Unit 2: Quadratics and Transformations

Content Area: **Mathematics**
Course(s): **Algebra 1**
Time Period: **Generic Time Period**
Length: **14 Weeks**
Status: **Published**

Unit Overview

This unit opens with a study of polynomials, including performing basic operations on polynomial expressions. From there, students will learn to factor polynomial expressions completely. They will then move into a study of quadratic functions--recognizing, graphing, writing, and solving them. Emphasis will be placed on seeing the relationships between quadratic functions presented in varied forms, as well as applying quadratic functions to real world situations. At the conclusion of the unit, students will explore the patterns of transformations of functions, focusing on both quadratic and absolute value functions.

By the end of January, administer the Linkit! Algebra I NJSLS Form B TEI AG

Transfer

Students will be able to independently use their learning to...

- Relate algebraic terminology to real life problems/applications.
- Apply knowledge of polynomials to solve real life problems.
- Apply polynomials, quadratics, and transformations to relevant situations that occur both inside and outside of their math classes.

Meaning

Understandings

Students will understand that...

- Critical vocabulary is necessary to communicate and understand mathematics in the real world.
- The symbolic language of algebra is used to communicate, analyze, and generalize patterns.
- Algebraic relationships can be represented graphically, numerically, symbolically, or verbally.
- In mathematical relationships, the value for one quantity depends on the value of another quantity.
- Non-linear change can be represented by graphs, mathematical models, and equations.
- Algebraic models are useful in describing real-life situations.
- Quadratic equations can be solved using a variety of methods.

- Introducing a constant into the equation of a function causes the graph of the function to shift, stretch, and/or reflect in the coordinate plane.

Essential Questions

Students will keep considering...

- How do mathematical ideas interconnect and build on one another to produce a coherent whole?
- How can critical vocabulary terms be used to better enhance the understanding of mathematics?
- What is the best method to solve a given quadratic equation?
- How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations?
- How can functions be used as tools to best describe and help explain real-life situations?
- How does the value of one variable affect the value of another variable?
- How can mathematical language be used to describe non-linear change?

Application of Knowledge and Skill

Students will know...

Students will know...

- Equivalent forms of expressions
- Different factoring techniques in order to factor completely
- Techniques of simplifying expressions
- Explanation of finding and graphing zeros of a polynomial
- Explanation of finding the vertex by completing the square
- Conversion of a quadratic equation in standard form to vertex form
- Deriving quadratic formula by using completing the square technique
- Various ways of solving a quadratic equation
- Reasoning for the answer to a square root will result in two answers
- When a solution to a quadratic equation has no real solution
- Function notation

- Ways to show zeros, extreme values (maxima/minima), and symmetry
- Introducing a constant to the equation of a function has the effect of shifting, stretching, or reflecting the graph of the function in the coordinate plane

Students will be skilled at...

Students will be skilled at...

- Simplifying polynomial expressions.
- Factoring polynomial expressions completely using techniques including greatest common factors, grouping, and difference of two squares.
- Finding zeros of polynomial functions.
- Completing the square in quadratic equations and functions.
- Deriving quadratic formula through completing the square.
- Solving quadratic equations in one variable.
- Determining how many real number solutions a quadratic equation has.
- Recognizing the relationship between zeros, roots, and x-intercepts of quadratic functions.
- Using factoring and completing the square to show zeros, extreme values (maxima/minima), and symmetry and interpret their meaning in terms of the context of the problem.
- Using patterns of transformations to graph and write equations for functions

Academic Vocabulary

quadratic equation/function in standard form	parabola	axis of symmetry	completing the square	maximum zeros
polynomial	binomial	degree of a polynomial	leading coefficient	factored form of a polynomial
factored completely	standard form of a polymomial	solving equations by factoring	perfect-square trinomial	difference of two squares
vertex	quadratic formula	perfect square	minimum	monomial
trinomial	FOIL pattern	zero product property	transformation	reflect
horizontal shift	vertical shift	maximum	quadratic	zero of a

			equation/function in vertex form	function
roots	zeros	x-intercepts	degree of a monomial	degreee of a polynomial
standard form of a polynomial				

