Unit 3: Normal Distribution, Confidence Intervals, and Hypothesis Testing

Content Area: **Mathematics**
Course(s): **Statistics**
Time Period: **April**
Length: **13 Weeks**
Status: **Published**

Unit Overview

This unit will present properties of a normal distribution, discuss its applications, and explain how a normal distribution curve can be used as an approximation to other distributions, such as the binomial distribution. The unit will also explain statistical procedures for estimating population mean, proportion, variance, and standard deviation. To conclude the unit, hypothesis testing and two specific statistical tests, used for hypotheses concerning means: the *z* test and the *t* test, will be presented.

Transfer

Students will be able to independently use their learning to...

- **Emphasize the characteristics of the theoretical normal distribution and show that it can be used as a model for real-life variables that are approximately normally distributed while knowing that no real-world distribution is perfectly normal.**
- **Apply the concept of making estimates to real life problems**
- **Relate hypothesis testing to real life problems/applications**

For more information, read the following article by Grant Wiggins.

http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60

Meaning

Understandings

Students will understand that...

- A normal distribution can be used to describe a variety of variables, such as heights, weights, and temperatures because the deviations from a normal distribution are very small.

- As a sample size increases without limit, the shape of the distribution of the sample means taken with replacement from a population will approach a normal distribution (this is called the central limit theorem).
- A normal distribution can be used to approximate other distributions, such as a binomial distribution, as long as two conditions are met ($np \geq 5$ and $nq \geq 5$).
- Estimation is an important aspect of inferential statistics and estimations of parameters of populations are accomplished by selecting a random sample from that population and choosing and computing a statistic that is the best estimator of the parameter.
- A decision-making process for evaluating claims about a population is called statistical hypothesis testing and that there are two types of statistical hypotheses for each situation: the null and the alternative hypotheses.
- There are two methods used to test hypotheses: the traditional method and the *P*-value method

Essential Questions

Students will keep considering...

- Is the bell-curve real?
- How is the area under the bell-curve related to the standard deviation?
- Does an ogive give the same information as a normal distribution?
- How is the area under a non-bell-curve related to standard deviation, median and mean?
- How can the area under the bell-curve be determined more accurately than by estimating?
- How can we tell if a distribution is normal and why would we care?
- Can we describe the shape of a distribution made up of many means from samples of the same size even without knowing what the population's distribution's shape is like?
- How large should the sample be in order to make an accurate estimate?
- How is the *t* distribution similar to the standard normal distribution and how is it different?
- How do you know whether the null hypothesis should be rejected?
- How does the *P*-value method for testing hypotheses differ from the traditional method?

Application of Knowledge and Skill

Students will know...

Students will know...

- When a distribution is symmetric, negatively or left-skewed, or positively or right-skewed.
- The properties of a normal distribution in order to solve problems involving distributions that are approximately normal.
- How to transform an original variable to a standard normal distribution variable using a formula in order to solve application problems.

- Which formula is to be used to gain information about an individual data value when the variable is normally distributed versus to gain information about a sample mean which the variable is normal distributed or when the sample size is 30 or more.
- The properties of a distribution of sample means.
- The steps for using the normal distribution to approximate a binomial distribution.
- The three properties of a good estimator.
- To make an accurate estimate, the size of the sample depends on three things: the margin of error, the population standard deviation, and the degree of confidence.
- That when the population standard deviation is known, the *z* value is used to compute the confidence interval but if the population standard deviation is unknown, the *t* value is used.
- The five steps for solving hypothesis-testing problems using the traditional method.
- The formula for the *z* test (in that it is the same as the formula shown in Chapter 6 for the situation where you are using a distribution of sample means).
- When to use the *z* test versus when to use the *t* test to test a hypothesis and that the procedure is the same except the *t* test uses Table F not E.
- The five steps for solving hypothesis-testing problems using the *P*-value method.

Students will be skilled at...

Students will be skilled at...

- Calculating the area under the standard normal distribution curve (to the left of any *z* value, to the right of any *z* value, and in between any two *z* values)
- Solving application problems using the standard normal distribution.
- Finding the probability that the mean of a randomly selected sample is a given value or the probability that sample mean is a certain given number.
- Approximating the binomial distribution of real life application problems using 6 steps (which include finding the mean and standard deviation for a binomial distribution, learned in chapter 5)
- Computing a confidence interval for a population mean.
- Constructing a confidence interval for a proportion, using a margin of error.
- Computing confidence intervals for variances and standard deviations using the chi-square distribution.
- Stating, for a given conjecture, the null and alternative hypotheses.
- Finding the critical values for specific α values and finding the critical value(s) for a given *t* test.
- Using diagrams to show the critical region (or regions) and using the traditional or *P*-value methods of hypothesis testing to complete real world problems.

