

Unit 5: Similarity, Slope, and Linear Relationships

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **9 Weeks**
Status: **Published**

Module 11: Angle Relationships

Unit Rationale

Angle relationships are fundamental to understanding geometric reasoning and are essential for students to progress in more advanced mathematical concepts, including proofs, transformations, and trigonometry. In this module, students investigate the properties and relationships between different types of angles, such as complementary, supplementary, vertical, and corresponding angles, and how these relationships arise in both simple and complex geometric contexts.

This unit is strategically placed to help students build a solid foundation for understanding the behavior of angles when intersecting lines or parallel lines are involved. These concepts are vital not only for geometry but also for algebraic reasoning, as they are often used to solve for unknown values in equations and real-world problems.

The study of angle relationships extends beyond theoretical knowledge and plays a significant role in real-world applications. Students will recognize how angles and their relationships are used in architecture, design, engineering, navigation, and various fields where geometry and spatial reasoning are important. Through hands-on activities and problem-solving, students will develop the ability to reason logically, apply geometric principles, and make informed decisions based on angle relationships.

Throughout the module, students will also engage with key Standards for Mathematical Practice, including:

- **MP2:** Reason abstractly and quantitatively
- **MP4:** Model with mathematics
- **MP7:** Look for and make use of structure

By the end of this module, students will be able to identify and apply the different types of angle relationships in both geometric figures and real-world contexts. They will gain confidence in solving problems involving parallel lines, transversals, and angle measures, and be well-prepared to apply these skills to more advanced geometric concepts in later studies. This unit reinforces the idea that angle relationships are not just abstract concepts but practical tools that help solve problems in the world around us.

Essential Questions

- What is the relationship between complementary and supplementary angles, and how can we use this knowledge to solve real-world problems?
- How do vertical angles help us understand the properties of intersecting lines?
- How are corresponding, alternate interior, and alternate exterior angles related when two parallel lines are cut by a transversal?
- How can angle relationships be used to prove that lines are parallel or to solve for unknown angles in geometric figures?
- What role do angle relationships play in understanding polygons, particularly triangles and quadrilaterals?
- How can angle relationships help us analyze and solve problems in geometric transformations, such as reflections and rotations?
- In what ways can we apply angle relationships to design and architecture, such as in the construction of buildings, bridges, or maps?

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)

- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning

- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)

- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks

- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Develop Angle Relationships for Triangles

Student Learning Intentions or We are learning to ... (WALT):

- We are learning to calculate the sum of interior angles in triangles.
 - We are learning to solve for missing angles in triangles using the Triangle Angle Sum Theorem.
 - We are learning to apply angle relationships (complementary, supplementary, and vertical) to triangles.
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Student Success Criteria ... "I can statements":

- I can define the Triangle Angle Sum Theorem and explain its meaning.
 - I can calculate the missing angles in a triangle using the Triangle Angle Sum Theorem.
 - I can solve real-world problems involving triangles and angle relationships.
 - I can use the concepts of complementary and supplementary angles to find missing angles in triangles.
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Instructional Strategies and Activities:

Introduction to Triangle Angle Relationships (10 minutes):

- **Direct Instruction:**

Begin by explaining that in every triangle, the sum of the interior angles is always 180° . Write the equation on the board:

$$\angle A + \angle B + \angle C = 180^\circ \quad \angle A + \angle B + \angle C = 180^\circ$$

Explain that this rule is the **Triangle Angle Sum Theorem**.

Discuss how this property helps us find missing angles when we know the measures of the other two angles in a triangle.

- **Example 1:**

Given a triangle with angles $\angle A = 40^\circ$ and $\angle B = 60^\circ$, ask the class to find $\angle C$.

○ **Solution:**

$$\angle C = 180^\circ - (40^\circ + 60^\circ) = 180^\circ - 100^\circ = 80^\circ$$

Visual Models:

● **Model the Concept:**

Show a diagram of a triangle on the board and label its angles. Display the relationship between the angles using a visual model. Show that regardless of the triangle's size or shape, the sum of the three interior angles is always 180° .

● **Interactive Visual:**

Use an interactive app or online tool (if available) to show how changing one angle impacts the other angles in the triangle while maintaining the 180° sum.

Guided Practice (15 minutes):

1. **Solving for Missing Angles in Triangles:**

Walk through a few more examples on the board. Involve the class by asking for suggestions and calculations.

○ **Example 2:**

Given $\angle A = 75^\circ$ and $\angle B = 50^\circ$, find $\angle C$.