Learning Goal 2.1: Polynomials

- Apply factoring techniques to solve problems.
- Factor Polynomials.
- Use arithmetic operations to simplify polynomials

MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Target 2.1.1 - Polynomial Terminology

- SWBAT decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts. (DOK: 2 - Comprehension)
- SWBAT identify the different parts of the expression and explain their meaning within the context of a problem. (DOK: 1 - Retrieval)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.A-APR.A	Perform arithmetic operations on polynomials

Target 2.1.2 - Polynomial Operations

- SWBAT add, subtract, and multiply polynomial expressions (DOK: 2 - Comprehension)
- SWBAT simplify polynomial expressions by writing in standard form (DOK: 1 - Retrieval)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.

MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Target 2.1.3 - Factoring Polynomials

- SWBAT factor polynomial equations using various methods, such as GCF, trinomial, difference of two squares, grouping, and completing the square. (DOK: 3 - Analysis)
- SWBAT solve and explain solutions of polynomial equations using various methods, such as GCF, trinomial, difference of two squares, taking square roots, grouping, and completing the square. (DOK: 4 - Knowledge Utilization)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.K-12.6	Attend to precision.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.K-12.7	Look for and make use of structure.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Learning Goal 2.2: Quadratics

- SWBAT graph and solve quadratic functions.
- SWBAT solve real-world problems using quadratic functions and equations.

MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.A-REI.B.4	Solve quadratic equations in one variable.

Target 2.2.1 - Using Standard Form

- SWBAT depict the domain and range of a quadratic function from its graph. (DOK: 3 - Analysis)
- SWBAT find the axis of symmetry and the vertex of a parabola. (DOK: 2 - Comprehension)

- SWBAT graph a quadratic function using a a table of values. (DOK: 2 - Comprehension)
- SWBAT identify quadratic functions and determine maximum or minimum. (DOK: 1 - Retrieval)
- SWBAT relate the roots, zeros, and x-intercepts of functions and their graphs. (DOK: 3 - analysis)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.5	Use appropriate tools strategically.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.A-REI.B.4	Solve quadratic equations in one variable.

Target 2.2.2 - Radicals

- SWBAT simplify radical expressions (DOK: 2 - Comprehension)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.7.EE.A	Use properties of operations to generate equivalent expressions.

Target 2.2.3 - Derive the Quadratic Formula

- SWBAT use completing the square to derive the quadratic formula. (DOK: 3 - Analysis)

MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.7	Look for and make use of structure.
MA.A-REI.B.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

Target 2.2.4 - Quadratic Formula

- SWBAT find and use the discriminant of a quadratic equation to determine the number of real roots (DOK: 2 - Comprehension)
- SWBAT solve and explain solutions of quadratic equations by using the quadratic formula (DOK: 3 -

Analysis)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.A-REI.B.4	Solve quadratic equations in one variable.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Target 2.2.5 - Real World Applications

- SWBAT examine real-world situations, based on graphs and equations, to create and solve quadratic functions. (DOK: 4 - Knowledge Utilization)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

Target 2.2.6 - Nonlinear Systems

SWBAT use technology to...

- SWBAT examine real-world situations, based on graphs and equations, to create and solve nonlinear systems. (DOK: 4 - Knowledge Utilization)
- SWBAT solve systems of equations in two variables in which one is linear and the other is quadratic. (DOK: 3 - Analysis)

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

Learning Goal 2.3: Transformations

- SWBAT explain transformations of functions in context

MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MA.F-BF.B	Build new functions from existing functions

Target 2.3.1 - Using Transformations to Graph Functions

Note: Technology-based exploration activity suggested to enable students to discover patterns of transformations

- SWBAT identify and describe the effect of the leading coefficient of a function on its graph (DOK: 2 - Comprehension)
- SWBAT identify and describe vertical and horizontal shifts of functions based on both equations and graphs of the functions (DOK: 2 - Comprehension)
- SWBAT use patterns of transformation to graph quadratic and absolute value functions. (DOK: 2 - Comprehension)

MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Target 2.3.2 - Using Transformations to Write Functions

- SWBAT use patterns of transformations to write the equations of graphs of quadratic and absolute value functions. (DOK: 2 - Comprehension)

MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MA.K-12.7	Look for and make use of structure.

