Unit 7: Area and Volume

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **9 Weeks**
Status: **Published**

Module 16: Analyze Figures to Find Circumference and Area

Unit Rationale

Understanding how to calculate the **circumference** and **area** of geometric figures is an essential skill in middle school mathematics, laying the groundwork for more advanced topics in geometry and measurement. In this module, students will focus on the specific application of these calculations to circles and other two-dimensional shapes. By learning to apply formulas for **circumference** and **area**, students will gain the tools necessary to solve problems in a wide range of mathematical and real-world contexts.

This module serves as an important component of the 7th-grade accelerated curriculum, preparing students for more complex geometry concepts, such as volume, surface area, and geometric transformations. The skills developed in this unit are also foundational for subjects like algebra and trigonometry, where spatial relationships play a crucial role. By mastering the ability to calculate **circumference** and **area**, students will be equipped with the knowledge to tackle problems that involve measurements, design, and various practical applications.

The module emphasizes the real-world relevance of these mathematical concepts by providing scenarios in areas like construction, navigation, and design. These contexts make the learning more engaging and show students how geometry can be applied in everyday situations, such as determining the length of a fence needed to enclose a circular garden or the area of a circular track.

Throughout the module, students will engage with key Standards for Mathematical Practice, including:

- **MP2:** Reason abstractly and quantitatively
- **MP4:** Model with mathematics
- **MP7:** Look for and make use of structure

By the end of this module, students will not only be able to calculate the **circumference** and **area** of circles with confidence, but they will also understand how these calculations apply to real-world problems. This knowledge will provide a solid foundation for students as they progress in their mathematical studies, enhancing their ability to apply mathematics in various fields and contexts.

Essential Questions

- **How can the circumference and area of a circle be calculated using formulas?**
- **What is the relationship between the radius, diameter, and circumference of a circle?**
- **How do the formulas for area and circumference apply to real-world problems, such as finding**

the perimeter of circular paths or the area of circular fields?

- **Why is the constant π\piπ important in calculations involving circles, and how is it used in determining circumference and area?**
- **How can understanding the area of a circle help in solving practical problems like designing circular objects or determining space in circular areas?**
- **What is the significance of accurate measurements when calculating the circumference and area of circles, and how do rounding and estimation affect the results?**
- **How are the concepts of area and circumference related to other geometric figures, and how do these concepts extend to more complex mathematical problems?**

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Derive and Apply Formulas for Circumference

Learning Intentions (WALT)

- We are learning to derive and apply formulas for the **circumference** of circles.
- We are learning to understand the relationship between the **diameter** and **radius** of a circle and how this impacts the formula for **circumference**.

Success Criteria (I Can Statements)

- I can explain the relationship between the **diameter** and **radius** of a circle.
- I can derive the formula for the **circumference** of a circle and apply it to solve real-world problems.
- I can calculate the **circumference** of circles using both the radius and the diameter.
- I can use the constant π\piπ to find the **circumference** of a circle.

Instructional Strategies and Activities

Engage (10 minutes)

- **Launch Activity:**
  - Display a circular object (e.g., a round table, a bicycle wheel, or a circular clock).
  - Ask students how they could measure the **distance around** the object (i.e., the **circumference**).
  - Introduce the concept of **circumference** as the perimeter or boundary of a circle.
- **Guiding Question:**
  - "If we know the **diameter** of a circle, how could we use it to find the **circumference**?"
- **Think-Pair-Share:**
  - Students discuss in pairs how they might calculate the **circumference** of the object. Afterward, the class shares their ideas.

Explore (20 minutes)

- **Hands-On Activity:**
  - **Measure Circumference:**
    - Give students a circular object with a known **diameter** (or provide a worksheet with various circle measurements).
    - Have students measure the **diameter** of the object using a ruler or tape measure.
    - In small groups, students will physically measure the **circumference** of the object using a piece of string or measuring tape, then compare the measured **circumference** with the known **diameter**.
    - **Record Findings:** Each group records their measurements and observations.
- **Group Discussion:**
  - Bring the class together and ask:
    - "What is the relationship between the **diameter** and the **circumference** you measured?"
    - "Can we find a pattern in the measurements? What is the ratio of the **circumference** to the **diameter**?"

Explain (15 minutes)

- **Teacher-Led Derivation of the Formula:**
  - Explain that the **circumference** of a circle is always about 3.14 times its **diameter**.
  - Introduce the constant π\piπ (approximately 3.14), and state that the formula for the **circumference** is:

    C=π×dC = \pi \times dC=π×d

    - Where CCC is the **circumference**, and ddd is the **diameter**.
  - Show how this formula can also be written in terms of the **radius**:

    C=2πrC = 2 \pi rC=2πr

    - Where rrr is the **radius** (half of the **diameter**).
- **Visuals and Interactive Exploration:**
  - Use a visual representation (e.g., drawing or interactive tool) of a circle to demonstrate the formulas and the relationship between radius, diameter, and circumference.

Elaborate (15 minutes)

- **Practice Problems:**
  - Provide students with a set of practice problems where they calculate the **circumference** using

both **diameter** and **radius**. Problems may include:

- Calculate the **circumference** of a circle with a **diameter** of 10 cm.
- Find the **circumference** of a circle with a **radius** of 7 inches.
- A real-world application: "A circular park has a **diameter** of 50 meters. How much fencing is needed to go around the park?"

