7.NS Differences and Distances

Alignments to Content Standards: 7.NS.A.1

Task

Conner and Aaron are working on their homework together to find the distance between two numbers, a and b, on a number line. Conner counts the units between the numbers, while Aaron subtracts the least number from the greatest. While both methods can give the correct answer, Conner and Aaron do not always apply them correctly.

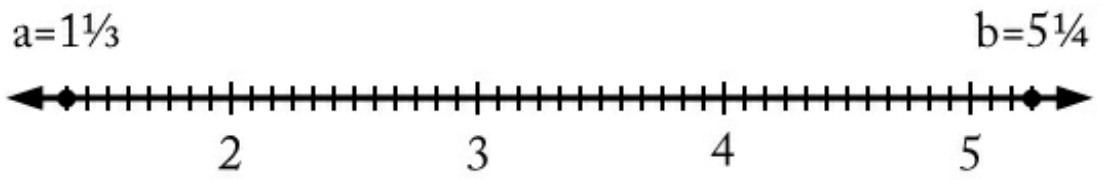
a. In the first question $a = 1 \frac{1}{3}$ and $b = 5 \frac{1}{4}$.

Conner finds the difference $b-a$.

$$\begin{align} b - a & = 5 \frac{1}{4} - 1 \frac{1}{3}\\ &= \frac{21}{4} - \frac{4}{3} \\ &= \frac{63}{12} - \frac{16}{12} \\ &= \frac{47}{12}\\ &= 3 \frac{11}{12} \end{align}$$

So Conner says that the distance between the two points is $3 \frac{11}{12}$.

Aaron marks the two numbers on the number line and counts 3 whole units between them. Then he adds $\frac{1}{4}$ and $\frac{1}{3}$ to account for the additional fractional distances. Since $\frac13+\frac14=\frac{7}{12}$ he says the distance between the two numbers is $3 \frac{7}{12}$.



Which, if either of them, is correct? Find and correct any incorrect work.

b. In the second question, $a = -3 \frac{1}{3}$ and $b = 2 \frac{2}{5}$.

Conner finds the difference $b-a$.

$$\begin{aligned} b - a &= 2 \frac{2}{5}-3 \frac{1}{3}\\ &= \frac{12}{5} - \frac{10}{3} \\ &= \frac{36}{15} - \frac{50}{15} \\ &= \frac{14}{15} \end{aligned}$$

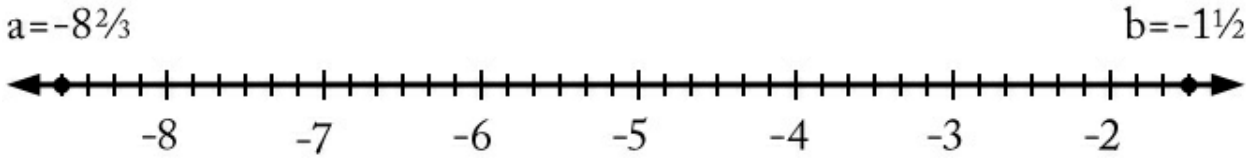
So Conner says that the distance between the two points is $\frac{14}{15}$.

Aaron marks the two numbers on the number line and counts 5 whole units between them. Then he adds $\frac{1}{3}$ and $\frac{2}{5}$ to account for the additional fractional distances. Since $\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$ he says that the distance between the two numbers is $5 \frac{11}{15}$.

Which, if either of them, is correct? Find and correct any incorrect work.

c. After talking with each other and understanding their mistakes, Aaron and Conner each obtain the correct answer with their preferred method when $a = -8 \frac{2}{3}$ and $b = - 1 \frac{1}{2}$.

Show how each student might have arrived at his answer.



IM Commentary

The purpose of this task is to help students connect the distance between points on a number line with the difference between the numbers. Students should already understand that this is true for positive numbers; see 2.MD.B.6, 3.MD.A.1, and 4.MD.A.2. Students should also have experience in earlier grades representing signed numbers on a number line; see 6.NS.C.6. This task assumes that students are familiar with the idea that differences between integers correspond to distances between them on the number line and asks them to analyze a situation involving non-integer quantities. The task could be used to launch a discussion about why this is true for any two numbers on the number line. It could also be used to segue into a discussion about seeing that subtracting a number is the same as adding it's opposite by representing sums and differences on a number line. In the process of looking at these two ways of finding the distance, students are asked to identify and correct common errors when these methods are employed, which encourages them to engage in MP.3, Construct viable arguments and critique the reasoning of others.

