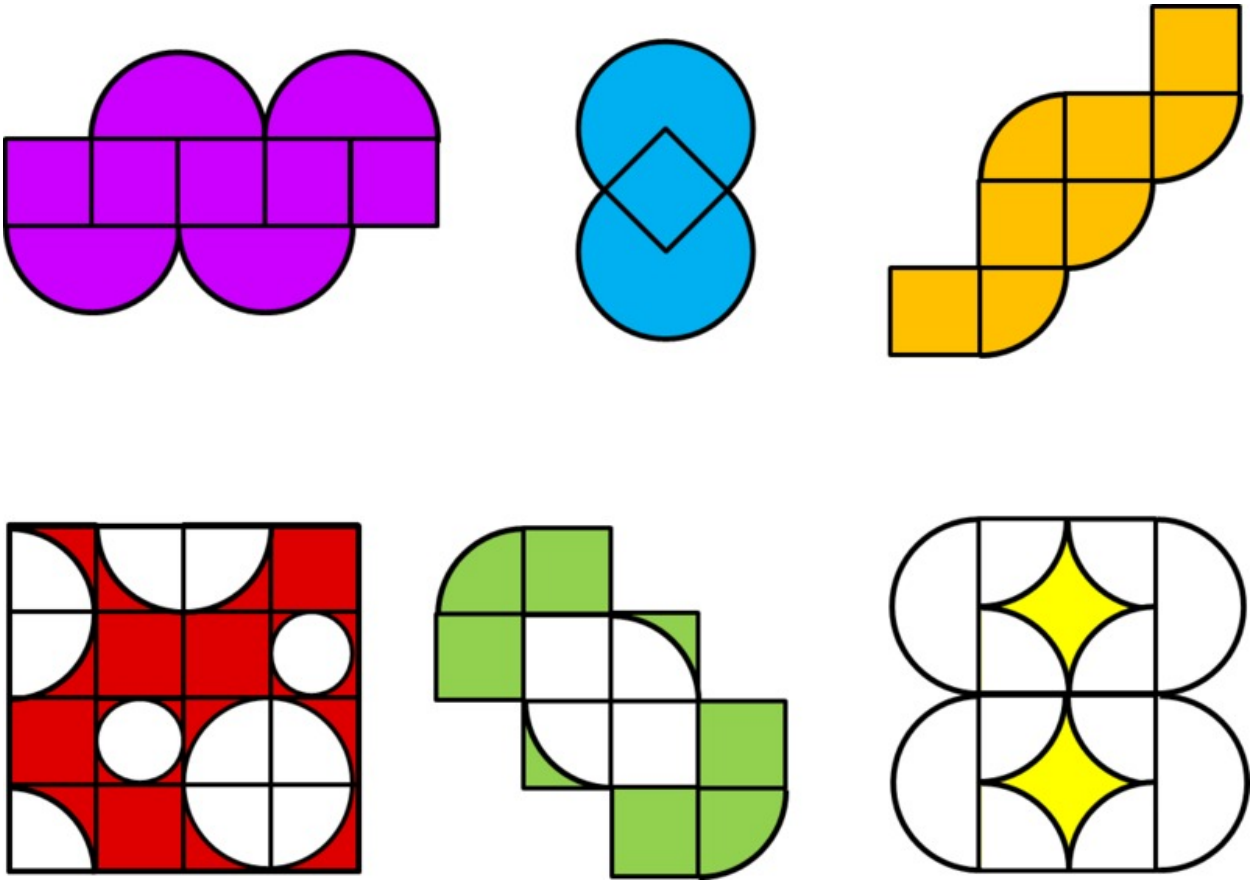
7.G Designs

Alignments to Content Standards: 7.G.B.4

Task

Find the area and perimeter of the colored part of each of the six figures below.



The purple, blue, orange, red, and green figures are composed of small squares with side-length 1 unit and curves that are an arc of a circle. The squares in the yellow figure are larger than the others and have a side-length of 2 units.

IM Commentary

The purpose of this task is for students to find the area and perimeter of figures composed of squares and fractions of circles. The first three figures only require addition of areas, while the last three require students to subtract the area of one region from another. Also, the red and green figures have more complex boundaries because they have "holes" in addition to the "outside" edges. The way to think about the perimeter of these kinds of figures is as the length of the boundary, where the boundary is the part of the figure that touches a colored region on one side and a white region on the other.

