

Unit 8: Data Analysis and Sampling

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **9 Weeks**
Status: **Published**

Module 18: Proportional Reasoning with Samples

Unit Rationale

Understanding how samples relate to populations is essential for students' success in data analysis and prepares them for more advanced statistical reasoning. In this module, students learn to apply proportional reasoning to estimate characteristics of a population based on sample data. They explore how proportional relationships underpin the connection between a representative sample and the entire group, and how samples can be used to make valid inferences.

This module is intentionally placed within the broader unit on data analysis and sampling to strengthen students' ability to reason about real-world situations where complete data is unavailable. Students extend their prior knowledge of ratios, rates, and proportions to interpret survey results, predict outcomes, and justify their conclusions based on proportional relationships. This work builds a bridge to future topics, such as probability, variability, and statistical analysis.

The real-world relevance of proportional reasoning with samples is emphasized through contexts such as polling, quality control, environmental studies, and market research. These examples not only deepen student engagement but also highlight the importance of mathematical reasoning in making informed decisions based on limited information.

Throughout the module, students will also engage with key Standards for Mathematical Practice, including:

MP2: Reason abstractly and quantitatively

MP4: Model with mathematics

MP6: Attend to precision

MP7: Look for and make use of structure

By the end of this module, students will be able to use proportional reasoning to make and justify predictions about populations, understand the importance of representative sampling, and communicate their findings clearly. This foundational skill set will support students as they progress into deeper statistical concepts and more sophisticated applications of mathematics in real-world contexts.

Essential Questions

- How can we use a sample to make predictions about a larger population?
- What makes a sample representative of a population?
- How do proportional relationships help us analyze and interpret sample data?

- Why is it important to consider sample size and method when making predictions?
- In what real-world situations might we need to make decisions based on samples rather than complete data?
- How can mathematical models help us communicate our conclusions based on sample data?

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)

- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning

- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)

- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks

- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Understand Representative Samples

Student Learning Intentions (WALT - We Are Learning To...)

- **WALT** define and understand what a representative sample is and why it's important in data analysis.
 - **WALT** identify key characteristics of a representative sample (randomness, sample size, diversity).
 - **WALT** use proportional reasoning to make predictions about a population based on a sample.
 - **WALT** analyze real-world situations to determine whether a sample is representative of a larger population.
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Student Success Criteria (I Can Statements)

- **I can** define a representative sample and explain its importance.
 - **I can** identify the characteristics that make a sample representative, such as randomness and diversity.
 - **I can** use proportional reasoning to predict population data from a sample.
 - **I can** assess whether a sample is representative and justify my reasoning with data.
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Instructional Strategies and Activities

1. Engage with a Real-World Problem:

- **Strategy:** Begin with a relevant real-world example, such as polling for a popular vote or gathering opinions on a new product. Engage students by asking them how they would collect data and if they could survey everyone.
- **Activity:** Show a video clip or news article about how companies or governments use samples to make decisions about large populations.

2. Direct Instruction:

- **Strategy:** Provide a clear definition of a representative sample and introduce vocabulary such as population, sample size, and bias. Use visuals (diagrams and charts) to illustrate the concept.
- **Activity:** Give a visual demonstration using a variety of sample groups and show which ones represent the population and which do not.

3. Guided Practice:

- **Strategy:** Walk through sample analysis as a class using a case study or a small dataset. Discuss how randomness, sample size, and diversity play a role in making the sample representative.
- **Activity:** In small groups, students review several sample data sets, analyzing them to decide if they are representative of a larger population, and justify their decisions using proportional

reasoning.

4. Collaborative Learning:

- **Strategy:** Have students work in pairs or small groups to discuss and solve sample analysis problems. Encourage students to share their reasoning with peers and challenge each other's ideas.
- **Activity:** Students create a chart on chart paper, comparing samples and explaining why one is more representative of the population than another.

5. Independent Practice:

- **Strategy:** Give students a worksheet with different scenarios where they identify whether a sample is representative and explain their reasoning.
- **Activity:** Students complete a set of sample analysis problems that require them to use proportional reasoning to predict the larger population's data.