- **Group Work:**
  - Have students work in pairs or small groups to solve more complex word problems involving the **circumference** of circles in real-world contexts (e.g., circular tracks, wheels, or round tables).

Closure (5 minutes)

- **Exit Ticket:**
  - Ask students to complete a brief exit ticket where they:
    1. Define **circumference**.
    2. Solve for the **circumference** of a circle with a **radius** of 4 cm.
    3. Explain how the formula C=2πrC = 2\pi rC=2πr is derived from the relationship between the radius and the **diameter**.

Formative Assessments

- **Observation:**
  - Throughout the lesson, observe student participation in group discussions, problem-solving, and hands-on activities to gauge understanding.
- **Exit Ticket Responses:**
  - Review students' exit ticket responses to check for accurate understanding of the **circumference** formula and its application.
- **Practice Problems:**
  - Assess the completion and accuracy of practice problems and group work to determine how well students are able to apply the formulas.

Instructional Materials and Resources

- Circular objects for hands-on measurement (e.g., round tables, lids, wheels)

- String or measuring tape for finding **circumference**
- Rulers for measuring **diameters**
- Whiteboard and markers for modeling and problem-solving
- Exit tickets for closure activity
- **Interactive tools** (e.g., online graphing tools, virtual manipulatives) for visualizing circle properties and relationships

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Reflections and Suggested Modifications

Reflections (post-lesson):

- Which students had difficulty understanding the relationship between the **radius** and **diameter**?
- Did students grasp the concept of using π\piπ in the **circumference** formula?
- How effective was the hands-on activity for reinforcing the formula and its application?

Suggested Modifications:

- For students needing extra support, provide visual aids or interactive apps that allow them to experiment with different circle sizes.
- For students who finish early, offer enrichment problems involving mixed applications of **circumference** and **area**, or explore how **circumference** relates to more complex figures (e.g., composite shapes).

Lesson 2: Derive and Apply a Formula for the Area of a Circle

Learning Intentions (WALT)

- We are learning to derive and apply the formula for the **area** of a circle.
- We are learning to understand the relationship between the **radius** of a circle and its **area**, and how this impacts the formula for **area**.

Success Criteria (I Can Statements)

- I can explain how the **radius** of a circle is related to the **area**.
- I can derive the formula for the **area** of a circle and apply it to solve real-world problems.

- I can calculate the **area** of a circle using the radius.
- I can use the constant π\piπ to find the **area** of a circle.

Instructional Strategies and Activities

Engage (10 minutes)

- **Launch Activity:**
  - Show students a circle drawn on a piece of graph paper.
  - Ask them how they might find the **area** of the circle (encourage brainstorming: dividing it into smaller shapes, counting square units, etc.).
- **Guiding Question:**
  - "If we know the **radius** of a circle, how can we find the **area** of the circle?"
- **Think-Pair-Share:**
  - Students discuss in pairs how they might find the **area** of the circle. Afterward, the class shares their ideas.

Explore (20 minutes)

- **Hands-On Activity:**
  - **Estimate Area by Counting Squares:**
    - Provide students with a circle drawn on a grid or graph paper. Students will count the number of squares inside the circle to estimate the **area**.
    - Once they've estimated, have them compare their counts and discuss the limitations of this method (e.g., squares don't always perfectly fit the curve).
- **Introduce the Concept of Area Formula:**
  - Ask students if there is a way to generalize the method for any circle, regardless of size. Introduce the idea of approximating the circle by breaking it into smaller sections, leading to the formula for **area**.
- **Discussion:**
  - Guide the students to recognize that the **area** of a circle is proportional to the square of its **radius**. Write the general formula for **area** as:

    A=πr2A = \pi r^2A=πr2

    - Where AAA is the **area** and rrr is the **radius**.

Explain (15 minutes)

- **Teacher-Led Derivation of the Formula:**
  - Discuss the reasoning behind the formula A=πr2A = \pi r^2A=πr2 by showing how the **radius** influences the **area**. Use a visual representation of the circle and the grid method to illustrate that **area** is the amount of space inside the circle, which increases as the square of the radius.
- **Modeling the Formula:**
  - Explain how the area of a circle is related to the **radius** in a way similar to how the area of a square is related to the length of its sides. The difference is that the circle's **area** grows in proportion to the square of its **radius** rather than its linear dimension.

Elaborate (15 minutes)

- **Practice Problems:**
  - Provide students with practice problems that involve calculating the **area** of circles using the formula A=πr2A = \pi r^2A=πr2. Problems could include:
    - Find the **area** of a circle with a **radius** of 4 cm.
    - Calculate the **area** of a circle with a **radius** of 10 inches.
    - Solve a real-world problem: "A circular swimming pool has a **radius** of 12 meters. What is its **area**?"
- **Real-World Application:**
  - Students apply the formula to real-life contexts, such as finding the **area** of a circular garden, the **area** of a circular tablecloth, or the **area** of a circular pizza. Emphasize the importance of **area** in practical decision-making, such as determining how much material is needed for a circular object or how much paint is required for a circular surface.

