nows
- Exit tickets
- Exponential Skittles Activity
- Homework
- Problem based learning
- student interviews
- Teacher observation
- whiteboard/communicator opportunities

Summative Assessment

- Link-It Exams
- Projects
- Quizzes
- student interviews
- Tests
- Unit Exam

21st Century Life and Careers

CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.

CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.

Technology

TECH.8.1.12.A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
TECH.8.1.12.B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
TECH.8.1.12.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

Accommodations and Modifications

- 504 Accommodations
- centers/stations
- challenge questions
- Common Core Workbook Reference 4-1 (Exponential Functions, Growth and Decay)
- Common Core Workbook Reference 4-2 (Inverses of Relations and Functions)
- Common Core Workbook Reference 4-3 (Logarithmic Functions)
- Common Core Workbook Reference 4-4 (Properties of Logarithms)
- Common Core Workbook Reference 4-5 (Exponential and Logarithmic Equations and Inequalities)
- Common Core Workbook Reference 4-6 (The Natural Base, e)
- Common Core Workbook Reference 4-7 (Transforming Exponential and Logarithmic Functions)
- Common Core Workbook Reference 5-6 (Radical Expressions and Rational Exponents)
- Common Core Workbook Reference 5-7 (Radical Functions)
- Common Core Workbook Reference 5-8 (Solving Radical Equations and Inequalities)
- Compound Interest and Simple Interest to relate real life world applications
- Graphing Calculator activity to reinforce transformations of graphs in relation to radical functions
- IEP Modifications
- manipulatives
- Maze activity involving equivalent expressions of exponentials
- Maze activity involving equivalent expressions of logarithms
- projects
- scaffolding questions
- small group instruction
- Unit 3 Exam Review
- use of technology

Unit Resources

- Desmos
- Explorations in Core Math for Common Core: Algebra 2 (Holt McDougal)
- Geometer sketchpad
- Kuta software
- NCTM website
- online textbook materials
- PARCC/NJSLA Released Questions
- SJMAP Resources
- Text