Illustrative Mathematics

8.F US Garbage, Version 1

Alignments to Content Standards: 8.F.A.1

Task

The following table shows the amount of garbage that was produced in the US each year between 2002 and 2010 (as reported by the EPA).

t **(years)**	2002	2003	2004	2005	2006	2007	2008	2009	2010
G **(million tons)**	239	242	249	254	251	255	251	244	250

Let's define a function which assigns to an input t (a year between 2002 and 2010) the total amount of garbage, G, produced in that year (in million tons). To find these values, you can look them up in the table.

a. How much garbage was produced in 2004?

b. In which year did the US produce 251 million tons of garbage?

c. Does the table describe a linear function?

d. Draw a graph that shows this data.

IM Commentary

Standard 8.F.1 states that students should "understand that a function is a rule that assigns to each input exactly one output." Often students think that a function has to be defined by an *algebraic* rule and that working with functions consists of plugging numbers into a formula. In this task, the rule of the function is more conceptual: We assign to a year (an input) the total amount of garbage produced in that year (the corresponding output). Even if we didn't know the exact amount for a year, it is clear that there will not be two different amounts of garbage produced in the same year. Thus, this makes sense as a "rule" even though there is no algorithmic way to determine the output for a given input except looking it up in the table.

Many situations can be described with a function even though there is no algebraic rule that defines it. Since functions are so powerful, we would like to analyze these situations with the tools that we have available when working with functions (e.g. associating inputs and outputs and graphing in 8th grade and function notation and rates of change in later years). This task presents an opportunity to explore a more general and non-algebraic view of functions.

Note that the details of this problem were chosen carefully. For one thing, the domain is naturally discrete (that is, it is hard to make sense of $G(2003.5)$) so there is no reason to think about values other than those that appear in the table. Later, students will be working with tables that only show some input-output pairs for a given function; sometimes they will be asked to find an algebraic expression that could extend the function to a larger domain. Here all input-output pairs for this function are contained in the table and there is no need to determine whether the function naturally extends to a larger domain. Also, the "rule" of the function was stated very explicitly to help students expand their notion of a rule to be more compatible with the more general idea of a function that they will encounter in later grades.

This task is appropriate as an example for instruction or for assessment.

Edit this solution

Solution

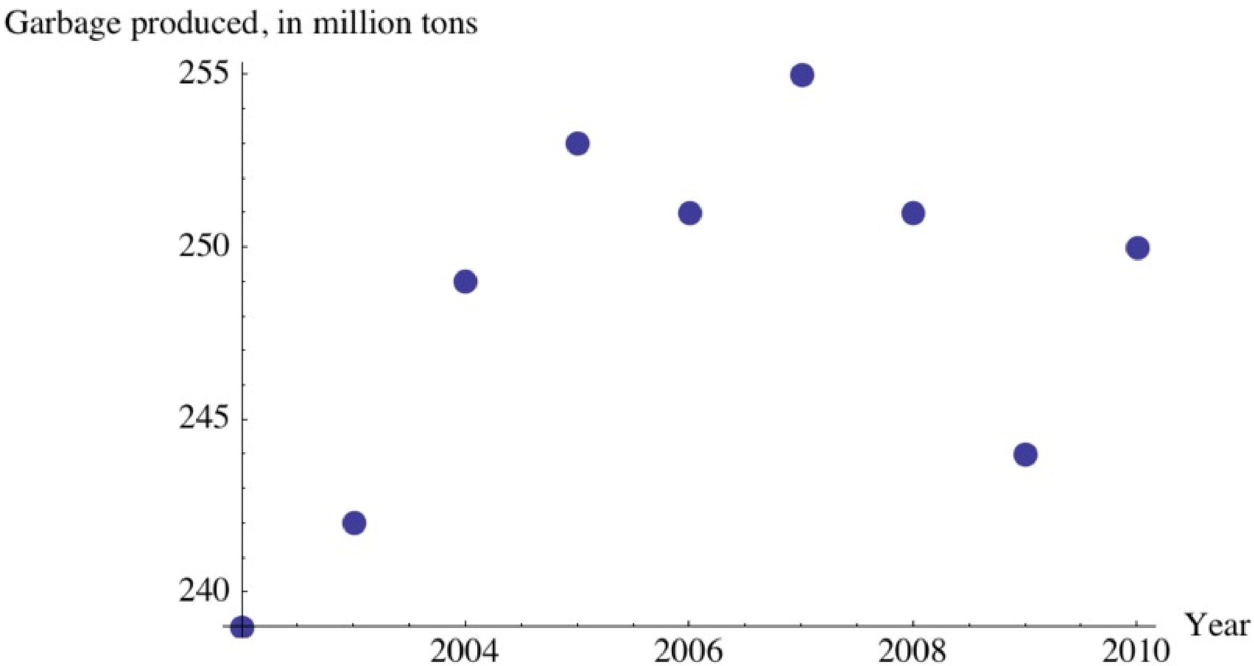
a. Finding $t = 2004$ in our table, we see that 249 million tons of garbage was produced in 2004.

b. Finding $G = 251$ in our table, we see that the US produced 251 million tons of garbage in two years that are listed in the table, 2006 and 2008.

c. For this function to be linear equal changes in input values have to correspond to equal changes in output values. Since our input values, t, are given for every year, we would want our output values, G, to be increasing by the same amount for each year.

We can see that from 2002 to 2003, G changed from 239 to 242 million tons, so it increased by 3 million tons. However, from 2003 to 2004, G changed from 242 to 249 million tons, meaning it increased by 7 million tons. From 2005 to 2006 the amount of garbage even decreased by 3 million tons. Therefore, our function is not linear.

d. By plotting the points given in the table, we arrive at the following graph, which also shows that the function is not linear because the points do not all lie on a line.



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