Unit 6: Applications of Real Numbers and Exponents

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **9 Weeks**
Status: **Published**

Module 13: Real Numbers

Unit Rationale

Understanding real numbers is a cornerstone of developing a strong number sense and building mathematical fluency. In this unit, students will explore the different types of real numbers—rational and irrational—and how these numbers are used in a variety of mathematical and real-world contexts. By investigating the properties and operations of real numbers, students will gain essential tools for solving problems and making connections between abstract mathematical concepts and everyday situations.

Students will build on their prior knowledge of integers, fractions, and decimals to extend their understanding of real numbers, including irrational numbers such as square roots and pi. By examining how these numbers interact with one another in various operations, students will deepen their understanding of mathematical relationships and develop the skills needed to solve more complex equations.

This unit emphasizes the importance of both rational and irrational numbers in representing quantities and solving problems. By learning to operate with real numbers and understand their properties, students will be better prepared to tackle advanced algebraic concepts, as well as apply mathematics in real-world settings such as measurement, finance, and science.

Throughout the unit, students will:

- Explore the classification of real numbers, including rational and irrational numbers.
- Apply operations (addition, subtraction, multiplication, and division) to real numbers.
- Investigate the properties of real numbers, such as commutative, associative, and distributive properties.
- Use absolute value and understand its importance in real-world contexts, such as distances and magnitudes.
- Solve real-world problems involving real numbers, applying appropriate mathematical strategies and operations.

This unit emphasizes several key Standards for Mathematical Practice, particularly:

- **MP2: Reason abstractly and quantitatively** — by interpreting real numbers and performing operations on them.
- **MP4: Model with mathematics** — by applying operations with real numbers to solve real-world problems.
- **MP6: Attend to precision** — by using clear and accurate notation when working with real numbers and their operations.

- **MP7: Look for and make use of structure** — by recognizing patterns in the properties of real numbers and their operations.

By the end of the unit, students will not only be able to perform operations with real numbers, but they will also develop a deeper understanding of their role in mathematics and real-world contexts. This knowledge is foundational for future studies in algebra, geometry, and beyond, as well as for solving practical problems in everyday life.

Essential Questions

- What are real numbers, and how do we classify them?
- How do operations with real numbers help us solve real-world problems?
- Why is it important to distinguish between rational and irrational numbers?
- How do the properties of real numbers (commutative, associative, distributive) simplify computations and problem-solving?
- What does absolute value represent, and how is it used in everyday situations?
- How can understanding the structure of real numbers support algebraic thinking?
- In what ways can we model real-world situations using real numbers?
- How can we ensure precision and accuracy when performing calculations with real numbers?

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.

- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Understand Rational and Irrational Numbers

Student Learning Intentions (WALT):

- We are learning to recognize and classify rational and irrational numbers.
- We are learning to explain the differences between rational and irrational numbers using examples and real-world contexts.

Student Success Criteria (I Can Statements):

- I can identify whether a number is rational or irrational.
- I can explain what makes a number rational or irrational using mathematical vocabulary.
- I can classify numbers correctly and support my reasoning with examples.

Instructional Strategies and Activities

Engage (10 minutes)

- **Real-World Launch Example:** Pose a situation such as calculating with money ($4.75) vs. estimating the value of π.

- **Think-Pair-Share:** Are these numbers rational or irrational? Why? What makes them different?

Explore (20 minutes) – *Three Rotations*

1. **Definition & Sort Station:** Students classify numbers (fractions, decimals, square roots, etc.) as rational or irrational.
2. **Number Line Station:** Estimate and place rational and irrational numbers on a number line.
3. **Scenario Station:** Real-world examples (e.g., measuring diagonals, using π, dealing with repeating decimals). Decide whether the numbers involved are rational or irrational.

Explain (10 minutes)

- **Class Discussion:** Groups share what patterns they noticed.
- **Teacher Modeling:** Define rational and irrational numbers. Use a Venn diagram or number line to show how numbers are related.

Elaborate (15 minutes)

- **Practice Activity:** Mixed exercises where students classify numbers, convert repeating decimals to fractions, and identify square roots as rational or irrational.

Closure (5 minutes)

- **Exit Ticket:** Provide three numbers (e.g., 4/5, √2, 0.333…) and ask students to label each as rational or irrational and explain their reasoning.

Formative Assessments:

- Observation and questioning at stations
- Responses on sorting and number line tasks
- Exit ticket responses
- Participation in class discussion

Instructional Materials and Resources:

- HMH *Into Math Grade 7 Accelerated* (if applicable)
- Number sorting cards
- Venn diagram or number line templates

- Whiteboards and markers
- Exit tickets
- Calculators (optional for decimal exploration)
- Real-world scenario cards

Reflections (Post-Lesson):

- Were students able to clearly explain the difference between rational and irrational numbers?
- Which types of numbers caused the most confusion (e.g., repeating decimals, roots)?
- Did students engage with all three stations and connect their learning across representations?

Suggested Modifications:

- **Support:** Provide a number classification chart with definitions and examples.
- **Extension:** Ask early finishers to write or present their own real-world example of an irrational number.
- **Engagement Boosters:** Include familiar contexts like sports stats (e.g., batting averages) or tech (e.g., screen resolution with √2).

Lesson 2: Investigate Roots

Student Learning Intentions (WALT):

- We are learning to find and estimate square roots and cube roots.
- We are learning to understand which roots are rational and which are irrational.

Student Success Criteria (I Can Statements):

- I can find exact square roots of perfect squares.
- I can estimate square roots of non-perfect squares.
- I can explain the difference between rational and irrational roots.

- I can solve real-world problems involving square and cube roots.

Instructional Strategies and Activities

Engage (10 minutes)

- **Quick Write:** What do you know about square roots?
- Show a calculator display of √2 or √10. Ask: "What do you notice?"
- **Discussion Prompt:** Are these numbers rational? How do you know?

Explore (20 minutes) – *Three Rotations*

1. **Perfect Squares Station:** Match numbers with their square roots (e.g., 36 → √36 = 6).
2. **Estimation Station:** Use number lines to estimate √8, √15, √50 between two perfect squares.
3. **Cube Root Station:** Introduce and explore ∛8, ∛27, ∛64 — identify if the root is rational.

Explain (10 minutes)

- **Teacher Modeling:**
  - Define **square root** and **cube root**.
  - Model estimating non-perfect square roots using a number line.
  - Discuss how square roots of perfect squares are rational and others are irrational.
  - Show the use of radicals and how to simplify them when possible.

Elaborate (15 minutes)

- **Partner or Individual Practice:**
  - Classify roots as rational or irrational.
  - Estimate roots to the nearest whole number or tenth.
  - Solve contextual problems (e.g., finding side lengths of squares or volumes of cubes).

Closure (5 minutes)

- **Exit Ticket:**
  - Identify whether $\sqrt{49}$, $\sqrt{20}$, and $\sqrt[3]{27}$ are rational or irrational.
  - Explain how you know.