Closure (5 minutes)

- **Exit Ticket:**
  - Ask students to complete an exit ticket where they:
    1. Define **area**.
    2. Solve for the **area** of a circle with a **radius** of 6 cm.
    3. Explain how the formula A=πr2A = \pi r^2A=πr2 is derived and how it relates to the **radius**.

Formative Assessments

- **Observation:**
  - Observe students during the hands-on activity and practice problems to assess their understanding of the **area** formula.
- **Exit Ticket Responses:**
  - Review the exit ticket responses to ensure students can correctly apply the **area** formula and explain its derivation.
- **Practice Problem Accuracy:**
  - Check the accuracy of students' solutions to practice problems, paying particular attention to their ability to use the **radius** to calculate **area**.

Instructional Materials and Resources

- Graph paper or grid paper for visualizing and estimating the **area** of a circle.
- Calculators for solving problems involving π\piπ.
- Whiteboard and markers for modeling the **area** formula.
- Exit tickets for closure activity.
- **Interactive tools** (e.g., online calculators or geometry tools) for visualizing the relationship between **radius** and **area**.

Reflections and Suggested Modifications

Reflections (post-lesson):

- Did students grasp the relationship between the **radius** and the **area** of a circle?
- Were students able to successfully apply the formula to real-world scenarios?
- How well did students understand the derivation of the formula A=πr2A = \pi r^2A=πr2?

Suggested Modifications:

- For students needing support, provide visual aids or interactive apps that allow them to experiment with the **area** of different circles.
- For enrichment, offer extension problems that involve the **area** of complex shapes, or challenge students to derive the formula for **area** by comparing it to other geometric shapes (such as a square or triangle).

Lesson 3: Areas of Composite Figures

Learning Intentions (WALT)

- We are learning to find the **area** of composite figures by breaking them into simpler shapes.
- We are learning to apply the formulas for the **area** of basic geometric shapes (rectangles, triangles, circles) to solve problems involving composite figures.

Success Criteria (I Can Statements)

- I can break down a composite figure into simpler shapes to find its **area**.
- I can apply the appropriate formulas for the **area** of rectangles, triangles, and circles to solve problems.
- I can calculate the **area** of composite figures accurately.

Instructional Strategies and Activities

Engage (10 minutes)

- **Launch Activity:**
  - Show a composite figure, such as a rectangle with a semicircle on top or a shape that consists of multiple basic shapes (like rectangles and triangles).
  - Ask students how they might find the **area** of the figure.
- **Guiding Question:**
  - "How can we break down complex shapes into simpler parts so we can find the total **area**?"
- **Think-Pair-Share:**
  - Have students discuss in pairs how they might approach finding the **area** of a composite figure. Afterward, invite a few students to share their strategies with the class.

Explore (20 minutes)

- **Hands-On Activity:**
  - **Break It Down:**
    - Provide students with a set of composite figures that are made up of rectangles,

triangles, and circles. Have them work in pairs to divide each figure into simpler shapes.

    - Students should then use the appropriate formulas to find the **area** of each individual shape and add them up to find the total **area** of the composite figure.

- **Guided Practice:**
  - Model a couple of examples on the board by breaking down composite shapes step by step, showing students how to use the formulas for **area** of basic shapes:
    - **Rectangle Area Formula:** A=l×wA = l \times wA=l×w
    - **Triangle Area Formula:** A=12×b×hA = \frac{1}{2} \times b \times hA=21×b×h
    - **Circle Area Formula:** A=πr2A = \pi r^2A=πr2
  - Ensure students understand how to calculate the **area** of each part and then add the areas together to get the total.

Explain (15 minutes)

- **Teacher-Led Explanation:**
  - Reinforce that composite figures can be decomposed into familiar geometric shapes. Show how the **area** of a composite figure is the sum of the **areas** of the individual shapes.
  - Use visual examples to highlight the process of decomposing a shape:
    - Example: For a composite shape that includes a rectangle and a triangle on top of it, find the **area** of the rectangle and the **area** of the triangle separately, then add them together.
- **Modeling:**
  - Work through an example with a composite figure that includes a rectangle and a semicircle. Break the figure into a rectangle and a circle, then apply the appropriate **area** formulas to find the total **area**.

Elaborate (15 minutes)

- **Practice Problems:**
  - Provide students with several composite figures and ask them to:
    - Break the figures into basic shapes (rectangles, triangles, circles).
    - Use the correct formulas to calculate the **area** of each individual shape.
    - Add the areas to find the **total area** of the composite figure.
  - Examples:

        - Find the **area** of a figure made up of a rectangle with a triangle on top.
        - Find the **area** of a figure made up of a rectangle with a semicircle attached to one side.
        - Calculate the **area** of a figure composed of a square and quarter-circle.
- **Real-World Application:**
    - Ask students to apply their knowledge of **area** to a real-world context. For example:
        - "A swimming pool is in the shape of a rectangle with a circular section at one end. Find the total area of the pool."
        - "A designer is creating a rug with a rectangular base and circular decoration. Find the **area** of the rug."

Closure (5 minutes)

- **Exit Ticket:**
    - Ask students to complete a quick exit ticket with one problem, where they:
        1. Break down a composite figure (such as a rectangle with a semicircle) into simpler shapes.
        2. Calculate the **area** of each shape.
        3. Add the areas to find the total **area** of the composite figure.