This task assumes students understand what area and perimeter are (which they first study in grade 3) and know how to find the areas and perimeters of squares and circles. It is meant as an instructional task and students who work in pairs or small groups can benefit from sharing their strategies for finding the area and perimeter of each figure.

Students will likely need some help understanding what is meant by the perimeter of the red and green figures, and the teacher might consider one of two approaches for providing it. One approach is to explain what is meant by the perimeter before the students start, showing, for example, two concentric circles with the region between them shaded in and explaining that the perimeter is the length of all the "edges" of the figure that border a shaded part and an unshaded part of the figure. Another approach is to let the students discuss for themselves what they think is meant by the perimeter of these figures when they get to it, stopping work on the tasks momentarily when everyone is ready to discuss it, and facilitating a conversation. If the students don't settle on it for themselves, they can be told what the convention is in cases like these where the figure has "holes." since the definition of perimeter is not obvious in these cases, allowing students to think about what might be a reasonable definition before giving it to them allows them to make sense of problems and persevere in solving them (MP1).

Edit this solution

Solution

All exact answers below are accompanied by a decimal approximation to help make the values more meaningful and allow for a check that the answers are reasonable.

- The purple figure is composed of 5 squares each with an area of one unit ^{2} and four half-circles with a radius of 1 unit. Putting two half circles together creates a whole circle with radius 1 unit which has an area of $\pi \cdot 1^2$ unit^2. Thus, the area of the purple figure is $5 + 2\pi \approx 11.28$ unit^2.

To find the perimeter of the purple figure, note that the boundary is composed of 4 half-circles with a radius of 1 unit and 4 segments of length 1 unit. Two half circles have a total length of $2\pi \cdot 1$ unit so the purple figure has a perimeter of $4 + 4\pi \approx 16.57$ units.

- The square in the blue figure has an area of 1 unit^2. The two arcs are both $\frac{3}{4}$ of a circle with radius 1 unit, so together their area is $\frac{3}{2}\pi \cdot 1^2$ unit^2. Thus, the area of the blue figure is $1 + \frac{3}{2}\pi \approx 5.71$ unit^2.

To find the perimeter of the blue figure, note that the boundary is composed of 2 arcs that are $\frac{3}{4}$ of a circle with a radius of 1 unit. These arcs have a total length of $\frac{3}{2} \cdot 2\pi \cdot 1$ unit so the purple figure has a perimeter of $3\pi \approx 9.42$ units.

- There are 4 squares in the orange figure and 4 quarter-circles. Thus, the area of the orange figure is $4 + \pi \approx 7.14$ unit^2.

To find the perimeter of the orange figure, note that the boundary is composed of 4 arcs that are $\frac{1}{4}$ of a circle and 8 segments of length 1. Thus, the orange figure has a perimeter of $8 + 2\pi \approx 14.28$ units.

- To find the area of the red figure, note that there are 16 squares which contain 1 large circle with a radius of 1 unit, two small circles with a radius of $\frac{1}{2}$ unit, 2 half large circles, and 1 quarter large circle that are removed from the area of the squares. Thus, the area of the red figure is

$$16 - \pi - 2 \cdot \frac{1}{4}\pi - 2 \cdot \frac{1}{2}\pi - \frac{1}{4}\pi$$

or $16 - \frac{11}{4}\pi \approx 7.36$ unit^2.

To find the perimeter of the red figure, note that the boundary is composed of 10 segments of length 1 unit and $2\frac{1}{4}$ circles of radius 1 unit and 2 circles of radius $\frac{1}{2}$ unit. Thus the perimeter is

$$10 + 2.25 \cdot 2\pi + 2 \cdot 2\pi \cdot \frac{1}{2}$$

or $10 + 6.5\pi \approx 30.52$ units.

- To find the area of the green figure, note that we can move the two quarter-circles to make complete square units in the middle. Thus, the area is 6 unit^2.

To find the perimeter of the green figure, note that the boundary is composed of 16 segments of length 1 unit and 4 quarter-circles with radius 1 unit. Thus the perimeter is $16 + 2\pi \approx 22.28$ units.

- To find the area of the yellow figure, note that we can subtract the area of a circle with radius 1 unit from a square with area 4 to get the area of one yellow "star." So the area is

$2 \cdot (4 - \pi) = 8 - 2\pi \approx 1.7$ unit^2.

To find the perimeter of the yellow figure, note that the boundary is composed of 8 quarter-circles of radius 1. Thus the perimeter is $2 \cdot 2\pi \approx 12.57$ units.