Formative Assessments

1. Exit Ticket:

- At the end of the lesson, have students write down one thing they learned about representative samples and one thing they still have a question about. This will gauge their understanding of the main concept.

2. Quick Formative Assessment During Guided Practice:

- Use questioning techniques to assess whether students can identify a representative sample in real-time. Prompt them to explain their reasoning for their answers.

3. Peer Review:

- After the group activity, students can pair up and provide feedback on each other's analysis of sample data, ensuring that they are identifying key characteristics of a representative sample.

4. Worksheet Check:

- Review completed worksheets to check for understanding. Ensure students can explain why certain samples are or are not representative, and see if they are using proportional reasoning correctly in their predictions.

Instructional Materials and Resources

1. **Whiteboard/Smartboard:** For interactive demonstrations of proportional reasoning and visual aids.
2. **Student Notebooks:** To record definitions, examples, and observations during the lesson.

3. **Sample Data Sets:** Real-life or fictional data that students will analyze to determine if it is representative.
 4. **Chart Paper/Markers:** For group work on comparing samples and creating visual representations.
 5. **Online Graphing Tools (Optional):** Tools for visualizing data and making proportional predictions.
 6. **Worksheets:** Pre-prepared problems or scenarios that students will solve independently or in groups.
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Reflections and Suggested Modifications

1. Reflection:

- **What went well?**
Reflect on whether students grasped the key concept of a representative sample. Did the real-world examples engage them? Were the explanations of randomness and bias clear?
- **What could be improved?**
If students struggled with proportional reasoning or identifying characteristics of a representative sample, consider revisiting these ideas in future lessons. Maybe more hands-on practice with diverse data sets could help.

2. Suggested Modifications:

- **For Struggling Students:**
Provide additional guided practice with visual aids. Consider simplifying the data sets or breaking down the proportional reasoning into smaller, manageable steps.
- **For Advanced Students:**
Challenge them with more complex data sets or ask them to design their own sample collection method. Have them explore how changing sample size can affect the representativeness of a sample.
- **Alternative Assessment Options:**
If students need more support with the formative assessments, consider providing options like oral presentations instead of written exit tickets or collaborative assessments where they work with a peer to analyze data.

Lesson 2: Make Inferences from a Random Sample

Student Learning Intentions (WALT - We Are Learning To...)

- **WALT** define and understand random sampling and how it is used in data analysis.
- **WALT** make inferences about a population based on a random sample and use proportional reasoning.
- **WALT** evaluate whether inferences based on random samples are reasonable and accurate.

Student Success Criteria (I Can Statements)

- **I can** explain what random sampling is and why it is important in making valid conclusions.
 - **I can** make inferences about a population from a random sample.
 - **I can** use proportional reasoning to predict population characteristics based on a random sample.
 - **I can** assess whether inferences based on a random sample are reasonable and accurate.
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Instructional Strategies and Activities

1. Engage with a Scenario:

- **Strategy:** Start by presenting a scenario that uses a random sample.
Example: "A school wants to estimate the number of students who prefer a certain school activity. They randomly survey 50 students. Can we use this data to estimate the preferences of the entire school?"
- **Activity:** Encourage students to discuss how they would use this small sample to make conclusions about the entire population.

2. Introduce Random Sampling:

- **Strategy:** Explain the concept of **random sampling**, emphasizing its importance in making unbiased, fair inferences about a population. Compare random sampling to biased sampling.
- **Activity:** Show examples of both biased and random samples using simple real-world data. For example, ask a few students for their favorite foods (biased sample) versus randomly drawing names from a hat to gather opinions (random sample).

3. Making Inferences from a Sample:

- **Strategy:** Model how to use proportional reasoning to make predictions from random samples.
- **Activity:** Use a random sample of 20 students who were asked about their favorite subject. If 15 students liked math, use proportional reasoning to estimate how many students out of 100 might like math.
- Work through the problem together as a class:
"If 15 out of 20 like math, what's the proportion, and how would you use that to estimate the number of math lovers in a larger group?"