Formative Assessments:

- Observation during station work
- Student responses during discussion
- Exit ticket explanations
- Work samples from practice problems

Instructional Materials and Resources:

- Square and cube root task cards
- Number line templates
- Calculators (optional for decimal estimation)
- Whiteboards and markers
- Exit tickets
- Math journals or notebooks

Reflections (Post-Lesson):

- Which students were able to estimate roots accurately?
- Were students confident distinguishing rational vs. irrational roots?
- Which root concept (square or cube) caused more difficulty?

Suggested Modifications:

- **Support:** Provide visual aids such as root ladders and charts of perfect squares and cubes.
- **Extension:** Challenge students to simplify square roots (e.g., $\sqrt{50} = \sqrt{(25\times2)} = 5\sqrt{2}$).
- **Engagement Boosters:** Use real-world examples (e.g., area of square garden, cube-shaped storage containers) to apply roots in context.

Lesson 3: Order Real Numbers

Student Learning Intentions (WALT):

- We are learning to compare and order real numbers, including rational and irrational numbers.
- We are learning to represent real numbers on a number line accurately.

Student Success Criteria (I Can Statements):

- I can compare and order rational and irrational numbers.
- I can locate real numbers on a number line.
- I can explain how to compare numbers with different forms (decimals, fractions, square roots, etc.).

Instructional Strategies and Activities

Engage (10 minutes)

- **Warm-Up:** Display a mix of real numbers (e.g., 1.5, $\sqrt{2}$, -3/4, π, -2). Ask:
  "Which of these is the greatest? Least?"
- **Think-Pair-Share:** How can we compare numbers when they are in different forms?

Explore (20 minutes) – *Three Rotations*

1. **Decimal Conversion Station:** Convert fractions and square roots to decimals (rounded) to aid comparison.
2. **Number Line Station:** Plot a variety of real numbers and label them.
3. **Ordering Challenge Station:** Order sets of real numbers from least to greatest using estimation, conversion, and reasoning.

Explain (10 minutes)

- **Teacher Modeling:**
  - Model converting a fraction and a square root to decimals (e.g., $\frac{3}{4} = 0.75$, $\sqrt{5} \approx 2.24$).
  - Show how to order a mixed set of real numbers and place them on a number line.
  - Emphasize rounding, estimation, and number sense over memorization.

Elaborate (15 minutes)

- **Partner or Independent Practice:**
  - Mixed sets of real numbers (fractions, decimals, square roots, π, negatives).
  - Tasks involve converting, comparing, and ordering, both numerically and on number lines.

Closure (5 minutes)

- **Exit Ticket:**
  - Given a set (e.g., $\sqrt{10}$, 3/2, π, 1.7), ask students to order the numbers and explain one comparison they made.

Formative Assessments:

- Station activity performance
- Observations and questioning during discussion
- Accuracy and explanation on exit tickets
- Correct placement of numbers on number lines

Instructional Materials and Resources:

- Calculators (for decimal estimation)
- Real number task cards

- Blank number lines
- Conversion reference charts
- Whiteboards and markers
- Exit tickets

Reflections (Post-Lesson):

- Which numbers were most challenging for students to compare or place?
- Did students rely on memorization or develop number sense through estimation?
- Were students able to explain their reasoning effectively?

Suggested Modifications:

- **Support:** Provide a chart of common square root approximations and fraction-to-decimal conversions.
- **Extension:** Ask students to create their own set of real numbers to order and challenge a peer.
- **Engagement Boosters:** Use contexts like sports stats, prices, and measurements to show practical relevance.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
    - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
    - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals**:
    - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.

- **Front-Loading Vocabulary**:
    - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
    - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
    - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.
- **Extra Time**:
    - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.
- **Oral Reading**:
    - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
    - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
    - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
    - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
    - Students with 504 plans are given extra time to complete assignments, ensuring they have

ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
  - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
  - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
  - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.
- **Peer Support**:
  - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
  - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
  - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
  - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
  - Provide concrete, hands-on activities to support at-risk students. This may include using tools

like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.

- **Tests/Assessments**:
  - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:
  - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
  - Fist to five: "How well do you understand what we talked about today?"
  - Fist to five: "How well did I teach this today?"
- **Classroom Activity**:
  - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
  - Color-coding ideas
  - Learning vocabulary in student languages
  - Visual and kinesthetic learning
  - Representations of learning without words
- **Classroom Activity**:
  - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
  - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
- **Classroom Activity**:
  - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
  - "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
  - Learn about different bases and numerical ideas:
    - Base 2 (binary) and connections to computer programming
    - How the Yoruba of Nigeria used base 20
    - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
  - "How might you all go about this?"
  - "What do you notice?"
- **Classroom Activity**:
  - Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web

rather than discrete sets of knowledge

- **Verbal Example**:
    - "How does this connect to what you’ve learned in the past?"
    - "How can you use that knowledge today?"
- **Classroom Activity**:
    - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
    - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"
- **Classroom Activity**:
    - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
    - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
    - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:

    - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
    - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
    - "Let’s get into partners and do a think-pair-share. We will incorporate everyone’s ideas and try to synthesize them."
- **Classroom Activity**:
    - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
    - "Come up with at least two answers that might solve this problem."
- **Classroom Activity**:
    - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
    - **Classroom Activity**:
        - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
    - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
  - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:
  - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:
  - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
  - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).
- **Climate Change Example**:
  - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.

MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MATH.9-12.A.REI	Reasoning with Equations and Inequalities

21st Century Life and Career

CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Integration of Career Readiness. Life Literacies and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.

Integration of Computer Science and Design ThinkingNew Section

CS.9-12.8.1.12.AP.1 Design algorithms to solve computational problems using a combination of original and existing algorithms.

CS.9-12.8.1.12.AP.5 Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

LA.RH.9-10.8	Assess the extent to which the reasoning and evidence in a text support the author's claims.
LA.RST.9-10.5	Analyze the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).
LA.RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual

	form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
LA.RST.9-10.8	Determine if the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.

Module 14: The Pythagorean Theorem

Unit Rationale

Understanding the Pythagorean Theorem is a foundational skill in middle school mathematics that bridges geometry, algebra, and real-world problem solving. This unit introduces students to the relationship between the side lengths of right triangles, empowering them to explore spatial relationships, reason quantitatively, and solve practical problems involving distance and measurement.

Students will build on their knowledge of squares and square roots to understand and apply the Pythagorean Theorem. Through exploration, modeling, and problem solving, they will learn to calculate unknown side lengths in right triangles and recognize how this principle applies in both mathematical contexts and real-world scenarios—such as construction, navigation, and design.

This unit emphasizes reasoning with irrational numbers, applying geometric formulas, and connecting mathematical concepts to everyday use. Students will learn that the Pythagorean Theorem is not just a formula to memorize, but a powerful tool for understanding how distance and structure work in two- and three-dimensional space.

