

# Unit 2: Probability, Counting Principles, and Discrete Probability Distributions

Content Area: **Mathematics**  
Course(s): **Statistics**  
Time Period: **January**  
Length: **12 Weeks**  
Status: **Published**

## Unit Overview

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- This unit will introduce probability – its meaning, how it is computed, and how to evaluate it in terms of the likelihood of an event actually happening. Students will aware if the process involved in determining sample spaces and calculating probability or odds of a given event.
- This unit will show students to inspect a problem and determine whether the basic counting principle (multiplication of choices), law of permutations, or law of combinations should be used to calculate the number of outcomes. Students will expand upon this concept in their discussion of mutually exclusive and independent events, the addition and multiplication rules of probability, and conditional probability.
- This unit will explain probability distributions: binomial, multinomial, Poisson, and hypergeometric distributions.

## Transfer

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Students will be able to independently use their learning to...

- Apply the concepts of probability to determine the chance of real life events occurring.

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For more information, read the following article by Grant Wiggins.

[http://www.authenticeducation.org/ae\\_bigideas/article.lasso?artid=60](http://www.authenticeducation.org/ae_bigideas/article.lasso?artid=60)

## Meaning

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## Understandings

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Students will understand that...

- There are established rules of logic that govern the study of probability.

- There are three types of probability (classical, empirical, and subjective) and they are used to solve a variety of problems in business, engineering, and other fields.
- The addition rules of probability are slightly different depending on if the events are mutually exclusive or not.
- The multiplication rules of probability can be used to find the probability of two or more events (independent and dependent) that occur in sequence.
- There are three rules that can be used to determine the number of all possible outcomes for a sequence of events: the fundamental counting rule, the permutation rule, and the combination rule.
- A discrete probability distribution consists of the values a random variable can assume and the corresponding probabilities of these values.
- The mean, variance, and standard deviation for a probability distribution are computed differently from the mean, variance, and standard deviation for samples (learned in Chapter 3).
- There are many types of distributions in statistics: binomial distribution, multinomial distribution, the Poisson distribution, and the hypergeometric distribution.

## **Essential Questions**

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Students will keep considering...

- Is anything in nature truly random?
- What is the probability that you can make a correct decision given partial information and what are the ramifications for group decisions and democracy?
- Are all disjointed events independent?
- What are conditional probabilities?
- Can humans simulate a random process and why is this an important issue?
- Can standard deviations be added?
- Do we live in a binary world?
- When is estimating how many tries before success an issue?

## **Application of Knowledge and Skill**

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### **Students will know...**

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Students will know...

- The three types of probabilities and the difference between each type.
- How to find the probability of events using the addition rules of probability.
- How to find the probability of independent and dependent events using the multiplication rules of probability and conditional probability.

- The difference between the permutation rule and the combination rule and when to utilize each rule.
- The formulas for the three counting rules.
- The two requirements for a probability distribution and be able to determine whether a given distribution is a probability distribution.
- The four requirements for a binomial experiment, the notation for the binomial distribution, and the formula for binomial probability.
- The differences between the different types of distributions: binomial distribution, multinomial distribution, the Poisson distribution, and the hypergeometric distribution.

### **Students will be skilled at...**

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Students will be skilled at...

- Classifying statements as an example of classical probability, empirical probability, or subjective probability.
- Calculating the probability of getting a given result.
- Determining if events are mutually exclusive.
- Calculating the probability of mutually exclusive events and events that are not mutually exclusive using addition rules.
- Stating whether events are independent or dependent.
- Calculating the probability of two independent events and the probability of two dependent events.
- Solving factorials, permutations, and combinations by hand and using the graphing calculator.
- Solving more-complex probability problems using the counting rules and probability rules.
- Constructing a probability distribution.
- Calculating the mean, variance, standard deviation, and expected value for a probability distribution.
- Calculating the probability of success in a binomial experiment.
- Calculating the mean, variance, and standard deviation for the variable of a binomial distribution.
- Calculating the probabilities for outcomes of variables using other types of distributions

### **Academic Vocabulary**

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Classical probability

Combination

Complement of an event

Compound event

Conditional probability

Dependent events

Empirical probability

Equally likely events

Event

Fundamental counting rule

Independent events

Law of large numbers

Mutually exclusive events

Outcome

Permutation

Probability

Probability experiment

Sample space

Simple event

Subjective probability

Tree diagram

Venn diagram

Binomial distribution

Binomial experiment

Discrete probability distribution

Expected value

Hypergeometric distribution

Multinomial distribution

Poisson distribution

Random variable

## Learning Goal 2.1

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To inspect a problem and determine whether the basic counting principle (multiplication of choices), law of permutations, or law of combinations should be used to calculate the number of outcomes.