▪ **Solution:**

$$\angle C = 180^\circ - (75^\circ + 50^\circ) = 180^\circ - 125^\circ = 55^\circ$$

○ **Example 3:**

Given $\angle A = 120^\circ$ and $\angle B = 30^\circ$, find $\angle C$.

▪ **Solution:**

$$\angle C = 180^\circ - (120^\circ + 30^\circ) = 180^\circ - 150^\circ = 30^\circ$$

Engage students by asking questions like:

- "What happens if we change the size of one angle?"
- "How do you know you've found the correct angle?"

2. **Angle Relationships:**

Discuss the application of **complementary** and **supplementary angles** in triangles.

- **Complementary Angles:** Two angles that add up to 90° . (Applicable for right triangles)
 - **Supplementary Angles:** Two angles that add up to 180° .
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Independent Practice (15 minutes):

- **Worksheet:**

Provide a set of problems where students calculate missing angles in triangles. Include both simple problems (where two angles are given) and slightly more complex problems (where they need to apply complementary or supplementary angles to find missing angles).

Example problems:

- A triangle has angles $\angle A = 45^\circ$, $\angle B = 80^\circ$. Find $\angle C$.
- A triangle has $\angle A = 90^\circ$, $\angle B = 45^\circ$. Find $\angle C$.
- A triangle has $\angle A = 30^\circ$ and $\angle B = 60^\circ$. What is $\angle C$?

Students will work independently to solve for the missing angles in the triangles.

Peer Collaboration (10 minutes):

- **Think-Pair-Share:**

Pair up students to compare answers to the worksheet problems. Each student explains their reasoning to the partner and checks if they made any calculation errors. After discussion, pairs will share their answers and reasoning with the class.

Encourage students to ask questions about any parts they did not understand, and guide them to clarify the concept of the Triangle Angle Sum Theorem.

Formative Assessments:

1. **Exit Ticket:**

After the lesson, ask students to solve two problems as an exit ticket:

- **Problem 1:** A triangle has $\angle A = 50^\circ$, $\angle B = 70^\circ$. What is $\angle C$?
- **Problem 2:** A triangle has $\angle A = 60^\circ$, $\angle B = 40^\circ$. What is $\angle C$?

Collect exit tickets to assess student understanding.

2. **Observation and Questioning:**

Walk around during independent practice and peer collaboration. Ask students how they are applying the Triangle Angle Sum Theorem and whether they are using angle relationships correctly.

Example question:

"How did you find the missing angle in this triangle? Can you explain the reasoning behind your steps?"

Instructional Materials and Resources:

- **Whiteboard/Smartboard:** For modeling and demonstrating angle relationships.
- **Worksheets/Problem Sets:** A set of practice problems for students to calculate missing angles in triangles.
- **Interactive Tools (Optional):** Online tools or apps for demonstrating triangle angle sum theorem dynamically.
- **Handouts/Reference Sheets:** A handout outlining the Triangle Angle Sum Theorem with a step-by-step guide for solving missing angles in triangles.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how students responded to solving for missing angles in triangles. Did they grasp the Triangle Angle Sum Theorem quickly? Were students able to apply the concept to real-world situations or more complex problems?
- Did students struggle with visualizing angle relationships, or was it easier for them to solve using numeric problems?

Suggested Modifications:

- **Differentiation:** For students who may struggle, provide additional practice with simpler triangles or use visual aids such as interactive tools where they can manipulate angles to see how the sum always equals 180° .
- **Enrichment:** For students who grasp the concept quickly, provide more challenging problems involving triangles in different contexts (e.g., non-right triangles or using angle relationships in geometric designs).

Lesson 2: Investigate Angle-Angle Similarity

Student Learning Intentions (WALT):

- We are learning to identify similar triangles using the Angle-Angle (AA) Similarity Criterion.
- We are learning to explain how angle congruence in triangles leads to similarity.
- We are learning to apply AA similarity to solve problems involving proportional side lengths in similar triangles.

Student Success Criteria (“I can” statements):

- I can explain what it means for two triangles to be similar using the AA Similarity Criterion.
 - I can identify corresponding angles and sides in similar triangles.
 - I can use proportions to solve for missing side lengths in similar triangles.
 - I can justify triangle similarity using angle relationships such as vertical angles and alternate interior angles.
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Instructional Strategies and Activities

Introduction to AA Similarity (10 minutes)

Direct Instruction:

- Begin by reviewing what makes triangles *similar* (same shape, but not necessarily same size).
- Introduce the **AA Similarity Criterion**: *If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.*
- Use diagrams to show two triangles with two pairs of congruent angles. Mark the congruent angles with arcs.
- Emphasize that if two angles are congruent, the third angle must also be congruent (Triangle Angle Sum Theorem).

