Illustrative Mathematics

8.NS Identifying Rational Numbers

Alignments to Content Standards: 8.NS.A.1

Task

Decide whether each of the following numbers is rational or irrational. If it is rational, explain how you know.

a. $0.33\overline{3}$

b. $\sqrt{4}$

c. $\sqrt{2} = 1.414213\dots$

d. 1.414213

e. $\pi = 3.141592\dots$

f. 11

g. $\frac{1}{7} = 0.\overline{142857}$

h. $12.34565656\overline{56}$

IM Commentary

The task assumes that students are able to express a given repeating decimal as a fraction. Teachers looking for a task to fill in this background knowledge could consider the related task "8.NS Converting Decimal Representations of Rational Numbers to Fraction Representations ."

There is a lot of interesting mathematics behind deciding questions about irrationality. There are a variety of arguments demonstrating that $\sqrt{2}$ is irrational (some of which would be quite accessible to a motivated middle school student), the first of which were discovered somewhere around the 5th century BC. And yet the irrationality of π was not proven until 1761, over *two millenia* later! Students who complete the task will probably be very close to being able to articulate the statement that a number is rational if and only if its decimal expansion is eventually periodic, in which case they could be posed problems like showing that the number

$$0.123456789101112131415161718192021....$$

is irrational. Note that even understanding the *statement* that $\sqrt{2}$ *equals* $1.414213\ldots$ is non-trivial, and partly addresses the part of the standard that says “Understand informally that every number has a decimal expansion."

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Solution

a. Since

$$0.33\overline{3} = \frac{1}{3}$$

$0.33\overline{3}$ is a rational number.

b. Since

$$\sqrt{4} = 2 = \frac{2}{1}$$

$\sqrt{4}$ is a rational number.

c. $\sqrt{2} = 1.414213\ldots$ is not rational. In eighth grade most students know that the square root of a prime number is irrational as a "fact," but few 8th grade students will be able to prove it. There are arguments that 8th graders can understand if they are interested.

d. Since

$$1.414213 = \frac{1414213}{100000},$$

1.414213 is a rational number.

e. $\pi = 3.141592\ldots$ is not rational. In eighth grade most students know that π is irrational as a "fact." The proof of this is quite sophisticated.

f. Since

$$11 = \frac{11}{1}$$

11 it is rational.

g. $\frac{1}{7} = 0.\overline{142857}$ is already written in a way that makes it clear it is a rational number, although some students might say it is irrational, possibly because the repeating part of the decimal is longer than many familiar repeating decimals (like $\frac{1}{3}$).

h. We have

$$12.3456565656\overline{56} = 12.34 + .00\overline{56} = \frac{1234}{100} + \frac{56}{9900} = \frac{1234 \cdot 99 + 56}{9900} = \frac{122222}{9900},$$

which is certainly rational.