4. Guided Practice:

- **Strategy:** Provide students with a sample data set and guide them through the process of making inferences.
- **Activity:** In pairs, students will analyze a random sample and use proportional reasoning to

make an inference about the larger population.

For example, provide data about students' favorite books (survey of 30 students) and have students predict how many students in the school would prefer the same book.

5. Independent Practice:

- **Strategy:** Have students complete an individual worksheet with several scenarios involving random sampling. They will make inferences using proportional reasoning and assess the reasonableness of their conclusions.
- **Activity:** Provide scenarios where they must predict population characteristics (e.g., voting preferences, product ratings, etc.) based on a random sample.

6. Closing Discussion:

- **Strategy:** Encourage students to reflect on the importance of using random samples for making inferences and how proportional reasoning helps ensure the predictions are valid.
- **Activity:** Have students share their predictions with the class and discuss the accuracy of their inferences.

Formative Assessments

1. Exit Ticket:

- At the end of the lesson, have students write one thing they learned about random sampling and one way they used proportional reasoning to make an inference. This will provide insight into their understanding of the lesson.

2. Quick Poll or Survey:

- Conduct a quick survey where students vote on a simple topic (e.g., favorite class). Use this real-time data to discuss how they could make inferences about the whole class.

3. Group Reflection:

- After the guided practice, ask each group to present their inferences and how they used proportional reasoning. Provide feedback on their reasoning.

4. Worksheet Assessment:

- Review students' independent work to assess if they can use proportional reasoning correctly and make reasonable inferences from random samples.

Instructional Materials and Resources

1. **Whiteboard/Smartboard:** To visually demonstrate random sampling and proportional reasoning.

2. **Student Notebooks:** For notes, definitions, and examples.
 3. **Sample Data Sets:** For practicing making inferences (these could be surveys about favorite foods, books, activities, etc.).
 4. **Online Graphing Tool (Optional):** To help visualize proportions and make predictions.
 5. **Worksheets:** Pre-prepared problems or scenarios for students to practice independently.
 6. **Chart Paper/Markers:** For group work and visualizing inferences made from random samples.
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Reflections and Suggested Modifications

1. Reflection:

- **What went well?**
Reflect on whether students were able to grasp the concept of random sampling and use proportional reasoning to make reasonable inferences. Were they able to differentiate between biased and random sampling?
- **What could be improved?**
If students struggled with making inferences, consider providing more scaffolded examples or breaking down the steps of proportional reasoning further. More practice with real-world examples may help reinforce these skills.

2. Suggested Modifications:

- **For Struggling Students:**
Provide additional scaffolding, such as guided questions or visual aids to help break down how proportional reasoning works with random samples.
 - **For Advanced Students:**
Challenge them by introducing larger sample sizes or requiring them to predict outcomes for multiple categories or more complex data sets. Ask them to consider how changing the sample size might impact the accuracy of the inferences.
 - **Alternative Assessment Options:**
For students who need more time or support, provide alternatives like group presentations, where they explain their reasoning and predictions.
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Sample Lesson Flow

1. **Engage:** Begin with an interesting scenario that involves a random sample (5-10 minutes).
2. **Direct Instruction:** Explain random sampling, its importance, and proportional reasoning (10-15 minutes).
3. **Guided Practice:** Walk through examples and work through data sets with students (15 minutes).

4. **Independent Practice:** Allow students to apply the concept on their own with a worksheet (10-15 minutes).
5. **Closing Discussion:** Recap the lesson and reflect on how random sampling and proportional reasoning help make valid predictions (5 minutes).

Lesson 3: Make Inferences from Repeated Random Samples

Student Learning Intentions (WALT - We Are Learning To...)

