8.F Foxes and Rabbits

Alignments to Content Standards: 8.F.A.1

Task

Given below is a table that gives the populations of foxes and rabbits in a national park over a 12 month period. Note that each value of t corresponds to the beginning of the month.

t, month	1	2	3	4	5	6	7	8	9	10	11	12
R, number of rabbits	1000	750	567	500	567	750	1000	1250	1433	1500	1433	1250
F, number of foxes	150	143	125	100	75	57	50	57	75	100	125	143

a. According to the data in the table, is F a function of R? Is R a function of F? Explain.

b. Are either R or F functions of t? Explain.

IM Commentary

There is a natural (and complicated!) predator-prey relationship between the fox and rabbit populations, since foxes thrive in the presence of rabbits, and rabbits thrive in the absence of foxes. However, this relationship, as shown in the given table of values, cannot possibly be used to present either population as a function of the other. This task emphasizes the importance of the "every input has exactly one output" clause in the definition of a function, which is violated in the table of values of the two populations. Noteworthy is that since the data is a collection of input-output pairs, no verbal description of the function is given, so part of the task is processing what the "rule form" of the proposed functions would look like.

The predator-prey example of foxes and rabbits is picked up again in F-IF Foxes and Rabbits 2 and 3 where students are asked to find trigonometric functions to model the two populations as functions of time.

This task could be used early on when functions are introduced. It illustrates examples of functions as well as relationships that are not functions. It could also be used as an assessment item.

This task is adapted from "Functions Modeling Change", Connally et al, Wiley 2007.

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Solution

a. The key is understanding that a function is a rule that assigns to each input exactly one output, so we will test the relationships in question according to this criterion:

For the first part, that is, for F to be a function of R, we think of R as the input variable and F as the output variable, and ask ourselves the following question: Is there a rule, satisfying the definition of a function, which inputs a given rabbit population and outputs the corresponding fox population. The answer is no: We can see from the data that when $R = 1000$, we have one instance where $F = 150$, and another where $F = 50$. Since this means that a single input value corresponds to more than one output value, F is not a function of R. In the language of the problem's context, this says that the fox population is not completely determined by the rabbit population; during two different months there are the same number of rabbits but different numbers of foxes.

Similarly, we can see that if we consider F as our input and R as our output, we have a case where $F = 100$ corresponds to both $R = 500$ and $R = 1500$, two different outputs for the same input. So R is not a function of F: There are two different months which have the same number of foxes but two different numbers of rabbits.

b. Letting t, months, be the input, we can clearly see that there is exactly one output R for each value of t. That is, the rule which assigns to a month t the population of rabbits during that month fits our definition of a function, and so R is a function of t. By the same reasoning F is also a function of t. Again, in the context of the situation it makes sense that at any given point in time, there is a unique number of foxes and a unique number of rabbits in the park.