Illustrative Mathematics

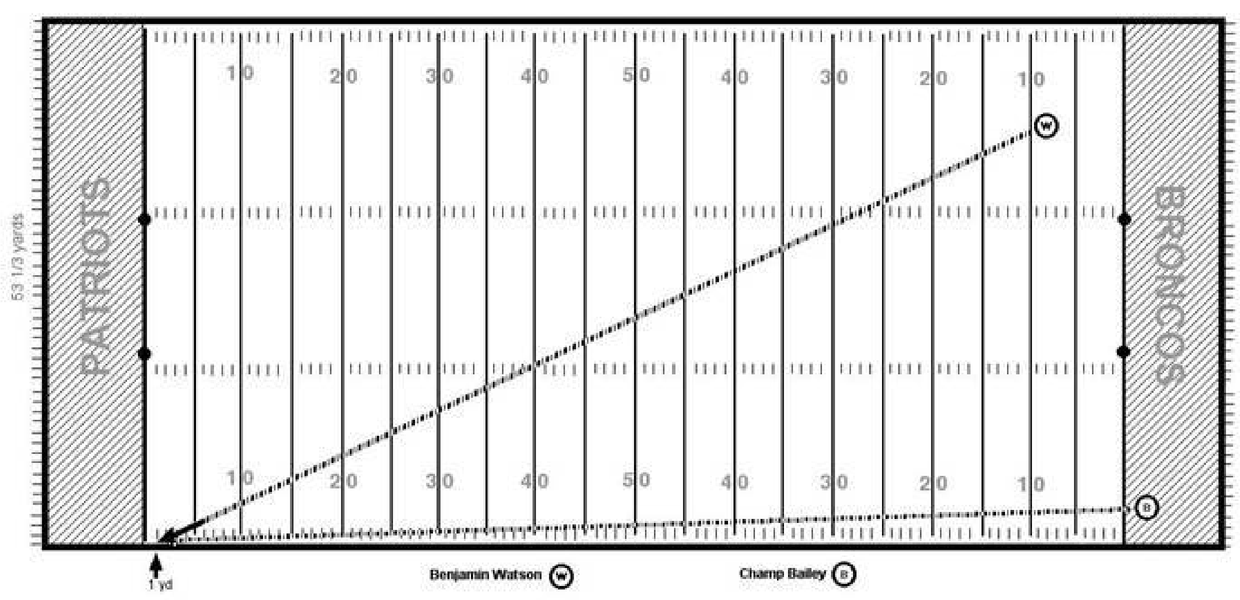
8.G.7 Running on the Football Field

Alignments to Content Standards: 8.G.B.7

Task

During the 2005 Divisional Playoff game between The Denver Broncos and The New England Patriots, Bronco player Champ Bailey intercepted Tom Brady around the goal line (see the circled B). He ran the ball nearly all the way to other goal line. Ben Watson of the New England Patriots (see the circled W) chased after Champ and tracked him down just before the other goal line.

In the image below, each hash mark is equal to one yard: note too the the field is $53\frac{1}{3}$ yards wide.



a. How can you use the diagram and the Pythagorean Theorem to find approximately how many yards Ben Watson ran to track down Champ Bailey?

b. Use the Pythagorean Theorem to find approximately how many yards Watson ran in this play.

c. Which player ran further during this play? By approximately how many more yards?

IM Commentary

Students need to reason as to how they can use the Pythagorean Theorem to find the distance ran by Ben Watson and Champ Bailey. The focus here should not be on who ran a greater distance, but on seeing how you can set up right triangles to apply the Pythagorean Theorem to this problem. Students must use their measurement skills and make reasonable estimates to set up the triangle and correctly apply the Theorem. According to our estimations and provided solutions both players ran about the same distance: we have used the tick marks on the football field to estimate these vertical and horizontal distances to the nearest yard.

Teachers should be prepared to help students with these estimations, particularly for Ben Watson. Careful counting of the small hashmarks running along the back of the end zone is one way to estimate how far he has run across the field. Another method would be to measure this distance with small ticks on a piece of paper and then line this paper up with the well marked yard lines on the football field. From the mathematical point of view, it is interesting to note that a few yards more or less in this direction have relatively little impact on the final result and this will hopefully come out as students will make different estimates. The students are likely to make slightly different estimations than those provided in the solution: this in turn will result in one player running a slightly greater distance than the other. If this happens, the teacher should engage the students in a discussion explaining their estimates and solution methods.

Students may engage in many of the CCSS standards for mathematical practices while working on this task. Consider giving students time either individually or in pairs to start the problem on their, initially with little guidance. By doing so you are allowing students the opportunity to engage in math practice one: “make sense of problems and persevere in solving them.” When students realize they can solve the problem by modeling with right triangles in order to apply the Pythagorean Theorem they are engaging in math practice number four: “model with mathematics.” There is a small range of lengths that could have been run by each player. This leaves the door open for a few different possible solutions. Consider allowing students to communicate their process and solutions. Doing so will engage students in math practice three: “construct viable arguments and critique the reasoning of others.”