- **WALT** explain how repeated random samples can provide more reliable inferences about a population.
 - **WALT** analyze variability in data from repeated random samples and use this information to make better inferences.
 - **WALT** use proportional reasoning to predict characteristics of a population and assess the reliability of those predictions.
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Student Success Criteria (I Can Statements)

- **I can** explain how repeated random sampling improves the accuracy of predictions about a population.
 - **I can** describe and interpret the variability in results from repeated random samples.
 - **I can** use proportional reasoning to make predictions and assess the reliability of my conclusions from repeated random samples.
 - **I can** recognize when repeated sampling shows a more reliable estimate than a single sample.
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Instructional Strategies and Activities

1. Engage with a Scenario:

- **Strategy:** Start by presenting a scenario where random sampling has been repeated multiple times.
Example: "A researcher wants to estimate the average height of students in a school by randomly selecting 10 students and recording their heights. How might the estimate change if the researcher repeats this process 10 times?"
- **Activity:** Have students predict how repeated samples might affect the reliability of the estimate.

2. Introduce Repeated Random Sampling:

- **Strategy:** Introduce the concept of **repeated random sampling** and explain how it helps reduce errors or bias in making inferences about a population.
- **Activity:** Demonstrate how repeated sampling from a population can lead to more accurate conclusions by comparing inferences made from one sample vs. multiple samples. Show how variability in data decreases with more samples.

3. Understanding Variability:

- **Strategy:** Teach students about **variability** in repeated random samples and how this variability affects the precision of inferences.
- **Activity:** Conduct a class activity where students collect data from repeated random samples of a population (e.g., heights, shoe sizes, etc.). Then, compare the results of each sample to see how close the estimates are to each other.
Discuss the role of variability and why repeated sampling helps refine the estimates.

4. Guided Practice:

- **Strategy:** Provide students with a series of random samples and have them analyze the variability in the results.
- **Activity:** Use a sample population (e.g., survey students about their favorite color or activity) and have students draw multiple random samples, calculate proportions for each sample, and compare the results.
- Discuss how the inferences might change if only one sample was used versus multiple samples.

5. Independent Practice:

- **Strategy:** Give students a set of repeated random samples and have them make inferences about the population. They should calculate averages and proportions, then analyze the variability in their results.
- **Activity:** Students can use a random number generator or simple methods like drawing names from a hat to simulate repeated sampling.
After collecting the data, students will calculate the mean or proportion for each sample and compare their estimates.

6. Closing Reflection:

- **Strategy:** Have students reflect on how repeated sampling improves the accuracy of their conclusions and why variability is an important concept in statistical analysis.
- **Activity:** Lead a class discussion on how the estimates from repeated samples compare to a single sample and why relying on just one sample might not give a reliable result.

Formative Assessments

1. Exit Ticket:

- At the end of the lesson, ask students to explain in one sentence how repeated sampling

improves the reliability of predictions.

2. **Sample Comparison:**

- After guided practice, have students compare the results of inferences made from one sample versus those made from multiple samples. This will allow you to assess their understanding of how variability impacts conclusions.

3. **Quick Poll or Survey:**

- Conduct a mini-survey in class (e.g., favorite sports, colors, etc.) and ask students to estimate the proportions of the class based on a single sample and multiple samples. Compare the results.

4. **Independent Worksheet:**

- Provide students with a worksheet that asks them to calculate estimates from multiple samples and interpret the variability. Review their work for understanding of variability and proportional reasoning.

Instructional Materials and Resources

1. **Whiteboard/Smartboard:** For illustrating how repeated random sampling reduces errors and shows variability.
2. **Student Notebooks:** For taking notes, calculating data, and reflecting on their learning.
3. **Random Sample Data Sets:** Use a population sample (e.g., random heights or test scores) to simulate repeated sampling.
4. **Graphing Tools:** For visualizing variability and comparing sample estimates (optional: online tools or apps).
5. **Worksheets:** Pre-prepared problems with repeated sampling scenarios and questions for reflection.
6. **Random Number Generator:** Or use other tools like drawing names or rolling dice to simulate random samples.

Reflections and Suggested Modifications

1. **Reflection:**

- **What went well?**
Reflect on whether students were able to understand the value of repeated random sampling and its impact on accuracy. Did they grasp the concept of variability and how it affects the reliability of their conclusions?
- **What could be improved?**
If students struggled with interpreting the variability or making inferences from repeated

samples, consider providing additional examples or visual aids to demonstrate how repeated sampling can help achieve a more reliable estimate.

