7.SP Red, Green, or Blue?

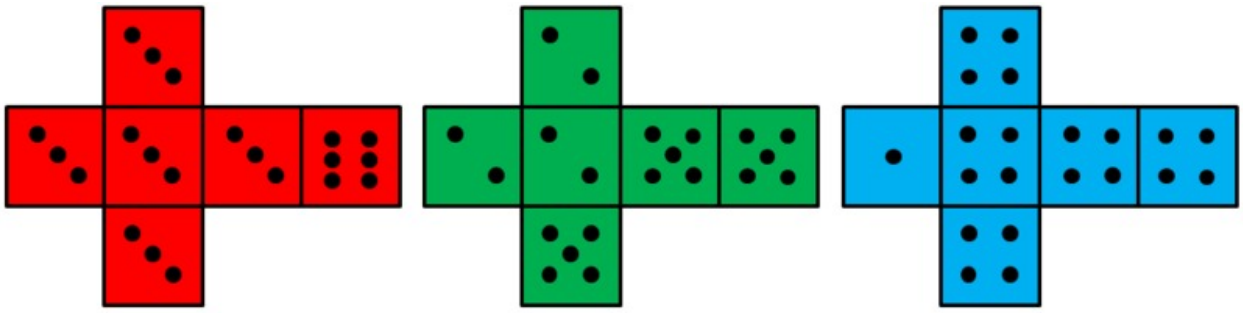
Alignments to Content Standards: 7.SP.C.8

Task

This is a game for two people.

You have three dice; one is red, one is green, and one is blue. These dice are different than regular six-sided dice, which show each of the numbers 1 to 6 exactly once. The red die, for example, has 3 dots on each of five sides, and 6 dots on the other. The number of dots on each side are shown in the table and picture below.

Red	3	3	3	3	3	6
Green	2	2	2	5	5	5
Blue	1	4	4	4	4	4



To play the game, each person picks one of the three dice. However, they have to pick different colors.

- The two players both roll their dice. The highest number wins the round.
- The players roll their dice 30 times, keeping track of who wins each round.
- Whoever has won the greatest number of rounds after 30 rolls wins the game.

a. Who is more likely to win when a person with the red die plays against a person with the green die? What about green vs. blue? What about blue vs. red?

b. Would you rather be the first person to pick a die or the second person? Explain.

IM Commentary

The purpose of this task is for students to find the probability of compound events using organized lists, tables, or tree diagrams. What is interesting about this task is that even though the dice all have the same expected value (3.5), red is more likely to beat green, green is more likely to beat blue, and blue is more likely to beat red. So this game is analogous to Rock-Paper-Scissors, which is a surprising result for most people. These three dice form what is called a set of "non-transitive dice"; to read more about such sets, see

http://en.wikipedia.org/wiki/Nontransitive_dice

It is possible to purchase these kinds of dice or to take white dice and alter the pips with white-out and a black sharpie marker and then color the edges with the appropriate colors. This task idea came from this blog:

http://protonsforbreakfast.wordpress.com/2013/01/08/amazing-dice-rediscovering-surprise/

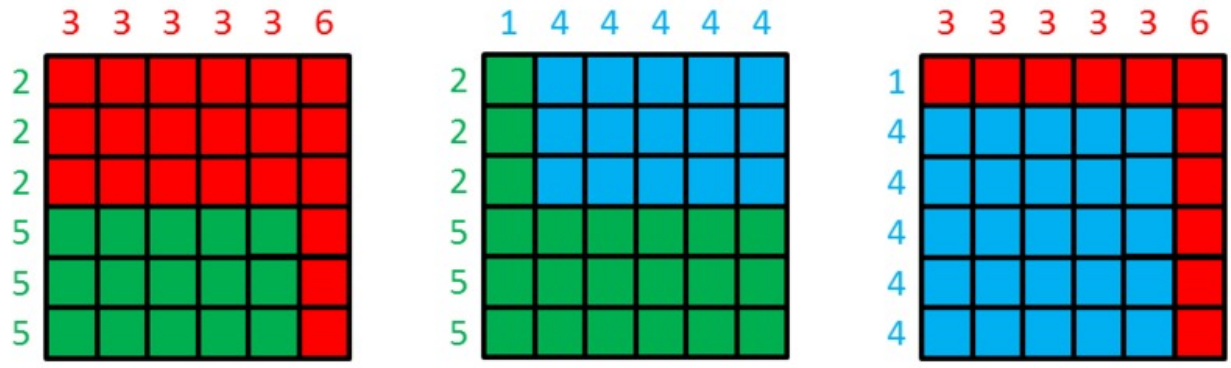
The probabilities calculated in the solution are theoretical probabilities based on analyzing equally likely outcomes for the dice. Experimental results can differ from the theoretical calculations and this is why the game includes 30 throws: this is enough so that the (approximately) 16% advantage red has over green on each throw becomes very difficult to overcome.

Note that the colored tables shown in the solution below look like the colored tables shown in the blog but were independently constructed by some teachers participating in the NSF-funded MCTP Summer Math Camp held at the University of New Mexico in July and August 2013.

Edit this solution

Solution

a. Here are three tables that are color-coded to see who will win which rolls in each of the three possible pairings:



When red plays green, the probability that red will win is

$$\frac{21}{36} \approx 0.58$$

so red is more likely to beat green after playing many rounds. Likewise, when green plays blue, the probability that green will win is

$$\frac{21}{36} \approx 0.58$$

so green is more likely to beat blue after playing many rounds. Similarly, when red plays blue, the probability that blue will win is

$$\frac{25}{36} \approx 0.69$$

so blue is more likely to beat red after playing many rounds.

b. It is better to choose your die second because if your opponent goes first, you can always choose a die that is more likely to beat the die he or she chose. If your opponent chooses red, you can choose blue. If your opponent chooses blue, you can choose green. If your opponent chooses green, you can choose red.