Throughout this unit, students will:

- Identify and label the parts of right triangles (legs and hypotenuse).
- Apply the Pythagorean Theorem to solve for missing side lengths.
- Estimate and simplify square roots to express exact and approximate answers.
- Solve real-world problems involving right triangles, including those on coordinate planes.
- Justify their solutions using mathematical reasoning and vocabulary.

This unit aligns with several Standards for Mathematical Practice, especially:

- **MP1: Make sense of problems and persevere in solving them** — as students interpret real-world and geometric situations.
- **MP4: Model with mathematics** — by applying the theorem to real-life contexts.
- **MP6: Attend to precision** — when labeling triangles, using square roots, and checking their work.
- **MP7: Look for and make use of structure** — as students recognize right triangles in varied contexts, including non-obvious diagrams.

By the end of this unit, students will not only be able to apply the Pythagorean Theorem correctly, but they will also understand its origin, importance, and practical applications—skills essential for further study in geometry, algebra, and science.

Essential Questions

- **What is the relationship between the sides of a right triangle?**
- **How can the Pythagorean Theorem be used to find unknown side lengths in right triangles?**
- **Why is the Pythagorean Theorem important in real-world applications, such as construction, navigation, and design?**
- **How can we estimate and simplify square roots to solve problems involving the Pythagorean Theorem?**
- **In what ways can we apply the Pythagorean Theorem to problems on a coordinate plane?**
- **How do we justify our solutions when using the Pythagorean Theorem in both mathematical and real-world contexts?**
- **What makes the Pythagorean Theorem applicable to two- and three-dimensional space?**
- **How does understanding the Pythagorean Theorem help us reason about distance, measurement, and spatial relationships?**
- **What role do irrational numbers play when using the Pythagorean Theorem?**
- **How can the Pythagorean Theorem be used to prove that a triangle is a right triangle?**

Pre-Assessments

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4. Lesson-Specific Checks
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Instructional Plan

Lesson 1: Prove the Pythagorean Theorem and Its Converse

Student Learning Intentions (WALT):

- We are learning to prove the Pythagorean Theorem using geometric reasoning.
- We are learning to prove the converse of the Pythagorean Theorem.

Student Success Criteria (I Can Statements):

- I can state the Pythagorean Theorem and apply it to solve problems.
- I can use a proof to show why the Pythagorean Theorem is true.
- I can explain the converse of the Pythagorean Theorem and apply it to determine whether a triangle is a right triangle.

Instructional Strategies and Activities

Engage (10 minutes)

- **Hook:**
  - Start with a visual: Present a right triangle with sides aaa, bbb, and hypotenuse ccc. Ask the class, "What do you think happens if we add the squares of the legs and compare that to the square of the hypotenuse?"
  - Show a few real-world examples where the Pythagorean Theorem is used, such as construction or navigation.
- **Introduction to the Pythagorean Theorem**
  - Introduce the statement of the **Pythagorean Theorem**:

    a2+b2=c2a^2 + b^2 = c^2a2+b2=c2

    where aaa and bbb are the lengths of the legs, and ccc is the length of the hypotenuse of a right triangle.
  - Discuss the importance of proving this theorem, setting the stage for why and how we prove geometric principles.

Explore (25 minutes)

- **Teacher Modeling: Proof of the Pythagorean Theorem**
  - **Proof by Rearrangement (Using Squares and Triangles):**
    - Create a large square with side length a+ba + ba+b, and divide it into smaller pieces, showing that the area can be rearranged to demonstrate the Pythagorean Theorem.
    - In the square, there are four right triangles, each with legs aaa and bbb, and a small square in the middle with side length ccc (the length of the hypotenuse).
    - Show how the area of the large square can be calculated in two ways:
      - By adding the areas of the four triangles and the small square:

        Area=4×12ab+c2=2ab+c2\text{Area} = 4 \times \frac{1}{2}ab + c^2 = 2ab + c^2Area=4×21ab+c2=2ab+c2
      - By squaring the side length of the large square:

        Area=(a+b)2=a2+2ab+b2\text{Area} = (a + b)^2 = a^2 + 2ab + b^2Area=(a+b)2=a2+2ab+b2
    - Set the two area expressions equal to each other:

      2ab+c2=a2+2ab+b22ab + c^2 = a^2 + 2ab + b^22ab+c2=a2+2ab+b2
    - Canceling 2ab2ab2ab from both sides, we are left with:

      c2=a2+b2c^2 = a^2 + b^2c2=a2+b2

        - This proves the Pythagorean Theorem.

- **Explore the Converse of the Pythagorean Theorem:**
    - State the **Converse of the Pythagorean Theorem**:
      "If a triangle has side lengths aaa, bbb, and ccc, and if a2+b2=c2a^2 + b^2 = c^2a2+b2=c2, then the triangle is a right triangle."
    - Provide a few examples where students test if a triangle is a right triangle using the converse (i.e., check if the sum of the squares of the legs equals the square of the hypotenuse).

  Example Problems for Converse:

    - Given a triangle with sides 777, 242424, and 252525, check if it's a right triangle.
        - 72+242=49+576=6257^2 + 24^2 = 49 + 576 = 62572+242=49+576=625, and 252=62525^2 = 625252=625, so the triangle is a right triangle.
    - Given a triangle with sides 888, 151515, and 202020, check if it's a right triangle.
        - 82+152=64+225=2898^2 + 15^2 = 64 + 225 = 28982+152=64+225=289, but 202=40020^2 = 400202=400, so it is not a right triangle.

Explain (10 minutes)

- **Class Discussion:**
    - Review the key takeaways:
        - The **Pythagorean Theorem** is a2+b2=c2a^2 + b^2 = c^2a2+b2=c2, where aaa and bbb are the legs, and ccc is the hypotenuse of a right triangle.
        - The **Converse** of the Pythagorean Theorem allows us to verify if a triangle is a right triangle based on its side lengths.
    - Encourage students to verbalize the steps in the proof and explain the logic behind the converse.
- **Teacher-guided Practice:**
    - Use a variety of triangles (right and non-right) and guide students through applying both the Pythagorean Theorem and its converse to solve problems.

Elaborate (15 minutes)

- **Partner Practice:**

    - Provide students with a set of triangles and ask them to:
        - Prove the Pythagorean Theorem for a few right triangles by verifying a2+b2=c2a^2 + b^2 = c^2a2+b2=c2.
        - Use the converse of the Pythagorean Theorem to determine if given triangles are right triangles.
    - Encourage students to work together, sharing their reasoning and justifications for their answers.

Closure (5 minutes)

- **Exit Ticket:**
    - Ask students to solve one problem where they must:
        - Determine if a given triangle is a right triangle using the converse of the Pythagorean Theorem.
        - Write a brief justification for their answer, referring to the mathematical reasoning behind the converse.

Formative Assessments:

- Observation during the proof activity and partner practice.
- Exit ticket responses.
- Student explanations of the Pythagorean Theorem and its converse.