Example 1 (Visual Proof):

- Show triangle ABC and triangle DEF with $\angle A \cong \angle D$ and $\angle B \cong \angle E$.
 - Ask: “What can you conclude about triangles ABC and DEF?”
 - Answer: By AA, triangle ABC \sim triangle DEF.
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Visual Models:

Model the Concept:

- Use colored overlays or tracing paper to align corresponding angles and visualize the similarity.
- Show that the triangles are not necessarily the same size, but angles match and sides are proportional.

Interactive Visuals (if available):

- Use an online geometry tool (like GeoGebra) to dynamically change triangle sizes while preserving two angles.
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Guided Practice (15 minutes)

Example 2: Given triangle XYZ and triangle MNO where $\angle X \cong \angle M$ and $\angle Y \cong \angle N$, prove the triangles are similar.

Solution: By AA Criterion, triangle XYZ \sim triangle MNO.

Example 3:

- Triangle ABC and triangle DEF share a vertex. $\angle A \cong \angle D$ (given), and $\angle C \cong \angle F$ (vertical angles).
- Ask: “Are the triangles similar?”
- Answer: Yes, by AA, the triangles are similar.

Guided Questions:

- How do vertical angles or alternate interior angles help us prove triangle similarity?
 - Why don't we need to compare all three angles?
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Angle Relationships in Similarity (5 minutes)

Mini-Review:

- Vertical Angles: Equal when two lines cross.
 - Alternate Interior Angles: Equal when parallel lines are cut by a transversal.
 - These angle relationships can be used to identify pairs of congruent angles in overlapping triangles or geometric diagrams.
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Independent Practice (15 minutes)

Worksheet Activities:

- Identify pairs of similar triangles in geometric figures using AA.
- Solve for missing side lengths in similar triangles by setting up and solving proportions.

Sample Problems:

1. Triangle PQR has angles 50° , 60° , and 70° . Triangle XYZ has angles 50° and 60° . Are they similar? Why?
 2. Triangle ABC \sim Triangle DEF. AB = 6 cm, AC = 9 cm, DE = 4 cm. Find DF.
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Peer Collaboration (10 minutes)

Think-Pair-Share:

- Pairs analyze diagrams, identify similar triangles using AA, and explain reasoning.
 - Share how they determined angle congruence (e.g., vertical, alternate interior, or given).
 - Present findings to class with justification.
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Formative Assessments

Exit Ticket:

1. Triangle A and triangle B have $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 5$. Are the triangles similar? Explain.
2. Triangle LMN \sim triangle XYZ. If LM = 8, MN = 6, and XY = 12, what is YZ?

Observation and Questioning:

- Ask students during peer work: “Which angles are congruent? What relationship do they have?”
 - Listen for accurate use of terminology (e.g., vertical, corresponding, similar).
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Instructional Materials and Resources:

- Diagrams of overlapping and parallel-line-based triangle figures
- Whiteboard/Smartboard

- Geometry software or interactive tools
 - Worksheets with triangle similarity identification and proportion problems
 - Student reference sheet: Angle relationships and AA Similarity notes
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Reflections and Suggested Modifications

Reflection:

- Did students connect angle congruence with triangle similarity?
- Were they able to justify similarity using visual evidence and vocabulary?

Modifications:

- **Support:** Provide angle relationship cards or scaffold diagrams with hints.
- **Enrichment:** Challenge early finishers with similarity in real-world contexts (e.g., shadows, scaled blueprints).

Lesson 3: Explore Parallel Lines Cut by a Transversal

Student Learning Intentions (WALT)

- We are learning to identify and define angle relationships formed when parallel lines are cut by a transversal.
 - We are learning to use angle relationships (corresponding, alternate interior, alternate exterior, and vertical angles) to find unknown angle measures.
 - We are learning to explain why certain pairs of angles are always congruent or supplementary.
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Student Success Criteria ("I can" statements)

- I can identify and name corresponding, alternate interior, alternate exterior, and vertical angles.
 - I can use angle relationships to solve for unknown angle measures.
 - I can explain why angles formed by parallel lines and a transversal are either congruent or supplementary.
 - I can use reasoning and mathematical vocabulary to justify my answers.
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Instructional Strategies and Activities

Introduction to Angle Relationships (10 minutes)

Direct Instruction:

- Define a **transversal** as a line that intersects two or more other lines.
- Draw two **parallel lines** cut by a transversal. Label all eight angles created.
- Introduce and define these angle relationships:
 - **Corresponding Angles**: Same relative position on each intersection.
 - **Alternate Interior Angles**: Inside the parallel lines, on opposite sides of the transversal.
 - **Alternate Exterior Angles**: Outside the parallel lines, on opposite sides of the transversal.
 - **Vertical Angles**: Opposite each other when two lines intersect.
 - **Same-Side Interior Angles** (also called consecutive interior): Inside the parallel lines and on the same side of the transversal.