Formative Assessments

- **Observation:**
    - Observe students as they work through the hands-on activity and practice problems. Ensure they are breaking down composite shapes correctly and applying the right formulas for **area**.
- **Exit Ticket Responses:**
    - Review the exit ticket responses to check for correct understanding of how to find the **area** of composite figures.
- **Practice Problem Accuracy:**
    - Assess the accuracy of the students' calculations as they solve practice problems, particularly their ability to correctly identify and apply the appropriate formulas.

Instructional Materials and Resources

- Graph paper or grid paper to help visualize and decompose composite figures.
- Whiteboard and markers for modeling.
- Calculators for solving problems involving **pi** or large numbers.
- Pre-prepared composite figure problems for practice.
- Exit tickets for closure activity.

Reflections and Suggested Modifications

Reflections (post-lesson):

- Did students understand how to break composite figures into simpler shapes?
- Were students able to correctly apply the formulas for the **area** of basic shapes to find the **area** of composite figures?
- Were students able to effectively solve real-world problems involving **area**?

Suggested Modifications:

- For students who struggle, provide additional guidance by giving them composite figures that already have labels for the individual shapes, so they can focus on applying the formulas.
- For students who finish early, provide enrichment problems that include more complex composite figures (e.g., figures that require multiple steps or additional shapes).
- Offer interactive online geometry tools that help students visualize composite figures and calculate **areas**.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
  - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
  - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

- **Visuals**:
    - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary**:
    - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
    - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
    - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.
- **Extra Time**:
    - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.
- **Oral Reading**:
    - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
    - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
    - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
    - Teachers will continuously check for understanding, ensuring students with accommodations

comprehend the lesson content.

- **Extra Time**:
    - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
    - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
    - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
    - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.
- **Peer Support**:
    - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
    - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
    - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
    - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols,

encouraging accountability.

- **Hands-On Tasks**:
  - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.
- **Tests/Assessments**:
  - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:
  - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
  - Fist to five: "How well do you understand what we talked about today?"
  - Fist to five: "How well did I teach this today?"
- **Classroom Activity**:
  - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
  - Color-coding ideas
  - Learning vocabulary in student languages
  - Visual and kinesthetic learning

    - Representations of learning without words
- **Classroom Activity**:
    - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
    - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
- **Classroom Activity**:
    - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
    - "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
    - Learn about different bases and numerical ideas:
        - Base 2 (binary) and connections to computer programming
        - How the Yoruba of Nigeria used base 20
        - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
    - "How might you all go about this?"
    - "What do you notice?"
- **Classroom Activity**:
    - Incorporate explorations where students interact with mathematics in a way that allows them to

"discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
  - "How does this connect to what you've learned in the past?"
  - "How can you use that knowledge today?"
- **Classroom Activity**:
  - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
  - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"
- **Classroom Activity**:
  - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
  - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
  - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
  - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
  - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
  - "Let's get into partners and do a think-pair-share. We will incorporate everyone's ideas and try to synthesize them."
- **Classroom Activity**:
  - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
  - "Come up with at least two answers that might solve this problem."
- **Classroom Activity**:
  - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
  - **Classroom Activity**:
    - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
  - "You recognized that you had to combine the constants 27 and 9, could you explain your

thinking?"

- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
    - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:
    - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:
    - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
    - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).
- **Climate Change Example**:
    - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

21st Century Life and Career

CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

New Jersey Student Learning Standards: Content Area

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MATH.9-12.A.REI	Reasoning with Equations and Inequalities

Integration of Career Readiness. Life Literacies and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.

Integration of Computer Science and Design ThinkingNew Section

CS.9-12.8.1.12.AP.1 Design algorithms to solve computational problems using a combination of original and existing algorithms.

CS.9-12.8.1.12.AP.5 Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

LA.RH.9-10.8	Assess the extent to which the reasoning and evidence in a text support the author's

	claims.
LA.RST.9-10.5	Analyze the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).
LA.RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
LA.RST.9-10.8	Determine if the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.

Module 17: Cross Sections, Surface Area, and Volume

Unit Rationale

Understanding cross sections, surface area, and volume is essential for students to comprehend the properties and relationships of three-dimensional objects. These concepts serve as a foundation for both real-world applications and more advanced topics in geometry, science, engineering, and architecture. In this module, students will explore how to calculate the surface area and volume of various 3D shapes, as well as how to analyze and interpret cross sections of these shapes.

This unit will introduce students to three-dimensional figures, starting with a focus on simple geometric solids such as cubes, rectangular prisms, spheres, cones, cylinders, and pyramids. By studying the **surface area** and **volume** of these shapes, students will develop essential problem-solving skills and a deeper understanding of how geometry applies to the world around them.

The real-world relevance of surface area and volume is emphasized throughout the module, where students will apply their knowledge to practical scenarios, such as determining how much paint is needed to cover a surface, calculating the capacity of containers, or estimating the amount of material needed for manufacturing.

Additionally, students will explore **cross sections**, which are the intersections formed when a three-dimensional figure is sliced. This allows them to visualize and understand the 2D representations of 3D objects, further deepening their conceptual understanding of spatial relationships and providing connections between geometry and algebra.