Instructional Materials and Resources:

- Whiteboard and markers for teacher demonstration.
- Paper and ruler for students to draw and explore triangles.
- Access to geometric tools (optional) such as a graphing calculator or online tools for verifying triangle properties.
- Exit tickets.
- Pre-prepared handouts with triangle problems for partner practice.

Reflections (Post-Lesson):

- Did students grasp the proof of the Pythagorean Theorem and the reasoning behind the converse?
- Were they able to apply the Pythagorean Theorem and its converse independently?
- Were there specific areas in the proof or converse that caused confusion, and how can I address them in future lessons?

Suggested Modifications:

- **For Support:**
    - Provide step-by-step guides for the proof of the Pythagorean Theorem. Use visuals and hands-on activities to reinforce understanding.
    - Offer more time for guided practice, with simpler triangles to start.
- **For Enrichment:**
    - Challenge students with more complex applications of the Pythagorean Theorem, such as using coordinate geometry to prove if a triangle is a right triangle based on its vertices.
    - Introduce the concept of finding missing side lengths algebraically by setting up equations based on the Pythagorean Theorem.

Lesson 2: Apply the Pythagorean Theorem

Student Learning Intentions (WALT):

- We are learning to apply the Pythagorean Theorem to solve for missing side lengths in right triangles.
- We are learning to use the Pythagorean Theorem in real-world problem-solving scenarios.

Student Success Criteria (I Can Statements):

- I can use the Pythagorean Theorem to find the length of a missing side in a right triangle.
- I can apply the Pythagorean Theorem to solve real-world problems.
- I can estimate and simplify square roots to express exact and approximate answers.

Instructional Strategies and Activities

Engage (10 minutes)

- **Hook:**
  - Start by showing a real-world scenario, such as measuring the diagonal distance across a rectangular park, or how engineers use the Pythagorean Theorem in building projects.
  - Ask the class: "If I know the length and width of a rectangular area, how could I find the distance between two opposite corners?"
- **Introduction to the Problem:**
  - Introduce a scenario where students are asked to apply the Pythagorean Theorem:
    - For example: "A ladder is leaning against a wall. The distance from the base of the ladder to the wall is 4 meters, and the height of the ladder from the ground to the top of the wall is 3 meters. How long is the ladder?"

Explore (25 minutes)

- **Teacher Modeling:**
  - Step 1: **Write the Pythagorean Theorem formula** on the board:

    a2+b2=c2a^2 + b^2 = c^2a2+b2=c2

    where aaa and bbb are the lengths of the legs (perpendicular sides), and ccc is the hypotenuse (the longest side).
  - Step 2: **Example Problem**
    - "We have a right triangle with legs a=3a = 3a=3 meters and b=4b = 4b=4 meters. We need to find the hypotenuse ccc."
    - Apply the Pythagorean Theorem:

      32+42=c23^2 + 4^2 = c^232+42=c2 9+16=c29 + 16 = c^29+16=c2 25=c225 = c^225=c2 c=25c = \sqrt{25}c=25 c=5 metersc = 5 \text{ meters}c=5 meters
    - Walk through each step and discuss how to simplify square roots to find the exact length of the hypotenuse.
  - Step 3: **Real-World Example:**
    - Apply the theorem to a real-world problem such as determining the length of a diagonal in a rectangular room, or finding the distance between two points on a coordinate plane.

Explain (10 minutes)

- **Class Discussion:**
  - Review the steps of solving a Pythagorean Theorem problem:
    1. Identify the legs and the hypotenuse of the triangle.
    2. Substitute the known values into the Pythagorean Theorem formula.
    3. Solve for the missing side by simplifying the equation and, if necessary, taking the square root.
    4. Check the solution by substituting the value back into the original equation.
  - Discuss what happens when the hypotenuse is unknown (how to find it) versus when a leg is unknown (how to solve for it).
- **Group Practice:**
  - Work through a few problems as a class, ensuring students understand how to manipulate the formula and solve for missing sides. Ask questions like:
    - What if we only know the hypotenuse and one leg? How would you solve for the missing leg?

Elaborate (15 minutes)

- **Partner Practice:**
  - **Task 1:** Give each pair of students a set of problems, where they apply the Pythagorean Theorem to find missing side lengths in right triangles. Encourage them to work together and compare answers. Example problems:
    - A right triangle has legs of length 6 meters and 8 meters. Find the hypotenuse.
    - A right triangle has a hypotenuse of 13 meters and one leg of 5 meters. Find the other leg.
    - A right triangle has legs of length 10 cm and 24 cm. Find the hypotenuse.
  - **Task 2:** Apply the Pythagorean Theorem to solve a real-world problem:
    - **Example:** "A dog is tied to a post with a leash of 15 feet. If the dog walks 9 feet away from the post, how far is the dog from the post along the ground?"
      - This is a real-world scenario that can be solved by using the Pythagorean Theorem, where 15 feet is the hypotenuse and 9 feet is one of the legs. Students will find the distance the dog has walked along the ground.

Closure (5 minutes)

- **Exit Ticket:**
  - Provide each student with a short problem where they must apply the Pythagorean Theorem to solve for the missing side.
  - Example Exit Ticket:
    - "A right triangle has legs of 5 inches and 12 inches. Find the hypotenuse."
    - "A right triangle has a hypotenuse of 17 cm and one leg of 8 cm. Find the other leg."
  - Collect the exit tickets to assess understanding.

Formative Assessments:

- Observation during class and partner practice.
- Student explanations during group practice.
- Exit ticket responses to assess individual understanding.

Instructional Materials and Resources:

- Whiteboard and markers for teacher demonstrations.
- Graph paper or plain paper for students to draw and solve problems.
- Access to calculators (optional) for estimating square roots.
- Exit tickets.
- Pre-prepared handouts with problems for partner practice.

Reflections (Post-Lesson):

- Which students struggled with applying the Pythagorean Theorem to real-world problems, and how can I support them in future lessons?
- Did students understand the importance of simplifying square roots for exact and approximate answers?
- Were students able to independently apply the Pythagorean Theorem to solve problems involving

missing side lengths?

Suggested Modifications:

- **For Support:**
  - Provide a step-by-step guide with a visual representation of the triangle for students who struggle with conceptualizing the problem.
  - Use smaller numbers for initial problems to help students build confidence with the formula.
- **For Enrichment:**
  - Challenge advanced students with non-right triangles and ask them to explore how the Pythagorean Theorem applies to them (introducing trigonometry concepts).
  - Have students explore problems where they have to set up the Pythagorean Theorem from word problems involving complex shapes (such as rectangles within a triangle).

Lesson 3: Apply the Pythagorean Theorem in the Coordinate Plane

Student Learning Intentions (WALT):

- We are learning to apply the Pythagorean Theorem to find distances between points in the coordinate plane.
- We are learning to use the Pythagorean Theorem in the coordinate plane to solve real-world problems involving distance.

Student Success Criteria (I Can Statements):

- I can find the distance between two points on a coordinate plane using the Pythagorean Theorem.
- I can apply the Pythagorean Theorem to solve real-world problems involving coordinate geometry.
- I can interpret the distance formula in the context of the coordinate plane.