Ask students to observe and describe the relationships between the angles. Which angles are equal? Which add up to 180° ?

Visual Models and Demonstrations

Model the Concept:

- Use color coding or overlays to highlight pairs of congruent and supplementary angles.
- Label all angles and show each angle pair visually with connecting lines or matching symbols.

Interactive Demonstration (optional):

- Use a geometry app or software like GeoGebra to adjust the position of the transversal and observe how angle relationships remain constant when lines are parallel.
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Guided Practice (15 minutes)

Example 1:

Given $\angle 1 = 65^\circ$, and two parallel lines cut by a transversal, find:

- $\angle 2$ (vertical angle) = 65°
- $\angle 5$ (corresponding angle) = 65°
- $\angle 4$ (alternate exterior) = 65°
- $\angle 3$ (supplementary to $\angle 1$) = 115°

Example 2:

If one angle is 110° , have students work through finding the remaining angle measures and identify their relationships.

Discussion Questions:

- How do you know which angles are congruent?
 - What clues tell you which angle relationships to use?
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Independent Practice (15 minutes)**Student Worksheet or Problem Set:**

- Identify all types of angle pairs in diagrams with parallel lines and a transversal.
- Solve for unknown angle measures using relationships.

Sample Problems:

1. Identify all pairs of corresponding angles in the given diagram.
 2. If $\angle 6 = 70^\circ$, find $\angle 1$, $\angle 3$, $\angle 4$, and $\angle 7$.
 3. Explain why $\angle 2$ and $\angle 8$ are congruent using appropriate mathematical vocabulary.
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Peer Collaboration (10 minutes)**Think-Pair-Share:**

- Students explain one angle relationship and how they used it to solve a problem.
 - Use sentence frames such as: “ $\angle A$ and $\angle B$ are alternate interior angles because they are inside the parallel lines and on opposite sides of the transversal.”
 - Share responses with the class and discuss.
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Formative Assessments

Exit Ticket:

1. Identify a pair of alternate interior angles in the diagram and explain why they are congruent.
2. Given $\angle 3 = 110^\circ$, find $\angle 6$ and justify your answer using the correct angle relationship.

Observation and Questioning:

- Walk around during practice and collaboration.
 - Ask students: “Which angle relationship are you using?” or “How do you know these two angles are equal?”
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Instructional Materials and Resources

- Whiteboard or Smartboard
 - Diagrams of parallel lines with a transversal
 - Angle relationship anchor chart
 - Geometry software or interactive tool (optional)
 - Worksheets with diagrams and problems on angle identification and calculation
 - Student handout or reference sheet summarizing angle pair definitions
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Reflections and Suggested Modifications**Reflection:**

- Did students correctly identify and describe angle relationships?
- Were they able to apply the correct rules to calculate unknown angle measures?
- Did they use mathematical vocabulary effectively when explaining their reasoning?

Modifications:

- **Support:** Use color-coded diagrams, pre-labeled figures, or a reference sheet with definitions.
- **Enrichment:** Provide real-world applications (e.g., architectural drawings or street maps) and challenge students to identify angle relationships in those contexts.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support:**

- The teacher provides auditory or written content to students in their native language.

- **Adjusted Speech:**

- The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

- **Visuals:**

- The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.

- **Front-Loading Vocabulary:**

- The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.
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Special Education Students

- **Chunking:**

- The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.

- **Checking for Understanding:**

- It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.

- **Extra Time:**

- The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.

- **Oral Reading:**

- The teacher will read work aloud to students, which can include class work, tests, and literature circles.

- **Timers:**

- The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking:**
 - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
 - **Checking for Understanding:**
 - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
 - **Extra Time:**
 - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.
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Gifted & Talented Strategies

- **Extensions/Enrichments:**
 - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
 - **Modify/Change Activities:**
 - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.
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Students at Risk of School Failure

- **Directions or Instructions:**
 - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.
- **Peer Support:**
 - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments:**
 - Teachers should consider modifying assignments for students at risk by simplifying tasks,

reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).

- **Increase One-on-One Time:**

- Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.

- **Contracts:**

- A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.

- **Hands-On Tasks:**

- Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.

- **Tests/Assessments:**

- Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.