By the end of this module, students will:

1. Understand and apply the formulas for the **surface area** and **volume** of 3D shapes, including rectangular prisms, cubes, cylinders, spheres, cones, and pyramids.
2. Identify and calculate the **cross sections** of 3D figures, recognizing how the shape of the cross section relates to the properties of the figure.
3. Apply their knowledge of surface area and volume in real-world contexts, such as packaging, construction, and design.
4. Develop mathematical reasoning and spatial visualization skills that will support them in future studies and problem-solving scenarios.

This module aligns with several Standards for Mathematical Practice, particularly:

- **MP4:** Model with mathematics — by applying surface area and volume formulas to real-world problems.

- **MP7:** Look for and make use of structure — by recognizing patterns and relationships in 3D shapes and their properties.
- **MP2:** Reason abstractly and quantitatively — by interpreting surface area, volume, and cross sections and solving related problems.

By the end of the module, students will not only be able to solve problems involving surface area, volume, and cross sections, but they will also appreciate the practical applications of geometry in everyday life. This knowledge will serve as a critical foundation for future work in more advanced mathematical concepts, engineering, architecture, and various scientific fields.

Essential Questions

- **What is the relationship between the surface area and volume of a three-dimensional shape?**
- **How can we calculate the surface area and volume of various three-dimensional figures (e.g., cubes, spheres, cylinders, cones, pyramids)?**
- **In what real-world situations can we apply the concepts of surface area and volume?**
- **How do cross sections help us understand the properties of three-dimensional objects?**
- **How does the shape of a cross section relate to the original three-dimensional figure?**
- **How can we use surface area to determine how much material is needed to cover a 3D object?**
- **How can we use volume to determine the capacity or storage space of a 3D object (e.g., a container, tank, or room)?**
- **What are some strategies for finding the surface area and volume of composite 3D figures (figures made up of multiple simple shapes)?**
- **How do different orientations of a 3D figure affect its cross sections and surface area?**
- **What is the significance of the formulas for surface area and volume, and how do they help us understand and manipulate the physical world?**

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Describe and Analyze Cross Sections of Solids

Student Learning Intentions (WALT)

- We are learning to describe and analyze the cross sections of 3D shapes (solids) such as cubes, cylinders, pyramids, and cones.
- We are learning to understand how the shape of a cross section depends on the orientation of the plane slicing the solid.

Student Success Criteria (I Can Statements)

- I can identify and describe the cross sections of different 3D shapes.
- I can predict and analyze the cross sections when a plane intersects different solids.

- I can explain how the shape of a cross section changes based on the angle of the cut.

Instructional Strategies and Activities

Engage (10 min)

- Launch the Lesson with Real-World Examples:
  - Show a variety of 3D objects (e.g., a cube, cone, cylinder, pyramid). Ask students: What do you think happens when we "slice" through a 3D object? What shapes might appear?
  - Explain that this lesson will focus on the cross sections of these 3D objects and how different slices can create different 2D shapes.
- Think-Pair-Share:
  - Ask students to work with a partner to discuss: What shape would you expect to see if we slice through a cube horizontally, vertically, or diagonally?
  - After a few moments, invite some pairs to share their predictions with the class.

Explore (20 min)

- Guided Practice with Hands-On Exploration:

  a. Cross Sections of a Cube (5 min):

  - Display a cube and ask students to observe different cross sections.
  - Slice the cube in different directions (horizontal, vertical, diagonal) using a transparent cube model or paper folding technique to simulate slicing.
  - Discuss how the cross sections of the cube might create different 2D shapes such as squares, rectangles, or triangles.

  b. Cross Sections of a Cylinder (5 min):

  - Show a cylindrical object (e.g., a can) and simulate slicing the cylinder in different directions.
  - Ask students to predict what shapes will appear: What happens if we slice the cylinder horizontally? What about vertically?
  - Discuss how horizontal slices create circles, while vertical slices create rectangles.

  c. Cross Sections of a Cone (5 min):

  - Use a model of a cone (or a 3D drawing) and show how slices through the cone produce different shapes.
  - Students predict and confirm that slicing the cone horizontally creates circles, while slicing vertically creates triangles.

d. Cross Sections of a Pyramid (5 min):

  - Present a square pyramid and discuss the different ways it can be sliced.
  - Students predict that horizontal slices create squares, and vertical slices produce triangles.

- Interactive Exercise:
  - Have students work in pairs to create paper models of a cylinder, cube, cone, and pyramid.
  - They will use scissors to cut the models at various angles and analyze the resulting cross sections.
  - Students will record their observations and share their findings with the class.

Explain (10 min)

- Class Discussion:
  - Review the cross sections that were explored in the hands-on activity.
  - Summarize the key points:
    - Cube: Horizontal/vertical slices produce squares or rectangles.
    - Cylinder: Horizontal slices produce circles; vertical slices produce rectangles.
    - Cone: Horizontal slices produce circles; vertical slices produce triangles.
    - Pyramid: Horizontal slices produce squares; vertical slices produce triangles.
- Teacher Modeling:
  - Draw a few 3D shapes on the board and demonstrate how different cross sections can be made by slicing the shape in different directions (horizontal, vertical, diagonal).
  - Discuss how the shape of the cross section depends on the angle and direction of the slice.

