7.SP Rolling Twice

Alignments to Content Standards: 7.SP.C.8

Task

A fair six-sided die is rolled twice. What is the theoretical probability that the first number that comes up is greater than or equal to the second number?

IM Commentary

The purpose of this task is for students to compute the theoretical probability of a compound event. Teachers may wish to emphasize the distinction between theoretical and experimental probabilities for this problem. For students learning to distinguish between theoretical and experimental probability, it would be good to find an experimental probability either before or after students have calculated the theoretical probability.

If this is the first time students have worked to compute the probability of a compound event, it is a good idea to run the experiment first to get a ball-park sense of the magnitude of the theoretical probability. A quick way to generate an experimental probability is to have students work in pairs to quickly roll the dice ten times and record the outcomes. Each pair can report on the number of times the event in question occurred and the teacher can compile the results quickly on the board or overhead. Then students should be challenged to determine the theoretical probability and comment on how close the experimental probability matched it.

If students are familiar with compound events, this task could be used to deepen their understanding of the relationship between theoretical and experimental probability. In this case, once the students have calculated the theoretical probability, they can perform the experiment and see how well the experimental probability agrees (or disagrees) with the theoretical probability. Because a common incorrect solution is $\frac{1}{2}$, the teacher can lead a discussion about the difficulty in using an experiment to distinguish between two probabilities that are close (like $\frac{7}{12}$ and $\frac{1}{2}=\frac{6}{12}$). This is a good opportunity to discuss the fact that the more times an experiment is performed, the closer the experimental probability and the theoretical probability are likely to be.

Students who are still operating on a more concrete as opposed to abstract level of cognitive development may choose to "draw" the outcomes as listed in the table. Teachers may want to make dice available to students who are struggling to visualize the possible outcomes. Braille dice are available for those with visual impairments.

This task was adapted from problem #18 on the 2011 American Mathematics Competition (AMC) 8 Test. The responses to the multiple choice answers for the problem had the following distribution:

Choice	Answer	Percentage of Answers
(A)	$\frac{1}{6}$	23.56
(B)	$\frac{5}{12}$	17.61
(C)	$\frac{1}{2}$	16.98
(D)*	$\frac{7}{12}$	23.38
(E)	$\frac{5}{6}$	11.20
Omit	-	7.02

Of the 153,485 students who participated, 72,648 or 47% were in 8th grade, 50,433 or 33% were in 7th grade, and the remainder were less than 7th grade.

Solutions

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Solution: 1 Plotting outcomes in a table

We can plot the different possible outcomes as the six sided die is rolled. One way of doing this is with a table as shown below.

1,1*	1,2	1,3	1,4	1,5	1,6
2,1*	2,2*	2,3	2,4	2,5	2,6
3,1*	3,2*	3,3*	3,4	3,5	3,6
4,1*	4,2*	4,3*	4,4*	4,5	4,6
5,1*	5,2*	5,3*	5,4*	5,5*	5,6
6,1*	6,2*	6,3*	6,4*	6,5*	6,6*

In the table the entry marked 1,3 means that the first throw was a 1 and the second throw a 3. An asterisk next to the entry means that the first number that came up is greater than or equal to the second number. There are 36 possibilities listed in the table, each equally likely. For the 21 starred cases, the first number was greater than or equal to the second number. So the probability that the first number is greater than or equal to the second number is $\frac{21}{36} = \frac{7}{12}$.

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Solution: 2 Plotting outcomes in a list

If the first number is a 1, this can only be bigger than or equal to the second number if the second number is a 1. If the first number is a 2, then this is bigger than or equal to 1 or 2. We can make a table for all possibilities that the first number is greater than or equal to the second:

Number on first throw	Possible numbers on second throw
1	1
2	1,2
3	1,2,3

4	1,2,3,4
5	1,2,3,4,5
6	1,2,3,4,5,6

Adding up the possibilities in the table, there are 21 ways that the number on the second roll can be greater than or equal to the number on the first roll. Since there are $6 \times 6 = 36$ total possible outcomes, the probability that the first number is greater than or equal to the second is $\frac{21}{36} = \frac{7}{12}$.

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Solution: 3 Abstract analysis of outcomes

There are 6 outcomes for the first roll and 6 for the second so the total number of possible outcomes is $6 \times 6 = 36$. There are 6 of these in which the two numbers are equal: (1,1), (2,2), (3,3), (4,4), (5,5), and (6,6). When the two numbers are not equal, half of these cases have a greater first number and half have a greater second number: this can be seen by reversing the order to the two throws, which exchanges these two cases. So of the 30 cases where the two numbers are different, 15 of them have a greater first number. This means the first number is greater than or equal to the second in $6 + 15 = 21$ of the 36 different possible outcomes. So the probability that the first throw is at least as big as the second is $\frac{21}{36} = \frac{7}{12}$.