Instructional Strategies and Activities

Engage (10 minutes)

- **Hook:**
    - Start by asking: "How would we measure the distance between two points on a map?"
    - Draw a simple coordinate plane on the board and place two points on it, asking students how they might calculate the distance between them.
- **Introduce the Concept of Distance on a Coordinate Plane:**
    - Discuss the idea that finding the distance between two points on the coordinate plane is similar to finding the length of the hypotenuse of a right triangle. The horizontal and vertical differences between the points form the legs of a right triangle, and the distance between the points is the hypotenuse.

Explore (20 minutes)

- **Teacher Modeling:**
    - **Problem Example:**
        - Given two points A(1,2)A(1, 2)A(1,2) and B(4,6)B(4, 6)B(4,6), find the distance between them.
        - **Step 1:** Label the coordinates on the coordinate plane.
        - **Step 2:** Draw a right triangle by connecting the points horizontally and vertically. The horizontal leg is the difference in the x-coordinates:

          4−1=34 - 1 = 34−1=3

          The vertical leg is the difference in the y-coordinates:

          6−2=46 - 2 = 46−2=4
        - **Step 3:** Use the Pythagorean Theorem to find the distance (hypotenuse):

          a2+b2=c2a^2 + b^2 = c^2a2+b2=c2 32+42=c23^2 + 4^2 = c^232+42=c2 9+16=c29 + 16 = c^29+16=c2 25=c225 = c^225=c2 c=25c = \sqrt{25}c=25 c=5c = 5c=5
        - The distance between points A(1,2)A(1, 2)A(1,2) and B(4,6)B(4, 6)B(4,6) is 5 units.
- **Class Discussion:**
    - Discuss the steps and reasoning behind finding the distance between two points. Highlight the relationship between the differences in the x and y coordinates and the legs of a right triangle.
    - Emphasize that the Pythagorean Theorem is used to solve for the hypotenuse, which in this case represents the distance between the two points.

Explain (10 minutes)

- **Distance Formula:**
  - Introduce the **Distance Formula** as a result of applying the Pythagorean Theorem in the coordinate plane:

    d=(x2−x1)2+(y2−y1)2d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}d=(x2−x1)2+(y2−y1)2

    where (x1,y1)(x_1, y_1)(x1,y1) and (x2,y2)(x_2, y_2)(x2,y2) are the coordinates of two points, and ddd is the distance between them.
  - Explain how this formula is derived from the Pythagorean Theorem, with the differences in the x and y coordinates corresponding to the legs of the right triangle.
  - Work through another example as a class, using the distance formula to find the distance between points C(3,5)C(3, 5)C(3,5) and D(7,9)D(7, 9)D(7,9).

Elaborate (15 minutes)

- **Partner Practice:**
  - Give pairs of students a set of problems where they must use the distance formula to calculate the distance between pairs of points on the coordinate plane.
  - Example problems:
    - Find the distance between A(2,3)A(2, 3)A(2,3) and B(6,7)B(6, 7)B(6,7).
    - Find the distance between P(−1,4)P(-1, 4)P(−1,4) and Q(3,−2)Q(3, -2)Q(3,−2).
    - Find the distance between M(0,0)M(0, 0)M(0,0) and N(4,3)N(4, 3)N(4,3).
- **Real-World Problem:**
  - Provide a real-world problem that requires using the distance formula:
    - **Example:** "A drone is flying from point A(2,3)A(2, 3)A(2,3) to point B(6,8)B(6, 8)B(6,8) in a city grid. How far does the drone travel?"
    - Students will calculate the distance between the points using the distance formula and apply it to the problem.

Closure (5 minutes)

- **Exit Ticket:**
  - Provide a final question for students to complete individually to check their understanding.

Example:

  - "Find the distance between the points A(1,−2)A(1, -2)A(1,−2) and B(4,1)B(4, 1)B(4,1)."

- Collect exit tickets to assess understanding.

Formative Assessments:

- Observation during the teacher modeling and class discussion.
- Partner practice responses.
- Exit ticket responses to assess individual understanding.

Instructional Materials and Resources:

- Whiteboard and markers for teacher demonstrations.
- Graph paper for students to draw coordinate planes and plot points.
- Pre-prepared handouts with problems for partner practice.
- Exit tickets.
- Calculators (optional, for more complex calculations).

Reflections (Post-Lesson):

- Did students grasp the relationship between the Pythagorean Theorem and the distance formula in the coordinate plane?
- Were students able to apply the distance formula in both mathematical and real-world contexts?
- Which types of problems did students find most challenging, and how can I address these areas in future lessons?

Suggested Modifications:

- **For Support:**
  - Provide more structured problems with pre-drawn coordinate grids to help students visualize the distance formula.

    - Use step-by-step guides to support students in applying the formula correctly.

- **For Enrichment:**
    - Challenge advanced students with more complex problems, such as finding the distance between points in 3D space or solving problems with larger coordinate values.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
    - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
    - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals**:
    - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary**:
    - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
    - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
    - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.
- **Extra Time**:

    - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.
- **Oral Reading**:
    - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
    - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
    - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
    - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
    - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
    - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
    - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
    - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.
- **Peer Support**:
    - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
    - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
    - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
    - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
    - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.
- **Tests/Assessments**:
    - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:
    - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and

instruction

- **Verbal Example**:
    - Fist to five: "How well do you understand what we talked about today?"
    - Fist to five: "How well did I teach this today?"
- **Classroom Activity**:
    - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
    - Color-coding ideas
    - Learning vocabulary in student languages
    - Visual and kinesthetic learning
    - Representations of learning without words
- **Classroom Activity**:
    - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
    - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
- **Classroom Activity**:
    - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:

- "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"

- **Classroom Activity**:
  - Learn about different bases and numerical ideas:
    - Base 2 (binary) and connections to computer programming
    - How the Yoruba of Nigeria used base 20
    - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
  - "How might you all go about this?"
  - "What do you notice?"
- **Classroom Activity**:
  - Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
  - "How does this connect to what you've learned in the past?"
  - "How can you use that knowledge today?"
- **Classroom Activity**:
  - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
  - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"

- **Classroom Activity**:
  - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
  - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
  - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
  - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
  - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
  - "Let’s get into partners and do a think-pair-share. We will incorporate everyone’s ideas and try to synthesize them."
- **Classroom Activity**:
  - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
    - "Come up with at least two answers that might solve this problem."
- **Classroom Activity**:
    - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
    - **Classroom Activity**:
        - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
    - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
    - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:
    - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:

    - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
    - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).
- **Climate Change Example**:
    - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MATH.8.G.B	Understand and apply the Pythagorean Theorem
MATH.8.G.B.6	Explain a proof of the Pythagorean Theorem and its converse.
MATH.8.G.B.7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
MATH.8.G.B.8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

21st Century Life and Career

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Integration of Career Readiness. Life Literacies and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.