2. Suggested Modifications:

- **For Struggling Students:**
Provide more scaffolded support by showing more examples of repeated random samples and discussing the results in-depth before moving to independent practice.
- **For Advanced Students:**
Challenge them to analyze more complex data sets or predict how the sample size might affect the variability and reliability of the inferences.
- **Alternative Assessment Options:**
If students struggle with the written format, consider having them present their findings verbally, discussing how variability and repeated sampling improve reliability.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support:**
 - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech:**
 - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals:**
 - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary:**
 - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking:**
 - The teacher presents information in a way that is easy for students to understand and remember.

Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.

- **Checking for Understanding:**

- It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.

- **Extra Time:**

- The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.

- **Oral Reading:**

- The teacher will read work aloud to students, which can include class work, tests, and literature circles.

- **Timers:**

- The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.
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Students with 504 Plans

- **Chunking:**

- The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.

- **Checking for Understanding:**

- Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.

- **Extra Time:**

- Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.
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Gifted & Talented Strategies

- **Extensions/Enrichments:**

- Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.

- **Modify/Change Activities:**

- Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.
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Students at Risk of School Failure

- **Directions or Instructions:**

- Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.

- **Peer Support:**

- Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.

- **Alternate or Modified Assignments:**

- Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).

- **Increase One-on-One Time:**

- Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.

- **Contracts:**

- A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.

- **Hands-On Tasks:**

- Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.

- **Tests/Assessments:**

- Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.

- **Seating:**

- Seat students near a helping peer or with quick access to the teacher. For students with hearing

or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example:**

- Fist to five: "How well do you understand what we talked about today?"
- Fist to five: "How well did I teach this today?"

- **Classroom Activity:**

- Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.
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Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies:**

- Color-coding ideas
- Learning vocabulary in student languages
- Visual and kinesthetic learning
- Representations of learning without words

- **Classroom Activity:**

- Multilingual Frayer Models for definitions or concepts
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Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example:**

- "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"

- **Classroom Activity:**

- Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.
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Incorporate the history of mathematics into lessons

- **Verbal Example:**

- "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"

- **Classroom Activity:**

- Learn about different bases and numerical ideas:
 - Base 2 (binary) and connections to computer programming
 - How the Yoruba of Nigeria used base 20
 - How the Mayans conceptualized the number 0 before the first recording of it
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Solicit student ways of thinking and processing

- **Verbal Example:**

- "How might you all go about this?"
- "What do you notice?"

- **Classroom Activity:**

- Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.
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Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example:**

- "How does this connect to what you've learned in the past?"
- "How can you use that knowledge today?"

- **Classroom Activity:**

- Learning webs that connect content
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Start with more complex math problems and scaffold as necessary

- **Verbal Example:**

- "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"

- **Classroom Activity:**

- When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.
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Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example:**

- "Show your thinking with words, pictures, symbols."
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Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example:**

- "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"
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Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example:**

- "Reflect on your day so far. What math have you already used today?"

- **Classroom Activity:**

- Community walks to engage with slope.
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Co-construct knowledge in the classroom

- **Verbal Example:**

- "Let's get into partners and do a think-pair-share. We will incorporate everyone's ideas and try

to synthesize them."

- **Classroom Activity:**

- Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.
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Choose problems that have complex, competing, or multiple answers

- **Verbal Example:**

- "Come up with at least two answers that might solve this problem."

- **Classroom Activity:**

- Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.

- **Classroom Activity:**

- **Deconstructed Multiple Choice:** Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.
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Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example:**

- "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"

- **Classroom Activity:**

- Error Analysis worksheets that highlight what is the right idea behind the mistake.
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Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example:**

- "What would a mathematician who is confused ask about this question?"

- **Classroom Activity:**

- After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective

questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example:**

- "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."

- **Classroom Activity:**

- Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).

