

# **N-RN, 8-NS Calculating the square root of 2**

Alignments to Content Standards: 8.NS.A N-RN.B

## **Task**

Jessica evaluates  $\sqrt{2}$  on her calculator which shows a value of

$$1.4142136.$$

She then writes

$$\sqrt{2} = 1.4142136.$$

- a. Is Jessica correct? Explain.
- b. Explain, in terms of the structure of the expression  $(1.4142136)^2$ , why it can not be equal to 2.
- c. Use the reasoning of part (b) to explain why  $\sqrt{2}$  can not be equal to any terminating decimal.

## **IM Commentary**

It is important in many situations to understand the level of accuracy of a calculator. Calculators round answers off so that they can display the best approximation possible on the screen. In the case of  $\sqrt{2}$ , an irrational number, it is impossible to display its full decimal expansion as this is infinite and non-repeating. The calculator displays the closest seven (or eight or nine) digit decimal number to  $\sqrt{2}$ .

This task is intended for instructional purposes so that students can become familiar and confident with using a calculator and understanding what it can and cannot do. This task gives an opportunity to work on the notion of place value (in parts (b) and (c)) and also to understand part of an argument for why  $\sqrt{2}$  is not a rational number. The argument is not complete as it is only shown here that  $\sqrt{2}$  does not have a terminating decimal expansion. For students familiar with working in bases other than base 10, however, the argument can be made general: if  $\sqrt{2} = \frac{p}{q}$  for whole numbers  $p$  and  $q$  then it has terminating "decimal" in base  $q$ . The argument of part (c), however, can be generalized to show that this is not possible.

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## Solution

a. It cannot be the case that  $\sqrt{2} = 1.4142136$  because  $\sqrt{2}$  is an irrational number while

$$1.4142136 = \frac{14142136}{10000000}$$

is a rational number. If students do not know that  $\sqrt{2}$  is an irrational number, then the reasoning of (b) and (c) below provides an alternative means of seeing why  $\sqrt{2}$  is not equal to *any* terminating decimal expansion. Developing an understanding of this reasoning is the goal of this task.

b. If we think about the multiplication algorithm to compute  $(1.4142136)^2$ , the last digit in its decimal expansion will be a 6, coming from the product of the two 6's in the one ten millionths' place (seven decimal places). This 6 will be in the one hundred trillionths' place (14 decimal places) and so this number can not be equal to 2.

c. The reasoning for part (b) can be applied for any number of decimal places. If we take a decimal number (with some non-zero digits to the right of the decimal) whose last digit is a 1, then its square will also have a last digit of 1 (and of course the last digit will be to the right of the decimal), and hence this value cannot equal 2. The argument is identical for any non-zero last digit of the proposed decimal number. We include below a table showing the last digit of a decimal number and the last digit of its square:

Last digit of terminating decimal number $x$	Last digit of terminating decimal number $x^2$
1	1

2	4
3	9
4	6
5	5
6	6
7	9
8	4
9	1

Notice that 0 is not listed in the table as a possible last digit of a terminating decimal. This is because for a terminating decimal, the last digit is understood to be the last non-zero digit: any extra zeroes may be removed from the decimal.



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