Academic Vocabulary

Central Limit Theorem

Correction for Continuity

Skewed Distribution

Normal Distribution

Standard Normal Distribution

Sampling Distribution of Sample Means

Sampling Error

Standard Error of the Means

Symmetric Distribution

Z-value score

Assumptions

Chi-square distribution

Confidence Interval

Confidence Level

Degrees of Freedom

Estimation

Estimator, Unbiased Estimator, Consistent Estimator, Relatively Efficient Estimator

Interval and Point Estimates

Margin of Error

Proportion

t distribution

α (alpha)

Alternative Hypothesis, Statistical Hypothesis, Null Hypothesis

β (beta)

Critical or rejection region

Critical value

Hypothesis Testing

Level of Significance

Noncritical or Nonrejection Region

One-tailed test, Left-tailed Test, Right-tailed Test

P-value

Statistical test

Test value

t test

z test

Learning Goal 3.1

To create, describe, and interpret types of distributions in statistics

Target 3.1.1--(Level of Difficulty: 2 Comprehension)

SWBAT:

- Identify distributions as symmetric or skewed
- Identify the properties of a normal distribution
- Calculate the area under the standard normal distribution, given various z values, by hand and by using the graphing calculators

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MA.K-12.1	Make sense of problems and persevere in solving them.
MA.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

Target 3.1.2--(Level of Difficulty: 3 Analysis)

SWBAT:

- Find the probabilities for a normally distributed variable by transforming it into a standard normal variable
- Find specific data values for given percentages, using the standard normal distribution

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MA.K-12.1	Make sense of problems and persevere in solving them.
MA.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.

Target 3.1.3--(Level of Difficulty: 2 Comprehension)

SWBAT:

- State the properties of the sampling distribution of sample means.
- Use the central limit theorem to solve problems involving sample means for large samples

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MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.7.SP.A.2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.
MA.7.SP.B.4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

Target 3.1.4--(Level of Difficulty: 3 Analysis)

SWBAT:

- State the steps for using the normal distribution to approximate the binomial distribution.
- Rewrite problems using the continuity correction factor.

- Use the normal approximation to compute probabilities for a binomial variable.

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.

Learning Goal 3.2

To construct confidence intervals for data drawn from a normal distribution

Target 3.2.1--(Level of Difficulty: 3 Analysis)

SWBAT:

- State the three properties of a good estimator
- Construct the confidence interval for the mean when σ is known
- Determine the minimum sample size for finding a confidence interval for the mean

MA.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
MA.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MA.S-IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
MA.S-IC.B.6	Evaluate reports based on data.

Target 3.2.2--(Level of Difficulty: 3 Analysis)

SWBAT:

- State the characteristics of the *t* distribution
- Construct the confidence interval for the mean when σ is unknown

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MA.S-IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
MA.S-IC.B.6	Evaluate reports based on data.

Target 3.2.3--(Level of Difficulty: 3 Analysis)

SWBAT:

- Construct the confidence interval for a proportion
- Determine the minimum sample size for finding a confidence interval for a proportion

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MA.S-IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Target 3.2.4--(Level of Difficulty: 3 Analysis)

SWBAT:

- Construct a confidence interval for a variance and a standard deviation

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.

MA.S-IC.B.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Learning Goal 3.3

To test hypotheses

Target 3.3.1--(Level of Difficulty: 2 Comprehension)

SWBAT:

- Understand the definitions used in hypothesis testing
- Sate the null and alternative hypotheses
- Find critical values for the *z* test
- State the five steps used in hypothesis testing (traditional method)

MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MA.S-IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
MA.S-IC.B.6	Evaluate reports based on data.

Target 3.3.2--(Level of Difficulty: 4 Knowledge Utilization)

SWBAT:

- Calculate the critical *t* value by using Table F and finding the α level and the degrees of freedom
- Test means when σ is unknown, using the *t* test

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.

MA.K-12.7	Look for and make use of structure.
MA.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MA.S-IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

Target 3.3.3--(Level of Difficulty: 4 Knowledge Utilization)

SWBAT:

- Test proportions, using the *z* test

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
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MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
MA.S-IC.B.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
MA.S-IC.B.6	Evaluate reports based on data.

Summative Assessment

- Quizzes
- Tests
- Unit Test #4

21st Century Life and Careers

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP2.1	Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
CRP.K-12.CRP4.1	Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others' time. They are excellent writers; they master conventions, word choice, and organization, and use

	effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.
CRP.K-12.CRP11.1	Career-ready individuals take personal ownership of their own education and career goals, and they regularly act on a plan to attain these goals. They understand their own career interests, preferences, goals, and requirements. They have perspective regarding the pathways available to them and the time, effort, experience and other requirements to pursue each, including a path of entrepreneurship. They recognize the value of each step in the education and experiential process, and they recognize that nearly all career paths require ongoing education and experience. They seek counselors, mentors, and other experts to assist in the planning and execution of career and personal goals.
CAEP.9.2.12.C.2	Modify Personalized Student Learning Plans to support declared career goals.

Technology

TECH.8.1.12.E.CS1	Plan strategies to guide inquiry.
TECH.8.1.12.E.CS3	Evaluate and select information sources and digital tools based on the appropriateness for specific tasks.
TECH.8.1.12.E.CS4	Process data and report results.

Formative Assessment and Performance Opportunities

- Application
- Classwork
- Closures/exit tickets
- cooperative groups (stations, jigsaw activities)
- Do nows
- Excel
- graphing calculators
- Homework
- Participation/Discussion
- problem based learning
- Reading
- Teacher directed Q & A
- Teacher Observations
- Whitebaord/communicator responses

Differentiation / Enrichment

- 504 accommodations
- challenge problems
- Extension of the Chapter/Critical Thinking problems
- Heterogeneous grouping
- IEP's
- Layered curriculum (math menu)
- Projects
- scaffolding questions
- small group instruction
- use of technology

Unit Resources

- • Textbook: Elementary Statistics: A Step by Step Approach by Bluman, 8th Edition, Publisher McGraw- Hill, Copyright 2012
- EXCEL
- nctm website
- Ti 83-84 graphing calculators