7.NS Equivalent fractions approach to non-repeating decimals

Alignments to Content Standards: 7.NS.A.2.d

Task

Malia found a "short cut" to find the decimal representation of the fraction $\frac{117}{250}$. Rather than use long division she noticed that because $250 \times 4 = 1000$,

$$\frac{117}{250} = \frac{117 \times 4}{250 \times 4} = \frac{468}{1000} = 0.468.$$

a. For which of the following fractions does Malia's strategy work to find the decimal representation?

$$\frac{1}{3}, \frac{3}{4}, -\frac{6}{25}, \frac{18}{7}, \frac{13}{8} \quad \text{and} \quad -\frac{113}{40}.$$

For each one for which the strategy does work, use it to find the decimal representation.

b. For which denominators can Malia's strategy work?

IM Commentary

This task is most suitable for instruction. The purpose of the task is to get students to reflect on the definition of decimals as fractions (or sums of fractions), at a time when they are seeing them primarily as an extension of the base-ten number system and may have lost contact with the basic fraction meaning. Students also have their understanding of equivalent fractions and factors reinforced.

If students need help connecting this method with that of long division, they can be asked to perform long division when the denominator is a power of ten.

The denominators identified in the second part, namely numbers which are factors of powers of ten (or, equivalently, numbers for which 2 and 5 are the only prime factors) are in fact the only ones whose decimal expansions terminate when the fraction is in reduced form. While the problem does not ask for this fact, it should be shared and can be explained readily: A terminating decimal is equal to $\frac{a}{10^n}$. In reduced form, the denominator must be a quotient - and thus a factor - of 10^n.

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Solution

a.
- The strategy does not work for $\frac{1}{3}$ because there are no multiples of 3 which are powers of 10.
- Because $4 \times 25 = 100$, $\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$.
- $-\frac{6}{25} = -\frac{24}{100} = -0.24$.
- The strategy does not work for $\frac{18}{7}$ because there are no multiples of 7 which are powers of 10.
- $\frac{13}{8} = \frac{13 \times 125}{8 \times 125} = \frac{1625}{1000} = 1.625$.
- $-\frac{113}{40} = -2\frac{37}{40} = -2 + (-\frac{37 \times 25}{40 \times 25}) = -2 + (-\frac{825}{1000}) = -2.825$.

b. The strategy can work for any denominator which is a factor of a power of 10. In this case one can multiply the numerator and denominator by the complementary factor (that is, the quotient of that power of 10 by the denominator) to obtain a fraction with denominator equal to that power of 10. Such fractions are represented by terminating decimals.