Integration of Computer Science and Design Thinking

CS.K-2.AP	Algorithms & Programming

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

LA.K-12.NJSLSA.R	Reading
	Note on range and content of student reading
SCI.9-12.5.1.12.D.2	Represent ideas using literal representations, such as graphs, tables, journals, concept maps, and diagrams.

Module 15: Exponents and Scientific Notations

Unit Rationale

The ability to work with exponents and scientific notation is essential for students to engage with large and small quantities efficiently and accurately—skills that are crucial in mathematics, science, and real-world applications. This unit equips students with the tools to represent, interpret, and operate with numbers using the laws of exponents and scientific notation, building a foundation for future algebraic and scientific work.

Students will explore how repeated multiplication leads to exponential expressions and how exponential notation offers a more efficient way to represent very large or very small numbers. They will apply the properties of integer exponents to simplify expressions and evaluate powers, ensuring they understand both the structure and logic behind the rules. Students will also learn how scientific notation allows us to express quantities in manageable ways—such as the size of a virus or the distance between planets—while performing operations like multiplication and division using exponent rules.

This unit supports students in developing number sense, precision, and fluency in mathematical communication. It emphasizes the conceptual understanding of exponents as well as procedural fluency in applying rules, making it a critical step toward success in algebra, physics, and other STEM fields.

Throughout this unit, students will:

- Use and understand the meaning of integer exponents.
- Apply the laws of exponents to simplify expressions.
- Convert between standard form and scientific notation.
- Perform operations with numbers in scientific notation.
- Solve real-world problems involving very large or very small quantities.

This unit aligns closely with several Standards for Mathematical Practice:

- **MP2: Reason abstractly and quantitatively** — as students make sense of exponential patterns and apply them to real contexts.
- **MP4: Model with mathematics** — by using scientific notation to represent real-world quantities.
- **MP6: Attend to precision** — when expressing numbers using correct notation and applying exponent

rules accurately.

- **MP7: Look for and make use of structure** — by recognizing and applying the consistent rules governing exponential expressions.

By the end of this unit, students will not only be able to manipulate and operate with exponential expressions and numbers in scientific notation, but also understand why these tools are necessary and how they are used in both academic and real-world contexts.

Essential Questions

- What are exponents, and how do they represent repeated multiplication?
- How can the properties of exponents be used to simplify and evaluate expressions?
- What is scientific notation, and why is it used to represent very large or very small numbers?
- How can we convert between standard form and scientific notation?
- In what ways do exponents and scientific notation help us solve real-world problems more efficiently?
- How do the operations of multiplication, division, and addition apply to numbers in scientific notation?
- Why is precision important when working with scientific notation and exponents?

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Know and Apply Properties of Exponents

Student Learning Intentions (WALT):

- We are learning to understand and apply the properties of exponents.
- We are learning to simplify expressions using the laws of exponents, including the product rule, quotient rule, and power of a power rule.

Student Success Criteria (I Can Statements):

- I can identify and use the properties of exponents to simplify expressions.
- I can apply the product rule, quotient rule, and power of a power rule to solve problems.
- I can evaluate expressions with exponents and explain my reasoning.

Instructional Strategies and Activities

Engage (10 minutes)

- **Hook:** Show a quick, relatable example: “If you have 3 copies of 3 and multiply them together, you get 3×3×33 \times 3 \times 33×3×3. But how can we write that more efficiently?”
- **Quick Discussion:** Introduce the concept of exponents.
  - *Example:* 33=3×3×33^3 = 3 \times 3 \times 333=3×3×3 and explain how exponents are used to

represent repeated multiplication.

- **Think-Pair-Share:** “What do you think happens when we multiply powers with the same base, like 23×222^3 \times 2^223×22?”

Explore (20 minutes)

- **Hands-On Activities:**
    1. **Product Rule Station:**
        - Introduce the product rule of exponents: am×an=am+na^m \times a^n = a^{m+n}am×an=am+n.
        - Have students simplify expressions like 32×343^2 \times 3^432×34.
        - Provide examples with different bases and exponents.
    2. **Quotient Rule Station:**
        - Explain the quotient rule: aman=am−n\frac{a^m}{a^n} = a^{m-n}anam=am−n.
        - Students will simplify expressions such as 5653\frac{5^6}{5^3}5356.
    3. **Power of a Power Station:**
        - Introduce the power of a power rule: (am)n=am×n(a^m)^n = a^{m \times n}(am)n=am×n.
        - Have students simplify expressions like (23)2(2^3)^2(23)2.
- **Activity Rotation:** Students rotate through each station, solving problems and discussing the rules in small groups.

Explain (10 minutes)

- **Teacher Modeling:**
    - Go over each exponent rule with detailed examples on the whiteboard:
        - Product Rule: 32×34=32+4=363^2 \times 3^4 = 3^{2+4} = 3^632×34=32+4=36.
        - Quotient Rule: 5653=56−3=53\frac{5^6}{5^3} = 5^{6-3} = 5^35356=56−3=53.
        - Power of a Power: (23)2=23×2=26(2^3)^2 = 2^{3 \times 2} = 2^6(23)2=23×2=26.
    - Reinforce how each rule applies to both numbers and variables (e.g., xa×xb=xa+bx^a \times x^b = x^{a+b}xa×xb=xa+b).

- **Whole-Class Discussion:**
  - Why do the rules work the way they do? Connect back to the idea of repeated multiplication.
  - Use visual aids (like diagrams or number lines) to show the simplification process.

Elaborate (15 minutes)

- **Partner Practice:**
  - Distribute a worksheet or set of problems with mixed exponent rules (product rule, quotient rule, power of a power).
  - Have students work together to simplify the expressions and check each other's work.
  - Problem Set Examples:
    - Simplify: 25×232^5 \times 2^325×23
    - Simplify: 7873\frac{7^8}{7^3}7378
    - Simplify: (42)3(4^2)^3(42)3
    - Simplify: 34×3−23^4 \times 3^{-2}34×3−2
- **Check for Understanding:**
  - Walk around the room to provide feedback and check for understanding.

Closure (5 minutes)

- **Exit Ticket:**
  - One problem: Simplify 33×323^3 \times 3^233×32.
  - One question: What rule do you use to simplify expressions like 57×545^7 \times 5^457×54? Explain why the rule works.

Formative Assessments:

- Observation and questioning during station rotations.
- Partner practice responses.
- Exit ticket responses.

- Whole-class discussion feedback.

Instructional Materials and Resources:

- Whiteboard and markers
- Graphing calculators (optional, for evaluating large exponents)
- Worksheets with mixed exponent problems
- Exit tickets
- Station task cards or slides for product rule, quotient rule, and power of a power rule
- Online resources (optional, for extra practice)

Reflections (Post-Lesson):

- Were students able to apply the exponent rules effectively to simplify expressions?
- Which rule did students find most challenging to apply?
- Did students struggle with conceptual understanding, or was it more about procedural fluency?
- Did all students actively engage in the station activities?