Elaborate (15 min)

- Independent Practice (10 min):
  - Provide students with a worksheet that includes several 3D shapes. For each shape, students must:
    - Predict the type of cross section they will get when slicing the object in different directions.
    - Draw the cross sections they predict.
- Group Collaboration (5 min):
  - In small groups, students will analyze a complex 3D shape (e.g., a triangular prism or a sphere).

They will predict and describe the cross sections of the shape. Each group will present their findings to the class.

Closure (5 min)

- Exit Ticket:
  - Ask students to answer the following question on an exit ticket:
    A cone is sliced horizontally near the top. What shape is the cross section? What would happen if the cone is sliced vertically through its apex?

Formative Assessments

- Observation during hands-on activities to check if students can identify and describe the cross sections correctly.
- Review of student predictions and drawings during independent practice.
- Exit ticket responses to assess individual understanding of how slicing shapes produces different cross sections.

Instructional Materials and Resources

- HMH Into Math Grade 7 Accelerated Teacher Edition and Student Book
- 3D models of cube, cone, cylinder, pyramid (physical models or digital representations)
- Scissors and paper for hands-on activity
- Worksheet with various 3D shapes and cross section questions
- Whiteboards and markers for students to draw cross sections
- Access to interactive 3D modeling tools (optional)

Reflections and Suggested Modifications

Reflections (post-lesson):

- Were students able to correctly predict and describe the cross sections of various 3D shapes?
- Did the hands-on activity help reinforce understanding, or did some students struggle with visualization?

- How well did students engage with the group collaboration task?

Suggested Modifications:

- For students needing additional support:
  - Provide 2D drawings of cross sections and ask students to match them with the correct 3D shapes.
  - Use interactive 3D software to help visual learners understand how cross sections are formed.
- For enrichment:
  - Challenge advanced students to analyze cross sections of more complex 3D shapes (e.g., a torus or a triangular prism) and describe how these shapes change depending on the orientation of the plane.
- Real-World Contexts:
  - Discuss real-world applications of cross sections in architecture, engineering, and art. For example, the cross section of a building's design or the slicing of fruits and vegetables in food industry design.

Lesson 2: Derive and Apply Formulas for Surface Areas of Cubes and Right Prisms

Student Learning Intentions (WALT)

- We are learning to derive the formulas for the surface area of cubes and right prisms.
- We are learning to apply the surface area formulas to solve real-world and mathematical problems.

Student Success Criteria (I Can Statements)

- I can derive the formula for the surface area of a cube and a right prism.
- I can apply the surface area formulas to calculate the surface area of cubes and right prisms.
- I can explain how the surface area relates to real-world problems, such as determining the amount of material needed to cover an object.

Instructional Strategies and Activities

Engage (10 min)

- **Launch the Lesson with Real-World Examples:**

    - Show a box (cube or rectangular prism) and ask: *How much material would we need to cover the entire surface of this box?*
    - Explain that this is a problem of surface area—finding the total area of all faces of a 3D object.
- **Think-Pair-Share:**
    - Ask students: *What do you think the surface area of a cube or rectangular prism is? How might we calculate it?*
    - Students share their ideas with a partner, and a few students share out loud with the class.

Explore (20 min)

- **Guided Exploration and Derivation of Surface Area Formula (10 min):**

  a. **Surface Area of a Cube:**

    - Display a cube and label the length of each side (s).
    - Discuss how the cube has six identical square faces.
    - Ask: *What is the area of one face of the cube?* (Answer: s^2)
    - Multiply the area of one face by 6 to find the total surface area of the cube:

      $$\text{Surface Area of Cube} = 6 \times s^2$$

  b. **Surface Area of a Right Rectangular Prism:**

    - Show a right rectangular prism and label the length (l), width (w), and height (h).
    - Discuss how the prism has two faces for each pair of dimensions (length × width, width × height, height × length).
    - Ask: *What is the area of each pair of faces?* (Answer:
        - $2 \times l \times w$ for the front and back,
        - $2 \times w \times h$ for the sides,
        - $2 \times h \times l$ for the top and bottom.)
    - Derive the formula for the surface area of the prism:

      $$\text{Surface Area of Rectangular Prism} = 2lw + 2wh + 2hl$$

- **Hands-On Activity (10 min):**
    - Provide students with cube and rectangular prism models (physical or digital) and measurement tools.

- Have students measure the length, width, and height of the objects and calculate their surface area using the derived formulas.
- Walk around to check for understanding and assist students with calculations as needed.

Explain (10 min)

- **Class Discussion:**
  - Ask students to share their findings from the hands-on activity.
  - Review the formulas for surface area and discuss the relationship between the dimensions of the shape and the total surface area.
  - Emphasize the importance of understanding surface area for practical applications, such as determining the amount of paint or wrapping paper needed to cover an object.
- **Teacher Modeling:**
  - Provide an example problem on the board: *A cube has a side length of 4 cm. What is its surface area?*
  - Solve the problem step-by-step, using the formula 6×s26 \times s^26×s2, and explain each step clearly to the students.

Elaborate (15 min)

- **Independent Practice (10 min):**
  - Provide students with a set of problems to solve, including:
    1. A cube with side length 5 cm.
    2. A right rectangular prism with length 3 cm, width 4 cm, and height 6 cm.
    3. A real-world problem such as determining the surface area of a box that needs to be painted (give dimensions).
  - Students will apply the formulas they derived to solve these problems.
- **Group Activity (5 min):**
  - In small groups, students will create a real-world surface area problem. Each group will choose an object (such as a box, package, or a building) and develop a surface area problem that includes given dimensions. Afterward, each group will share their problem with the class, and the class will solve it together.

