7.SP Sitting across from Each Other

Alignments to Content Standards: 7.SP.C.8.a 7.SP.C.8.b

Task

Angie, Bridget, Carlos, and Diego are seated at random around a square table, one person to a side. What is the theoretical probability that Angie and Carlos are seated opposite each other?

IM Commentary

The purpose of this task is for students to compute the theoretical probability of a seating configuration. There are 24 possible configurations of the four friends at the table in this problem. Students could draw all 24 configurations to solve the problem but this is time consuming and so they should be encouraged to look for a more systematic method. Two systematic ways of counting are presented in the solutions below. The first organizes the 24 possibilities by first looking at where Angie is sitting and then counting possibilities starting from there. The second solution is more abstract. It also places Angie in one of the four seats to begin but then the basic idea is that once Angie has been placed, Carlos is equally likely to be in any of the three remaining places so there is a one in three chance that he will be opposite Angie at the table. A common mistake is to assume that since there are four people, then there is a $\frac{1}{4}$ chance of Angie and Carlos seating opposite one another.

The event of interest is a compound event because it consists of more than a single outcome in the sample space for this chance experiment.

This task was adapted from problem #12 on the 2011 American Mathematics Competition (AMC) 8 Test. The responses to the multiple choice answers for the problem had the following distribution:

Choice	Answer	Percentage of Answers
(A)	$\frac{1}{4}$	34.58
(B)*	$\frac{1}{3}$	28.03
(C)	$\frac{1}{2}$	26.54
(D)	$\frac{2}{3}$	5.14
(E)	$\frac{3}{4}$	3.73
Omit	-	1.94

Of the 153,485 students who participated, 72,648 or 47% were in 8th grade, 50,433 or 33% were in 7th grade, and the remainder were less than 7th grade.

Solutions

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Solution: 1

Angie will be at one of the four seats of the table, leaving three open spots for Bridget, Carlos, and Diego. We can list the possible placements of the other three, proceeding clockwise from Angie: BCD, BDC, CBD, CDB, DBC, or DCB. In 2 of these 6 possibilities Carlos is opposite Angie. This list can be repeated 4 times for the different four seats Angie might occupy. So there are 24 total possible seating arrangements and in 8 of them Carlos and Angie are opposite one another. Therefore the probability that Carlos and Angie will be seated opposite one another is $\frac{8}{24} = \frac{1}{3}$.

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Solution: 2

Once Angie is seated, there are three possibilities for where Carlos might be seated. Only one of these is across from Angie, so the probability that Carlos is seated across from Angie is $\frac{1}{3}$.

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Solution: 3 Symmetry

Once we choose Angie's seat, there are three possible positions for Carlos. He can be seated to her right, to her left, or across the table from her. For each of these scenarios, there are two possibile arrangements since Bridget and Diego can occupy the remaining two seats in two different ways. So Carlos is equally likely to be seated to Angie's left, right, or across from her. This means that the probability that Angie and Carlos are across from one another at the table is $\frac{1}{3}$.

There is a nice geometric way to see that it is equally likely that Carlos is sitting to Angie's left or to her right: if the table is reflected about the line through Angie's seat and the seat opposite her, this exchanges the seats to her left and right. So to each possibility where Carlos is to her left there is a corresponding possible arrangement where he is to her right (and vice versa).

Comparing the situations where Carlos is to Angie's left (or right) with those where he is across from her requires a different argument because there is no symmetry of the square which leaves Angie's place alone and switches the place opposite her with one of the adjacent places.