### Target 2.1.1--(Level of Difficulty: 2 Comprehension)

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SWBAT:

- Determine the sample spaces
  - Find the probability of an event, using classical probability or empirical probability
  - Find the complement of an event
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MA.K-12.2

Reason abstractly and quantitatively.

MA.K-12.8

Look for and express regularity in repeated reasoning.

MA.S-CP.A.1

Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

### Target 2.1.2--(Level of Difficulty: 3 Analysis)

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SWBAT:

- Determine which events are mutually exclusive and which are not
  - Evaluate the probability of compound events, using addition rules
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MA.K-12.1

Make sense of problems and persevere in solving them.

MA.K-12.4

Model with mathematics.

MA.K-12.7

Look for and make use of structure.

MA.S-CP.B.7

Apply the Addition Rule,  $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ , and interpret the answer in terms of the model.

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### Target 2.1.3--(Level of Difficulty: 3 Analysis)

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SWBAT:

- Define independent and dependent events
- Find the probability of compound events, using multiplication rules
- Calculate the conditional probability of any event using the formula for conditional probability

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MA.K-12.1

Make sense of problems and persevere in solving them.

MA.K-12.2

Reason abstractly and quantitatively.

MA.K-12.4

Model with mathematics.

MA.K-12.7

Look for and make use of structure.

MA.S-CP.A.2

Understand that two events  $A$  and  $B$  are independent if the probability of  $A$  and  $B$  occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

MA.S-CP.A.3

Understand the conditional probability of  $A$  given  $B$  as  $P(A \text{ and } B)/P(B)$ , and interpret independence of  $A$  and  $B$  as saying that the conditional probability of  $A$  given  $B$  is the same as the probability of  $A$ , and the conditional probability of  $B$  given  $A$  is the same as the probability of  $B$ .

MA.S-CP.B.8

Apply the general Multiplication Rule in a uniform probability model,  $P(A \text{ and } B) = [P(A)] \times [P(B|A)] = [P(B)] \times [P(A|B)]$ , and interpret the answer in terms of the model.

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### Target 2.1.4--(Level of Difficulty: 2 Comprehension)

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SWBAT:

- Find the total number of outcomes in a sequence of events, using the fundamental counting rule
  - Find the number of ways  $r$  objects can be selected from  $n$  objects, using the permutation rule
  - Find the number of ways  $r$  objects can be selected from  $n$  objects, using the combination rule
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MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.

### **Target 2.1.5--(Level of Difficulty: 3 Analysis)**

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SWBAT::

- Find the probability of an event, using the counting rules

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.6	Attend to precision.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.

### **Target 2.2.1--(Level of Difficulty: 3 Analysis)**

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SWBAT:

- Construct a probability distribution for a random variable

MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-MD.A.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.

### **Target 2.2.2--(Level of Difficulty: 3 Analysis)**

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SWBAT:

- Find the mean, variance, standard deviation, and expected value for a discrete random variable

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

**Target 2.2.3--(Level of Difficulty: 3 Analysis)**

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SWBAT:

- Find the exact probability for  $X$  successes in  $n$  trials of a binomial experiment
- Find the mean, variance, and standard deviation for the variable of a binomial distribution

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MA.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
MA.S-MD.A.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.

**Target 2.2.4--(Level of Difficulty: 3 Analysis)**

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SWBAT:

- Find probabilities for outcomes of variables of multinomial distributions, Poisson distributions, and hypergeometric distributions

MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.4	Model with mathematics.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.



MA.S-MD.B.5

Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

## Summative Assessment

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- Quizzes
- Tests
- Unit Test #3

## 21st Century Life and Careers

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CRP.K-12.CRP2.1

Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

CRP.K-12.CRP8.1

Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

CAEP.9.2.12.C.1

Review career goals and determine steps necessary for attainment.

CAEP.9.2.12.C.2

Modify Personalized Student Learning Plans to support declared career goals.

## Technology

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TECH.8.1.12.E.CS4

Process data and report results.

## Formative Assessment and Performance Opportunities

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- applications
- Classwork
- Closures/exit tickets
- cooperative groups(jigsaw activities, stations)
- Do nows
- Excel
- graphing calculator
- Homework

- participation/discussion
- problem based learning
- Readings
- Teacher observation
- Whiteboard/communicator responses

## **Differentiation / Enrichment**

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- 504 accommodations
- Extending the Concepts problems/Critical Thinking Challenges
- Heterogeneous groupings
- IEP's
- layered curriculum (math menu)
- Scaffolding questions
- small group instruction
- use of technology

## **Unit Resources**

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- • Textbook: Elementary Statistics: A Step by Step Approach by Bluman, 8th Edition, Publisher McGraw- Hill, Copyright 2012
- Excel
- nctm website
- TI 83-84 graphing calculator