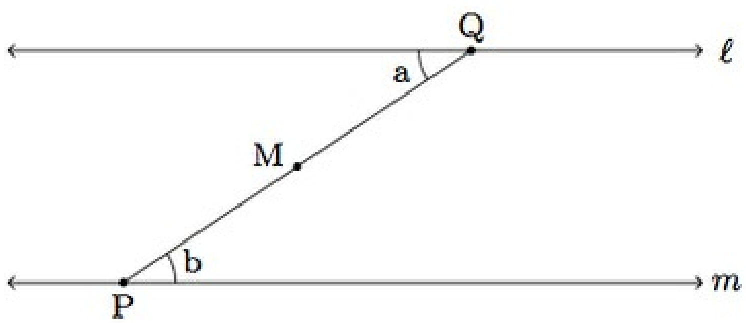
# 8.G Congruence of Alternate Interior Angles via Rotations

Alignments to Content Standards: 8.G.A.5

## Task

Suppose $\ell$ and $m$ are parallel lines with $Q$ a point on $\ell$ and $P$ a point on $m$ as pictured below:




Also labelled in the picture is the midpoint $M$ of $\overline{PQ}$ and a pair of angles. Explain why rotating the picture about $M$ by 180 degrees demonstrates that angles $a$ and $b$ are congruent.

## IM Commentary

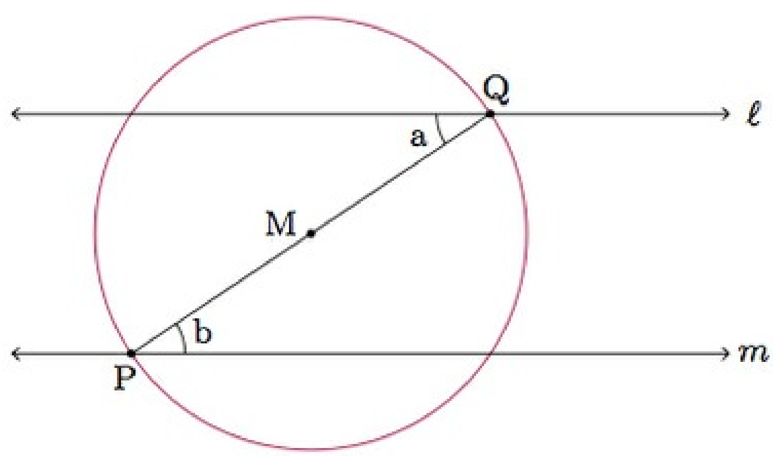
This goal of this task is to experiment with rigid motions to help visualize why alternate interior angles (made by a transverse connecting two parallel lines) are congruent: this result can then be used to establish that the sum of the angles in a triangle is 180 degrees (see http://www.illustrativemathematics.org/illustrations/1501). Students will explore the impact of rotation by 180 degrees on lines in a carefully chosen setting. Teachers should expect informal arguments as students are only beginning to develop a formal understanding of parallel lines and rigid motions. This task provides a good opportunity to explore with geometry software (if available) or with physical manipulatives. Students working on this problem will engage in MP5 ''Use Appropriate Tools Strategically'' whether they use geometry software or physical manipulatives.

This task is very closely related to the euclidean parallel postulate. To see why, note that lines $\ell$ and $m$ are parallel by assumption. Suppose $k$ is a line through $M$ parallel to $\ell$. This 180 degree rotation maps $k$ to itself and so the images of $m$ and $\ell$ are parallel to $k$. The 180 degree rotations of $m$ and $\ell$ are parallel to $k$ and pass through $Q$ and $P$ respectively. The parallel postulate says that there is only one line through $Q$ (respectively $P$) parallel to $k$ and this is $\ell$ (respectively $m$). This reasoning implies that the rotation maps $m$ to $\ell$ and $\ell$ to $m$.

Edit this solution

## Solution

The point $M$ is the center of the rotation and so it does not move. We next examine the impact of the rotation on $P$ and $Q$ and for this it is helpful to draw the circle with center $M$ and radius $|MP| = |MQ|$:




The rotation is through 180 degrees or half of a circle so it takes each point on the circle to its ''opposite,'' that is a point $A$ will map to the point $B$ so that $\overline{AB}$ is a diameter (and similarly $B$ will map to $A$). Applying this reasoning to our picture, we can see that the 180 degree rotation will interchange $P$ and $Q$.

Next we need to study what the 180 degree rotation does to $\ell$ and $m$. Since rotation by 180 degrees takes parallel lines to parallel lines this means that rotation by 180 degrees takes $\ell$ and $m$ to a pair of parallel lines that pass through the points $P$ and $Q$. Using the supplied application (or experimenting by hand or with other software), we can check that the new set of parallel lines is still $\ell$ and $m$ where $\ell$ has taken the place of $m$ and vice versa. This rotation interchanges the angles marked $a$ and $b$ so they are congruent.