- **Climate Change Example:**

- Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MATH.9-12.A.REI	Reasoning with Equations and Inequalities

21st Century Life and Career

CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Integration of Career Readiness. Life Literacies and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.

Integration of Computer Science and Design ThinkingNew Section

CS.9-12.8.1.12.AP.1 Design algorithms to solve computational problems using a combination of original and existing algorithms.

CS.9-12.8.1.12.AP.5 Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSLs for ELA, Social Studies, Science and/or Math

LA.RH.9-10.8	Assess the extent to which the reasoning and evidence in a text support the author's claims.
LA.RST.9-10.5	Analyze the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).
LA.RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
LA.RST.9-10.8	Determine if the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.

Module 19: Use Statistics and Graphs to Compare Data

Unit Rationale

In Module 19, students focus on using statistical methods and graphical representations to compare data. This

module builds on foundational concepts of data analysis and introduces students to more advanced techniques for interpreting and comparing data sets. It provides students with tools to analyze data trends, make informed decisions, and draw conclusions based on statistical evidence.

Understanding and comparing data is essential not only in mathematics but also in everyday life, where students encounter various forms of data in media, business, and personal decision-making. This module aims to equip students with the ability to use statistics and graphs to understand relationships between variables, draw meaningful conclusions from data sets, and communicate their findings clearly and accurately.

The module begins with a review of basic statistical concepts such as mean, median, and mode, and then moves into more complex forms of comparison, such as using range, interquartile range, and standard deviation to measure the spread and variation within data. Students will also explore different types of graphs, including bar graphs, histograms, line graphs, and box plots, learning how to choose the appropriate graph for the data being analyzed.

Through hands-on activities and real-world applications, students will develop a deeper understanding of how to use statistics and graphs to compare data, as well as the importance of interpreting data with caution. The module will encourage students to consider the implications of their conclusions and the potential biases or limitations of the data they are analyzing.

This module is crucial for students as they advance in their mathematical journey and will serve as a foundation for more complex topics, such as probability, inferential statistics, and data science. By the end of the module, students will have the tools to effectively compare data sets, critically evaluate statistical information, and use graphs and measures of central tendency and variation to support their analyses.

Key Standards for Mathematical Practice addressed in this module include:

- **MP4:** Model with mathematics
- **MP5:** Use appropriate tools strategically
- **MP6:** Attend to precision

Students will also develop critical thinking skills by evaluating how different types of data representations can highlight or obscure important features of a dataset, leading to better decision-making and more accurate conclusions in future studies and real-world situations.

Essential Questions

- **How can different statistical measures (mean, median, mode, range, etc.) be used to compare two or more data sets?**
- **What role do graphs (bar graphs, histograms, line graphs, box plots, etc.) play in representing and comparing data, and how do we choose the right type of graph for a given situation?**
- **How do we interpret the spread and variation in a data set, and why is it important to understand measures of variability (e.g., range, interquartile range, standard deviation)?**
- **How can we use statistical data to make informed decisions or predictions in real-world**

situations?

- **What are the limitations and potential biases in data collection and representation, and how do they affect the conclusions we draw from data?**
- **In what ways can comparing data sets help us identify trends, patterns, or relationships between different variables?**
- **How does understanding the difference between correlation and causation help us avoid incorrect conclusions when comparing data sets?**

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)

- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning

- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)

- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks

- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 3: Compare Means Using Mean Absolute Deviation and Repeated Sampling

Student Learning Intentions (WALT):

- **We are learning to** compare means using mean absolute deviation (MAD).
- **We are learning to** understand how repeated sampling affects the comparison of means and variability.

Student Success Criteria ("I can" statements):

- **I can** calculate the mean absolute deviation (MAD) for a data set.
 - **I can** compare the means of two data sets using MAD to determine which has less variability.
 - **I can** interpret the impact of repeated sampling on the means and MAD.
 - **I can** explain how repeated sampling helps us estimate the reliability of a data set and its variability.
-

Instructional Strategies and Activities:

1. Introduction to Mean Absolute Deviation (MAD) (15 minutes):

- Introduce the concept of **Mean Absolute Deviation (MAD)**, explaining that MAD measures the average distance between each data point and the mean. It helps us understand the spread of the data, showing how much the individual data points vary from the mean.
- Demonstrate the formula for MAD:

$$MAD = \frac{1}{n} \sum_{i=1}^n |x_i - \text{mean}|$$

Where x_i represents each data point and n = total data points

- Model the steps of calculating MAD using a small data set, going through each part of the formula to ensure students understand how to find the mean and how to calculate the absolute deviations.