Closure (5 min)

- **Exit Ticket:**
  - Ask students to solve one surface area problem on their own: *A right rectangular prism has a*

length of 7 cm, width of 2 cm, and height of 5 cm. What is the surface area?

Formative Assessments

- **Observation:**
  Monitor students during the hands-on activity and independent practice to check if they can accurately apply the surface area formulas.
- **Exit Ticket:**
  Review the exit ticket responses to ensure students can calculate surface area correctly.
- **Partner or Group Practice:**
  Listen to student explanations during the group activity to assess their understanding of surface area and how it can be applied to real-world problems.

Instructional Materials and Resources

- HMH Into Math Grade 7 Accelerated Teacher Edition and Student Book
- Cube and rectangular prism models (physical or digital)
- Rulers or measuring tools for hands-on activity
- Worksheets with surface area problems
- Whiteboards and markers for students to solve problems
- Calculators (optional)

Reflections and Suggested Modifications

Reflections (post-lesson):

- Were students able to derive and apply the formulas for surface area of cubes and right prisms?
- Did students struggle with applying the formulas to solve real-world problems?
- How effectively did the group activity support student learning and engagement?

Suggested Modifications:

- **For students needing support:**
  - Provide visual aids such as labeled diagrams of cubes and rectangular prisms with highlighted

faces to guide students in applying the formulas.

- Offer practice problems with step-by-step solutions for students who need extra practice with surface area calculations.

- **For enrichment:**
  - Challenge advanced students by introducing surface area problems involving composite 3D shapes or irregular prisms.
  - Provide problems with non-integer dimensions and have students work through the formulas with decimal or fractional measurements.

Lesson 3: Derive and Apply a Formula for the Volume of a Right Prism

Student Learning Intentions (WALT)

- We are learning to derive the formula for the volume of a right prism.
- We are learning to apply the volume formula to solve real-world and mathematical problems involving right prisms.

Student Success Criteria (I Can Statements)

- I can derive the formula for the volume of a right prism.
- I can apply the volume formula to calculate the volume of a right prism.
- I can explain the concept of volume and how it relates to real-world situations, such as determining the amount of space inside an object.

Instructional Strategies and Activities

Engage (10 min)

- **Launch the Lesson with Real-World Example:**
  - Show a box or container (rectangular prism) and ask: *How much space is inside this box?*
  - Explain that this is a volume problem—finding the amount of space inside a 3D object.
- **Think-Pair-Share:**
  - Ask students: *What do you think the volume of a right prism is? How might we calculate it?*
  - Students share their ideas with a partner, and a few students share out loud with the class.

Explore (20 min)

- **Guided Exploration and Derivation of Volume Formula (10 min):**

  a. **Volume of a Right Prism:**

    - Display a rectangular prism (or right prism with a rectangular base) and label the dimensions: length (l), width (w), and height (h).
    - Discuss how the volume of a prism is the area of the base (which is a rectangle) times the height (the distance between the two bases).
    - Ask: *What is the area of the base?* (Answer: l×wl \times wl×w).
    - Multiply the area of the base by the height to find the volume:

      Volume of Right Prism=Base Area×Height=l×w×h\text{Volume of Right Prism} = \text{Base Area} \times \text{Height} = l \times w \times hVolume of Right Prism=Base Area×Height=l×w×h

    - Explain that the formula works for any right prism, not just rectangular prisms, as long as we find the area of the base and multiply by the height.

  b. **Teacher Modeling:**

    - Provide an example problem: *A right rectangular prism has a length of 4 cm, width of 3 cm, and height of 5 cm. What is the volume of the prism?*
    - Solve the problem step-by-step:
      - First, find the area of the base: 4×3=12 cm24 \times 3 = 12 \, \text{cm}^24×3=12cm2
      - Then, multiply the base area by the height: 12×5=60 cm312 \times 5 = 60 \, \text{cm}^312×5=60cm3
    - Explain the steps to students clearly, emphasizing that the unit for volume is cubic units (cm3\text{cm}^3cm3).

- **Hands-On Activity (10 min):**
    - Provide students with models of right prisms (either physical or virtual). Each prism should have different dimensions.
    - Students will measure the length, width, and height of each prism and calculate the volume using the formula V=l×w×hV = l \times w \times hV=l×w×h.
    - Walk around the room to check for understanding and provide support as necessary.

Explain (10 min)

- **Class Discussion:**

    - Ask students to share their findings from the hands-on activity.
    - Discuss how the formula for volume applies to different types of right prisms (e.g., cubes, rectangular prisms).
    - Emphasize that the volume represents the amount of space inside an object, and it's measured in cubic units.
- **Teacher Modeling:**
    - Provide another example where the base of the prism is a triangle (e.g., a triangular prism).
    - Derive the volume formula for this type of prism:

      Volume of Triangular Prism=12×Base×Height of Triangle×Height of Prism\text{Volume of Triangular Prism} = \frac{1}{2} \times \text{Base} \times \text{Height of Triangle} \times \text{Height of Prism}Volume of Triangular Prism=21 ×Base×Height of Triangle×Height of Prism
    - Solve the problem step-by-step, showing how to find the area of the triangular base first, and then multiplying by the height of the prism.