Suggested Modifications:

- **For Support:**
  - Provide step-by-step guides on how to apply each exponent rule.
  - Offer visual aids (like charts or diagrams) showing examples of each rule.
- **For Enrichment:**
  - Include more complex problems involving negative exponents or zero exponents.
  - Have students create their own problems to challenge peers with varying bases and exponents.

Lesson 2: Understand Scientific Notation

Student Learning Intentions (WALT):

- We are learning to understand and use scientific notation to represent very large and very small numbers.
- We are learning to convert numbers between standard form and scientific notation.

Student Success Criteria (I Can Statements):

- I can explain what scientific notation is and why it is used.
- I can convert numbers from standard form to scientific notation and vice versa.
- I can perform operations (multiplication and division) with numbers in scientific notation.
- I can express numbers in scientific notation when appropriate for large or small values.

Instructional Strategies and Activities

Engage (10 minutes)

- **Hook:**
  - Ask students, "What is the size of the universe?" or "How small are atoms?"
  - Display a large number like the distance from the Earth to the Sun (approximately 93 million miles) or the mass of a proton (about 0.00000000000000000000000000000000000167 kg).
  - Introduce the challenge: *How can we express these very large or very small numbers more easily?*
- **Introduction to Scientific Notation:**
  - Explain that scientific notation is used to write such numbers in a more manageable form, using powers of 10.

Explore (20 minutes)

- **Hands-On Activity:**
  - **Scientific Notation Station:**
    - Introduce scientific notation: a×10na \times 10^na×10n, where aaa is a number between 1 and 10, and nnn is an integer.

        - Show examples:
            - 5,000=5×1035,000 = 5 \times 10^35,000=5×103
            - 0.0000003=3×10−70.0000003 = 3 \times 10^{-7}0.0000003=3×10−7
        - Have students practice converting numbers between standard form and scientific notation.
            - Examples:
                - Convert 400,000 to scientific notation.
                - Convert 2.5×1042.5 \times 10^42.5×104 to standard form.
        - Provide task cards or worksheets with various numbers for students to convert.
            - **Example:** Convert 0.0000420.0000420.000042 to scientific notation.

- **Partner Work:**
    - After initial practice, students work in pairs to convert a series of numbers, first from standard form to scientific notation, and then from scientific notation back to standard form.
    - Afterward, partners explain to each other how they arrived at their solutions.

Explain (10 minutes)

- **Teacher Modeling:**
    - Show the step-by-step process for converting large and small numbers between standard form and scientific notation.
    - Use visual aids, such as a number line or place-value chart, to help students understand how to move the decimal point when converting to scientific notation.
    - **Example 1 (Large Number):** Convert 120,000 to scientific notation:
        - Move the decimal 5 places left: 1.2×1051.2 \times 10^51.2×105.
    - **Example 2 (Small Number):** Convert 0.0000078 to scientific notation:
        - Move the decimal 6 places right: 7.8×10−67.8 \times 10^{-6}7.8×10−6.
- **Clarify:**
    - Emphasize that the exponent tells how many places the decimal point is moved.
    - Explain that for large numbers, the exponent is positive, and for small numbers, the exponent is negative.

Elaborate (15 minutes)

- **Independent Practice:**
    - Provide a worksheet or interactive activity where students practice converting various numbers to and from scientific notation.
        - Examples:
            - Convert 3,200 to scientific notation.
            - Convert 7×10−37 \times 10^{-3}7×10−3 to standard form.
            - Convert 0.00012 to scientific notation.
- **Real-World Application Problem:**
    - **Example Problem:**
        - "The distance from Earth to the nearest star (other than the Sun) is about 4.22 light-years. Write this distance in scientific notation. Then, explain how you could use this in a real-world context (e.g., space travel, astronomy)."
    - Have students work through this in pairs or individually and then share their solutions.

Closure (5 minutes)

- **Exit Ticket:**
    - One problem: Convert the number 0.00065 to scientific notation.
    - One reflective question: Why is scientific notation useful, especially in scientific and mathematical contexts?

Formative Assessments:

- Observation and questioning during partner work.
- Review of student practice during independent work.
- Exit ticket responses to gauge individual understanding.

Instructional Materials and Resources:

- Whiteboard and markers
- Scientific notation worksheets or task cards
- Digital resources (optional, for extra practice)
- Exit tickets
- Calculators (optional, for verifying large or small numbers)

Reflections (Post-Lesson):

- Did students grasp the process of converting between scientific notation and standard form?
- Which types of problems (large numbers vs. small numbers) did students find more challenging?
- Were students able to explain the importance and application of scientific notation?

Suggested Modifications:

- **For Support:**
    - Provide a reference chart with steps for converting between standard form and scientific notation.
    - Use visual aids like number lines or place-value grids to help students understand how to move the decimal.
- **For Enrichment:**
    - Ask students to explore more advanced problems, such as adding or subtracting numbers in scientific notation.
    - Challenge them to find real-world examples where scientific notation is applied, such as in scientific research or technology.

Lesson 3: Compute with Scientific Notation

Student Learning Intentions (WALT):

- We are learning to perform arithmetic operations (multiplication, division) with numbers in scientific notation.
- We are learning to apply the properties of exponents to simplify computations involving scientific

notation.

Student Success Criteria (I Can Statements):

- I can multiply numbers in scientific notation.
- I can divide numbers in scientific notation.
- I can simplify expressions involving scientific notation by applying exponent rules.

Instructional Strategies and Activities

Engage (10 minutes)

- Hook:
  - Present a problem: "NASA's Voyager 1 spacecraft is currently about 2.7 billion miles from Earth. Write this distance in scientific notation."
  - Ask students to think about how they might compute with such large numbers.
  - Introduce the challenge of multiplying and dividing numbers written in scientific notation to make calculations simpler.
- Introduction to Operations with Scientific Notation:
  - Review how scientific notation simplifies large and small numbers, and explain that performing arithmetic operations is also easier using scientific notation.

Explore (20 minutes)

- Hands-On Activity: Multiplying Scientific Notation
  - Teacher Modeling:
    - Example: Multiply (2×103)×(4×102)(2 \times 10^3) \times (4 \times 10^2)(2×103)×(4×102)
      - Step 1: Multiply the numbers (2 × 4 = 8).
      - Step 2: Apply the rule for exponents: 103×102=103+2=10510^3 \times 10^2 = 10^{3+2} = 10^5103×102=103+2=105.
      - Result: 8×1058 \times 10^58×105.
    - Discuss how the exponents of 10 are added when multiplying numbers in scientific

notation.