2. Guided Practice: Comparing Means and MAD (20 minutes):

- Provide students with two data sets. Ask them to:
 1. Calculate the mean for each data set.

2. Calculate the MAD for each data set.
3. Compare the two means and explain which data set has the greater or lesser variability based on MAD.

- As students work, encourage them to think about how MAD helps us measure spread and why it's a useful tool for comparing data sets with the same or different means.

3. **Introduction to Repeated Sampling** (15 minutes):

- Explain the concept of **repeated sampling**, where we take multiple random samples from a population to understand the variability of the mean. The more times we sample, the more reliable our estimate of the mean becomes.
- Show an example of repeated sampling using a simulation or by taking multiple random samples from a small data set and calculating the mean and MAD for each sample. Discuss how the variability in the sample means can give us a better sense of the data's reliability and variability.

4. **Guided Practice: Comparing Means Using Repeated Sampling** (20 minutes):

- Provide students with a data set and ask them to simulate repeated sampling. They should:
 1. Take 5 random samples from the data set.
 2. Calculate the mean and MAD for each sample.
 3. Compare the variability in the means of the samples and analyze how the MAD changes with each sample.
- Encourage students to observe how the means converge and whether the MAD changes significantly with each additional sample.

5. **Independent Practice: Comparing Means Using MAD and Repeated Sampling** (20 minutes):

- Give students a worksheet with a larger data set. Have them perform the following tasks:
 1. Calculate the mean and MAD of the entire data set.
 2. Take 10 random samples from the data set, calculate the mean and MAD for each sample, and then compare the results.
 3. Write a brief conclusion about how repeated sampling impacts the variability and reliability of the mean.

6. **Class Discussion and Reflection** (10 minutes):

- Lead a class discussion where students reflect on how repeated sampling helps us understand the variability of data. Discuss questions such as:
 - "How does repeated sampling help us make more reliable conclusions about the data?"
 - "What does a larger MAD tell us about the data set?"

- "Why is it important to compare means and MAD when analyzing data?"
-

Formative Assessments:

- **Exit Ticket:** At the end of the lesson, give students an exit ticket with a data set. Ask them to:
 1. Calculate the mean and MAD.
 2. Compare the MAD of two different data sets and explain which one has more variability.
 3. Write a brief explanation of how repeated sampling can help estimate the true mean and reduce variability.
 - **Quick Check during Guided Practice:** As students work on comparing means and MAD, circulate and ask guiding questions like:
 - "What does a large MAD tell you about the data set?"
 - "How do you think repeated sampling might impact the reliability of your mean?"
 - "Do you notice any patterns in how the sample means change as you take more samples?"
-

Instructional Materials and Resources:

- Whiteboard or interactive board to demonstrate calculations and concepts.
 - Printed worksheets with data sets for practice.
 - Calculators for calculating the mean and MAD (optional).
 - Access to a random sampling tool or software (optional) to simulate repeated sampling.
-

Reflections and Suggested Modifications:

- **Reflection:** After the lesson, assess how well students understood the concept of MAD and how repeated sampling affects variability. If students struggled with calculating MAD or understanding the impact of repeated sampling, additional practice with simpler data sets or more guided explanations may be needed.
- **Modifications:**
 - **For struggling students:** Provide additional visual aids or examples to help them understand how to calculate MAD. Break the process down into smaller steps and provide a practice set with fewer data points.
 - **For advanced students:** Encourage them to explore how the number of samples affects the

accuracy of the mean and MAD. You could also introduce the concept of standard deviation and discuss how it compares to MAD for more advanced analysis.