Many professional athletes (football players, soccer players, and tennis players for example) must develop a very fine sense of geometry in order to know when and where to meet the ball (or opposing player in this case). This comes through experience. In this case, if Ben Watson had aimed to catch Champ Bailey at the 50 yard line he would have failed and Bailey would have scored a touchdown. The teacher may wish to have students make calculations for this case as well to see how much further Ben Watson would have to run (compared to Champ Bailey) if they were to meet at the 50 yard line.

This task submitted by Brian Marks and Leslie Lewis at Yummymath.com.

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Solution

a. The triangle formed by Watson’s run, the sideline and the perpendicular line from Watson to the sideline, make a right triangle. You could use the Pythagorean Theorem to find out approximately how far Watson ran. You would have to estimate how much of the 160 feet (53 1/3 yard) width of the field represented his vertical starting point. You can get a good estimate using the marks going along the end zone.

b. Looking at Ben Watson's position on the football field, it appears as if he has run between 90 and 92 yards up the field in order to reach Champ Bailey at the one yard line. Counting the number of hashmarks at the back of the end zone between Ben Watson's position and the top of the end zone, there appear to be between 9 and 12. Disregarding the $\frac{1}{3}$ of a yard in the football field width (because Champ Bailey was within the field when Watson tackled him), this means that Watson ran between 53-12=41 and 53-9= 44 yards across the field. We can now apply the Pythagorean theorem to estimate how far Ben Watson has run: the straight line path from Watson to the one yard line where he meets Bailey is the hypotenuse while the two legs are a vertical line going from Watson's initial position down to the side line where he meets Bailey and a horizontal line going all the way up the sideline where Bailey ran. For our lower end estimate for the distance Watson has run, these legs are 41 yards and 90 yards respectively. Applying the Pythagorean theorem gives

$$\begin{aligned}(90\text{ yards})^2 + (41\text{ yards})^2 &= (\text{run distance})^2\\ 8,100\text{ yards}^2 + 1,681\text{ yards}^2 &= (\text{run distance})^2\\ 9781\text{ yards}^2 &= (\text{run distance})^2\end{aligned}$$

Using these lengths Watson's run distance would be approximately 99 yards or about 297 feet.

Applying this same reasoning to obtain a high end reasonable estimate for the right triangle that models Watson's run, we get a right triangle with legs of 44 yards and 92 yards. This gives the following equation for Watson's run:

$$\begin{aligned}(92\text{ yards})^2 + (44\text{ yards})^2 &= (\text{run distance})^2\\ 8,464\text{ yards}^2 + 1,936\text{ yards}^2 &= (\text{run distance})^2\\ 10,400\text{ yards}^2 &= (\text{run distance})^2\end{aligned}$$

Using these lengths Watson's run distance would be approximately 102 yards or about 306 feet. So a reasonable estimate for the distance of Watson's run would be between 99 and 102 yards or 297 and 306 feet.

c. The same process can be applied to find out how far Bailey has run though here the ambiguity in how far Bailey has run across the field is far less because he starts off very close to the sideline. Champ Bailey started from inside the Bronco's end zone and ran to the Patriot's 1-yard line. According to the picture, Bailey started from roughly one to three yards behind the goal line in the end zone. He also started about three or four yards from the sideline. Just as we did for Ben Watson, we can create a right triangle and use the hypotenuse to model Bailey's run: the legs for the run are the short line from Bailey's position in the end zone to the sideline and the long line all the way up the field to the one yard line where he is tackled. For a low end estimate for the length of this hypotenuse we use legs of lengths 3 yards and 100 yards respectively:

$$\begin{aligned}(100\text{ yards})^2 + (3\text{ yards})^2 &= (\text{interception run distance})^2\\ 10,000\text{ yards}^2 + 9\text{ yards}^2 &= (\text{interception run distance})^2\\ 10,009\text{ yards}^2 &= (\text{interception run distance})^2\end{aligned}$$

Using these lengths Bailey's run distance would be approximately 100 yards or 300 feet.

On the high end a good estimate for the length of the hypotenuse that models Bailey's run would be:

$$\begin{aligned}(102 \text{ yards})^2 + (4 \text{ yards})^2 &= (\text{interception run distance})^2\\ 10,404 \text{ yards}^2 + 16 \text{ yards}^2 &= (\text{interception run distance})^2\\ 10,420 \text{ yards}^2 &= (\text{interception run distance})^2\end{aligned}$$

Using these lengths Bailey's run distance would be approximately 102 yards or 306 feet. So our estimates show that Bailey ran between 100 and 102 yards.

The range in possible run lengths for each player is similar. Both players ran roughly the same distance give or take a yard. Depending on how each student measures or estimates the lengths of the triangle it is possible that they might find that Watson ran a greater distance, that Bailey ran a greater distance or that they ran the same distance. The focus here should be on the process of modeling the problem with right triangles, how we came to the lengths of the right triangles, how we computed the distances as well as communicating and critiquing the reasoning of others.

It is interesting to note that although we have applied the same technique to estimate the distance each player has run, the range of values is only two yards for Bailey while it is three for Watson. This is because Bailey is so close to the sideline that the estimate for this distance does not influence the overall estimate for how far he has run. For Watson, the estimates for both the horizontal and the vertical distance impact the overall estimate for how far he has run.