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, only one practice connection will be discussed in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This task helps illustrate Mathematical Practice Standard 3, “Construct viable arguments and critique the reasoning of others.” The teacher might display this task to the class and allow individuals to read and work through the problems. A whole group discussion could then be used to verbalize the correct reasoning from the flawed reasoning. The teacher could then direct the students to create their own formal arguments, using the task and progressing from the pictorial referents to their symbolic representations. Typical guiding questions are: “Is this way of finding the solution most effective? Why or why not?” “What connections can you make between

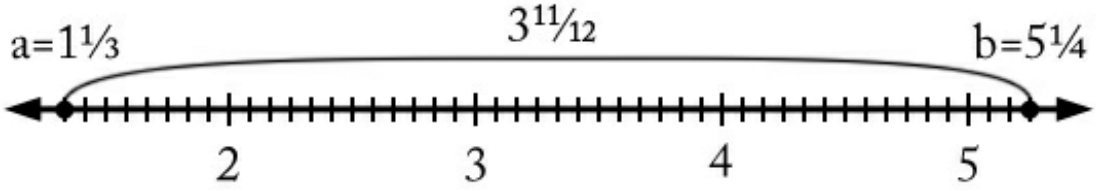
the pictorial and symbolic representation?" "What mathematical evidence would support your solution?"

Edit this solution

Solution

a. Conner's answer - and application of his method - are correct.

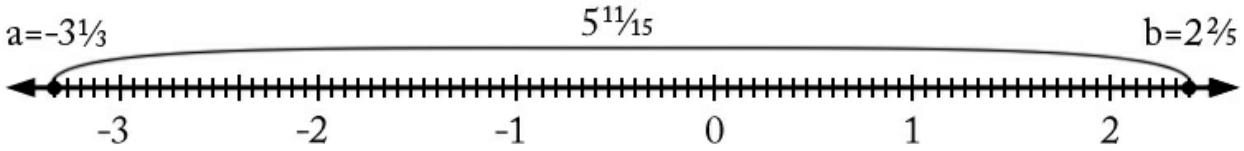
Aaron correctly identifies 3 whole units between 2 and 5, along with $\frac{1}{4}$ of a unit between 5 and $5 \frac{1}{4}$. But between $1 \frac{1}{3}$ and 2 are $\frac{2}{3}$ of a unit, so by his method one should be adding 3 and $\frac{1}{4}$ and $\frac{2}{3}$ to get $3 \frac{11}{12}$.



b.

Aaron's answer - and application of his method - are correct.

Conner makes two errors in subtraction. The biggest error is taking $\frac{12}{5} - \frac{10}{3}$ rather than $\frac{12}{5} - \frac{-10}{3}$. This correct difference is equal to $\frac{12}{5} + \frac{10}{3} = \frac{36}{15} + \frac{50}{15} = \frac{86}{15} = 5 \frac{11}{15}$. Conner makes a second error after this incorrect set-up, subtracting the smaller number $\frac{36}{15}$ from the larger $\frac{50}{15}$, probably in order to get a positive answer. If he had performed the subtraction appropriately to get a negative number, he might have caught his first error.



c. Conner subtracts the least number from the greatest: $\begin{aligned} -1 \frac{1}{2} - (-8 \frac{2}{3}) &= 8 \frac{2}{3} - 1 \frac{1}{2}\\ &= \frac{26}{3} - \frac{3}{2}\\ &= \frac{52 - 9}{6}\\ &= 7 \frac{1}{6}\end{aligned}$ Aaron counts the units between the numbers.

There are 6 units between -8 and -2. There are also \frac{2}{3} and \frac{1}{2} of a unit distance between a and b and these integers. The total distance is 6 + \frac{4}{6} + \frac{3}{6} = 7 \frac{1}{6}.