- Practice:
  - Have students work through multiplication problems in pairs or small groups:
    - (3×104)×(2×105)(3 \times 10^4) \times (2 \times 10^5)(3×104)×(2×105)
    - (6×102)×(3×103)(6 \times 10^2) \times (3 \times 10^3)(6×102)×(3×103)
    - (5×106)×(2×107)(5 \times 10^6) \times (2 \times 10^7)(5×106)×(2×107)

- Hands-On Activity: Dividing Scientific Notation
  - Teacher Modeling:
    - Example: Divide (6×104)÷(3×102)(6 \times 10^4) \div (3 \times 10^2)(6×104)÷(3×102)
      - Step 1: Divide the numbers (6 ÷ 3 = 2).
      - Step 2: Apply the rule for exponents: 104÷102=104−2=10210^4 \div 10^2 = 10^{4-2} = 10^2104÷102=104−2=102.
      - Result: 2×1022 \times 10^22×102.
    - Discuss how the exponents of 10 are subtracted when dividing numbers in scientific notation.
  - Practice:
    - Have students work through division problems in pairs or small groups:
      - (9×106)÷(3×103)(9 \times 10^6) \div (3 \times 10^3)(9×106)÷(3×103)
      - (5×108)÷(2×104)(5 \times 10^8) \div (2 \times 10^4)(5×108)÷(2×104)
      - (7×105)÷(7×102)(7 \times 10^5) \div (7 \times 10^2)(7×105)÷(7×102)

Explain (10 minutes)

- Teacher Modeling:
  - Revisit the key rules for multiplying and dividing numbers in scientific notation:
    - Multiplying: Multiply the coefficients and add the exponents of 10.
    - Dividing: Divide the coefficients and subtract the exponents of 10.
  - Emphasize that when multiplying or dividing, the base (10) remains constant, and only the exponents change.

- Class Discussion:
  - Discuss the concept of scientific notation consistency:
    - For example, 8×1058 \times 10^58×105 is equivalent to 0.8×1060.8 \times 10^60.8×106, because the exponent can shift and the coefficient can adjust accordingly.
  - Example: Convert 8×1058 \times 10^58×105 to 0.8×1060.8 \times 10^60.8×106, showing how it keeps the number in scientific notation but adjusts for consistency.

Elaborate (15 minutes)

- Real-World Application Problem:
  - Example Problem:
    - The diameter of the Milky Way galaxy is approximately 1.0×1051.0 \times 10^51.0×105 light-years. The distance between the Earth and the nearest galaxy (Andromeda) is approximately 2.5×1062.5 \times 10^62.5×106 light-years. What is the ratio of the distance from the Earth to the nearest galaxy to the diameter of the Milky Way galaxy?
  - Have students solve this by dividing the two numbers in scientific notation:
    - 2.5×1061.0×105\frac{2.5 \times 10^6}{1.0 \times 10^5}1.0×1052.5×106
- Partner Practice:
  - After reviewing the solution, provide a set of problems for students to complete independently or in pairs:
    - (4×103)×(3×102)(4 \times 10^3) \times (3 \times 10^2)(4×103)×(3×102)
    - (8×106)÷(4×102)(8 \times 10^6) \div (4 \times 10^2)(8×106)÷(4×102)
    - (2.5×108)×(5×10−3)(2.5 \times 10^8) \times (5 \times 10^{-3})(2.5×108)×(5×10−3)
  - Circulate around the room to assist with any challenges.

Closure (5 minutes)

- Exit Ticket:
  - One problem: Multiply (2×103)×(5×102)(2 \times 10^3) \times (5 \times 10^2)(2×103)×(5×102).
  - One reflective question: How would you explain to a friend the steps you take to multiply two numbers in scientific notation?

Formative Assessments:

- Observation and questioning during hands-on activities.
- Student responses to real-world application problems.
- Exit ticket responses to assess individual understanding.

Instructional Materials and Resources:

- Whiteboard and markers
- Scientific notation worksheets with multiplication and division problems
- Calculators (optional, for verifying large or small numbers)
- Exit tickets
- Visual aids or charts for reference

Reflections (Post-Lesson):

- Did students understand the process of multiplying and dividing scientific notation?
- Were students able to apply exponent rules correctly to simplify scientific notation operations?
- How did students respond to real-world application problems—did they see the value of scientific notation?

Suggested Modifications:

- For Support:
  - Provide a step-by-step guide for multiplying and dividing scientific notation.
  - Use visual aids (such as number lines or charts) to reinforce the steps involved in the operations.
- For Enrichment:
  - Introduce problems that require students to adjust the coefficient to maintain proper scientific notation (e.g., converting a result like 12×10512 \times 10^512×105 to 1.2×1061.2 \times 10^61.2×106).

- Challenge students to apply scientific notation operations in complex, real-world scenarios (e.g., comparing measurements in space research).

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
  - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
  - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals**:
  - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary**:
  - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
  - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
  - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.
- **Extra Time**:
  - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.

- **Oral Reading**:
  - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
  - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
  - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
  - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
  - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
  - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
  - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
  - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check

back to ensure they haven't forgotten.

- **Peer Support**:
    - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
    - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
    - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
    - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
    - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.
- **Tests/Assessments**:
    - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:
    - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
    - Fist to five: "How well do you understand what we talked about today?"

- Fist to five: "How well did I teach this today?"

- **Classroom Activity**:
  - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
  - Color-coding ideas
  - Learning vocabulary in student languages
  - Visual and kinesthetic learning
  - Representations of learning without words
- **Classroom Activity**:
  - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
  - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
- **Classroom Activity**:
  - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
  - "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
  - Learn about different bases and numerical ideas:

        - Base 2 (binary) and connections to computer programming
        - How the Yoruba of Nigeria used base 20
        - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
    - "How might you all go about this?"
    - "What do you notice?"
- **Classroom Activity**:
    - Incorporate explorations where students interact with mathematics in a way that allows them to “discover” or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
    - "How does this connect to what you’ve learned in the past?"
    - "How can you use that knowledge today?"
- **Classroom Activity**:
    - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
    - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"
- **Classroom Activity**:
    - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
  - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
  - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
  - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
  - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
  - "Let's get into partners and do a think-pair-share. We will incorporate everyone's ideas and try to synthesize them."
- **Classroom Activity**:
  - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
  - "Come up with at least two answers that might solve this problem."

- **Classroom Activity**:
    - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
    - **Classroom Activity**:
        - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
    - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
    - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:
    - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:
    - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
    - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).
- **Climate Change Example**:
  - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MATH.8.EE.A.4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

21st Century Life and Career

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Integration of Career Readiness, Life Literacies and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP7	Employ valid and reliable research strategies.

Integration of Computer Science and Design Thinking

	Individuals use computing devices to perform a variety of tasks accurately and quickly. Computing devices interpret and follow the instructions they are given literally.
	Data can be used to make predictions about the world.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

	Language: System and structure, effective use, and vocabulary
ELA.K-12.R.CT	Comparison of Texts: By the end of grade 12, analyze and reflect on how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.
ELA.K-12.W.RW	Range of Writing: By the end of grade 12, write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.
SCI.9-12.5.2.12	All students will understand that physical science principles, including fundamental ideas about matter, energy, and motion, are powerful conceptual tools for making sense of phenomena in physical, living, and Earth systems science.