By the end of this lesson, students will be able to compare means using MAD, understand how repeated sampling can provide more reliable estimates of the mean, and interpret the variability of data more effectively.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support:**

- The teacher provides auditory or written content to students in their native language.

- **Adjusted Speech:**

- The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

- **Visuals:**

- The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.

- **Front-Loading Vocabulary:**

- The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.
-

Special Education Students

- **Chunking:**

- The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.

- **Checking for Understanding:**

- It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.

- **Extra Time:**

- The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.

- **Oral Reading:**

- The teacher will read work aloud to students, which can include class work, tests, and literature circles.

- **Timers:**

- The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.
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Students with 504 Plans

- **Chunking:**

- The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.

- **Checking for Understanding:**

- Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.

- **Extra Time:**

- Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.
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Gifted & Talented Strategies

- **Extensions/Enrichments:**

- Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.

- **Modify/Change Activities:**

- Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.
-

Students at Risk of School Failure

- **Directions or Instructions:**

- Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.

- **Peer Support:**

- Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.

- **Alternate or Modified Assignments:**

- Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).

- **Increase One-on-One Time:**

- Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.

- **Contracts:**

- A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.

- **Hands-On Tasks:**

- Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.

- **Tests/Assessments:**

- Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.

- **Seating:**

- Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and

instruction

- **Verbal Example:**

- Fist to five: "How well do you understand what we talked about today?"
- Fist to five: "How well did I teach this today?"

- **Classroom Activity:**

- Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.
-

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies:**

- Color-coding ideas
- Learning vocabulary in student languages
- Visual and kinesthetic learning
- Representations of learning without words

- **Classroom Activity:**

- Multilingual Frayer Models for definitions or concepts
-

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example:**

- "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"

- **Classroom Activity:**

- Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.
-

Incorporate the history of mathematics into lessons

- **Verbal Example:**

- "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"

- **Classroom Activity:**

- Learn about different bases and numerical ideas:
 - Base 2 (binary) and connections to computer programming
 - How the Yoruba of Nigeria used base 20
 - How the Mayans conceptualized the number 0 before the first recording of it
-

Solicit student ways of thinking and processing

- **Verbal Example:**

- "How might you all go about this?"
- "What do you notice?"

- **Classroom Activity:**

- Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.
-

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example:**

- "How does this connect to what you've learned in the past?"
- "How can you use that knowledge today?"

- **Classroom Activity:**

- Learning webs that connect content
-

Start with more complex math problems and scaffold as necessary

- **Verbal Example:**

- "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"

- **Classroom Activity:**

- When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.
-

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example:**

- "Show your thinking with words, pictures, symbols."
-

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example:**

- "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"
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Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example:**

- "Reflect on your day so far. What math have you already used today?"

- **Classroom Activity:**

- Community walks to engage with slope.
-

Co-construct knowledge in the classroom

- **Verbal Example:**

- "Let's get into partners and do a think-pair-share. We will incorporate everyone's ideas and try to synthesize them."

- **Classroom Activity:**

- Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.
-

Choose problems that have complex, competing, or multiple answers

- **Verbal Example:**

- "Come up with at least two answers that might solve this problem."

- **Classroom Activity:**

- Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.

- **Classroom Activity:**

- **Deconstructed Multiple Choice:** Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.
-

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example:**

- "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"

- **Classroom Activity:**

- Error Analysis worksheets that highlight what is the right idea behind the mistake.
-

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example:**

- "What would a mathematician who is confused ask about this question?"

- **Classroom Activity:**

- After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.
-

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example:**

Interdisciplinary Connections: NJSLs for ELA, Social Studies, Science and/or Math

SOC.K-12.6	Language: System and structure, effective use, and vocabulary Engaging in Civil Discourse and Critiquing Conclusions
ELA.K-12.R.IT	Interactions Among Text Elements: By the end of grade 12, analyze how and why individuals, events, and ideas develop and interact over the course of a text.
SCI.9-12.5.1.12.C.a	Refinement of understandings, explanations, and models occurs as new evidence is incorporated.