Elaborate (15 min)

- **Independent Practice (10 min):**
    - Provide students with a set of problems to solve, including:
        1. A rectangular prism with a length of 7 cm, width of 5 cm, and height of 8 cm.
        2. A triangular prism with a triangular base (base = 6 cm, height = 4 cm) and a height of 10 cm.
        3. A real-world problem where students must calculate the volume of a shipping box to determine how much space is available for packing.
- **Group Activity (5 min):**
    - In small groups, students will create their own volume problem based on a real-world scenario (e.g., determining how much liquid a container can hold, calculating the volume of a box to pack items for shipping, etc.).
    - After creating the problem, students will exchange problems with another group to solve.

Closure (5 min)

- **Exit Ticket:**
    - Ask students to solve a volume problem:
      A right rectangular prism has a length of 8 cm, width of 4 cm, and height of 6 cm. What is its volume?

- Students will complete the exit ticket and turn it in as they leave the class.

Formative Assessments

- **Observation:**
  Walk around the room during the hands-on activity to check for understanding. Make sure students are correctly applying the volume formula and measuring dimensions accurately.
- **Exit Ticket:**
  Review the exit ticket responses to ensure students can apply the volume formula correctly.
- **Partner or Group Practice:**
  Observe the group discussions as students create and solve volume problems. Assess if they can apply the formula in real-world scenarios and explain their reasoning.

Instructional Materials and Resources

- HMH Into Math Grade 7 Accelerated Teacher Edition and Student Book
- Models of right prisms (physical or digital)
- Rulers or measuring tools for hands-on activity
- Worksheets with volume problems
- Whiteboards and markers for students to solve problems
- Calculators (optional)

Reflections and Suggested Modifications

Reflections (post-lesson):

- Were students able to derive and apply the formula for the volume of a right prism?
- Did the hands-on activity help students understand how volume is calculated?
- Were students able to explain the concept of volume and its real-world applications?

Suggested Modifications:

- **For students needing support:**
  - Provide visual aids with labeled prisms and step-by-step instructions for calculating volume.

- Offer problems with smaller numbers or easier dimensions to help students build confidence before moving on to more complex problems.

- **For enrichment:**
  - Challenge advanced students by providing problems involving composite 3D shapes (e.g., a right prism with a base that is a pentagon or hexagon).
  - Ask students to find the volume of a prism with fractional or decimal measurements and to round the final answer to the nearest tenth.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
  - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
  - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals**:
  - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary**:
  - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
  - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
  - It is important to consistently check for understanding, especially for students who have

accommodations, to ensure they comprehend the concepts in a way that makes sense to them.

- **Extra Time**:
    - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.
- **Oral Reading**:
    - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
    - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
    - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
    - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
    - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
    - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
    - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
  - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.
- **Peer Support**:
  - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
  - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
  - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
  - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
  - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.
- **Tests/Assessments**:
  - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:
  - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
  - Fist to five: "How well do you understand what we talked about today?"
  - Fist to five: "How well did I teach this today?"
- **Classroom Activity**:
  - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
  - Color-coding ideas
  - Learning vocabulary in student languages
  - Visual and kinesthetic learning
  - Representations of learning without words
- **Classroom Activity**:
  - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
  - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
- **Classroom Activity**:
  - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
  - "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
  - Learn about different bases and numerical ideas:
    - Base 2 (binary) and connections to computer programming
    - How the Yoruba of Nigeria used base 20
    - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
  - "How might you all go about this?"
  - "What do you notice?"
- **Classroom Activity**:
  - Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
  - "How does this connect to what you've learned in the past?"
  - "How can you use that knowledge today?"
- **Classroom Activity**:
  - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
  - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we

need to consider now?"

- **Classroom Activity**:
    - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
    - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
    - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
    - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
    - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
    - "Let's get into partners and do a think-pair-share. We will incorporate everyone's ideas and try to synthesize them."
- **Classroom Activity**:
    - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
  - "Come up with at least two answers that might solve this problem."
- **Classroom Activity**:
  - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
  - **Classroom Activity**:
    - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
  - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
  - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
  - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:
  - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:

    - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
    - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).
- **Climate Change Example**:
    - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MATH.8.G	Geometry

21st Century Life and Career

CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.

Integration of Career Readiness. Life Literacies and Key Skills

CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Integration of Computer Science and Design Thinking

CS.CS	Computing Systems
	Real world information can be stored and manipulated in programs as data (e.g., numbers, words, colors, images).

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

	Key Ideas and Details
LA.K-12.NJSLSA.W1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
	K-12 To build a foundation for college and career readiness, students need to learn to use writing as a way of offering and supporting opinions, demonstrating understanding of the subjects they are studying, and conveying real and imagined experiences and events. They learn to appreciate that a key purpose of writing is to communicate clearly to an external, sometimes unfamiliar audience, and they begin to adapt the form and content of their writing to accomplish a particular task and purpose. They develop the capacity to build knowledge on a subject through research projects and to respond analytically to literary and informational sources. To meet these goals, students must devote significant time and effort to writing, producing numerous pieces over short and extended time frames throughout the year.