Target 2.3.3 - Using Vertex Form

- SWBAT use completing the square to rewrite quadratic equations in vertex form. (DOK: 2 - Comprehension)
- SWBAT use patterns of transformations to graph and identify key features of quadratic equations written in vertex form. (DOK: 2 - Comprehension)

MA.K-12.6	Attend to precision.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value

	of the function it defines.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Formative Assessment and Performance Opportunities

- academic games
- classroom discussions
- classwork
- Closure
- Do Nows
- group work
- homework
- stations
- student/teacher discussions
- think-pair-share

Summative Assessment

- LinkIt Exam
- performance task
- test/quiz
- Unit Exam

21st Century Life and Careers and Technology

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.
CAEP.9.2.12.C.3	Identify transferable career skills and design alternate career plans.
TECH.8.1.12.A.CS1	Understand and use technology systems.
TECH.8.1.12.A.CS2	Select and use applications effectively and productively.

TECH.8.1.12.D.CS2 Demonstrate personal responsibility for lifelong learning.

Accommodations and Modifications

- 504 accomodations
- Common Core Workbook Reference 6-3 (Polynomials)
- Common Core Workbook Reference 6-4 (Adding and Subtracting Polynomials)
- Common Core Workbook Reference 6-5 (Multiplying Polynomials)
- Common Core Workbook Reference 7-1 (Factors and GCF)
- Common Core Workbook Reference 7-2 (Factoring by GCF)
- Common Core Workbook Reference 7-3 (Factoring x^2 + bx + c)
- Common Core Workbook Reference 7-4 (Factoring ax^2 + bx + c)
- Common Core Workbook Reference 7-6 (Choosing a Factoring Method)
- Common Core Workbook Reference 8-1 (Identifying Quadratic Functions)
- Common Core Workbook Reference 8-1 (Non-Linear Systems)
- Common Core Workbook Reference 8-2 (Characteristics of Quadratic Functions)
- Common Core Workbook Reference 8-3 (Graphing Quadratic Functions)
- Common Core Workbook Reference 8-4 (Transforming Quadratic Functions)
- Common Core Workbook Reference 8-5 (Solving Quadratic Functions by graphing)
- Common Core Workbook Reference 8-6 (Solving Quadratic Equations by Factoring)
- Common Core Workbook Reference 8-7 (Solving Quadratic Equations by using Square Roots)
- Common Core Workbook Reference 8-8 (Completing the Square)
- grouping
- IEPs
- use of technology - TI 83 Calculator to graph quadratics, find zeros, y-intercepts, max or min

Unit Resources

- Desmos
- Explorations in Core Math for Common Core: Algebra 1 (Holt McDougal)
- Graphing Calulators
- Illustrative Mathematics
- Khan Academy
- Kuta software
- PARCC/NJSLA released questions
- Textbook

Interdisciplinary Connections

Real world applications involving quadratic functions helps students to find the max, zeros, and y-intercepts

for an object that rises and falls. (MA.9-12.F-IF.C.7a)

Real world applications involving mulitplying polynomials to find the area which helps students analyze engineering decisions (MA.9-12.A.APR.D.6)

9-12.HS-ETS1-1.1.1	Analyze complex real-world problems by specifying criteria and constraints for successful solutions.
9-12.HS-PS2-4.5.1	Use mathematical representations of phenomena to describe explanations.