- **Seating:**

- Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example:**

- Fist to five: "How well do you understand what we talked about today?"
- Fist to five: "How well did I teach this today?"

- **Classroom Activity:**

- Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.
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Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies:**
 - Color-coding ideas
 - Learning vocabulary in student languages
 - Visual and kinesthetic learning
 - Representations of learning without words
 - **Classroom Activity:**
 - Multilingual Frayer Models for definitions or concepts
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Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example:**
 - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
 - **Classroom Activity:**
 - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.
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Incorporate the history of mathematics into lessons

- **Verbal Example:**
 - "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"
 - **Classroom Activity:**
 - Learn about different bases and numerical ideas:
 - Base 2 (binary) and connections to computer programming
 - How the Yoruba of Nigeria used base 20
 - How the Mayans conceptualized the number 0 before the first recording of it
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Solicit student ways of thinking and processing

- **Verbal Example:**

- "How might you all go about this?"
- "What do you notice?"

- **Classroom Activity:**

- Incorporate explorations where students interact with mathematics in a way that allows them to “discover” or experience mathematics.
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Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example:**

- "How does this connect to what you’ve learned in the past?"
- "How can you use that knowledge today?"

- **Classroom Activity:**

- Learning webs that connect content
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Start with more complex math problems and scaffold as necessary

- **Verbal Example:**

- "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"

- **Classroom Activity:**

- When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.
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Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example:**

- "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example:**

- "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example:**

- "Reflect on your day so far. What math have you already used today?"

- **Classroom Activity:**

- Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example:**

- "Let's get into partners and do a think-pair-share. We will incorporate everyone's ideas and try to synthesize them."

- **Classroom Activity:**

- Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example:**

- "Come up with at least two answers that might solve this problem."

- **Classroom Activity:**

- Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.

- **Classroom Activity:**

- Deconstructed Multiple Choice: Given a set of multiple-choice answers, students

discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example:**

- "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"

- **Classroom Activity:**

- Error Analysis worksheets that highlight what is the right idea behind the mistake.
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Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example:**

- "What would a mathematician who is confused ask about this question?"

- **Classroom Activity:**

- After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.
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Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example:**

- "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."

- **Classroom Activity:**

- Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.
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Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).**

- **Climate Change Example:**

- Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MATH.9-12.A.REI	Reasoning with Equations and Inequalities

21st Century Life and Career

CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Integration of Career Readiness, Life Literacies and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.

Integration of Computer Science and Design ThinkingNew Section

CS.9-12.8.1.12.AP.1 Design algorithms to solve computational problems using a combination of original and

existing algorithms.

CS.9-12.8.1.12.AP.5 Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSL Standards for ELA, Social Studies, Science and/or Math

LA.RH.9-10.8	Assess the extent to which the reasoning and evidence in a text support the author's claims.
LA.RST.9-10.5	Analyze the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).
LA.RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
LA.RST.9-10.8	Determine if the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.

Module 12: Linear Relationships

Unit Rationale

Understanding linear relationships is essential for developing algebraic thinking and preparing students to model real-world scenarios using mathematics. In this unit, students explore how two quantities change in relation to each other through consistent, predictable patterns. By investigating and interpreting tables, graphs, and equations, students learn to recognize and represent linear relationships.

Students will build on their prior experiences with ratios, proportions, and coordinate graphing to develop a formal understanding of how constant rates of change—slopes—relate to linear equations. They will examine how changes in one variable affect another, and use this understanding to make predictions, compare relationships, and solve problems in context.

Representing linear relationships across multiple forms (verbal, graphical, tabular, and algebraic) supports deeper conceptual understanding and flexibility in thinking. Rather than focusing on memorizing procedures, students will develop reasoning skills to interpret patterns and express relationships clearly. These foundational skills prepare students for more advanced topics in algebra and data analysis.

Throughout the unit, students will:

- Explore how changes in input produce consistent changes in output (rate of change).
- Represent linear relationships using tables, graphs, and equations.
- Identify the slope and y-intercept from various forms of data and explain their meanings.

- Distinguish between proportional and non-proportional linear relationships.
- Apply linear models to solve real-world and mathematical problems.

This unit emphasizes several key Standards for Mathematical Practice, particularly:

MP2: Reason abstractly and quantitatively — by interpreting real-world relationships and connecting representations.

MP4: Model with mathematics — by applying linear equations to represent and solve contextual problems.

MP6: Attend to precision — by using correct notation and mathematical language when describing rate of change and intercepts.

MP7: Look for and make use of structure — by recognizing the structure of linear equations and the meaning of slope and intercept.

By the end of the unit, students will have developed a strong foundation in understanding and representing linear relationships. These skills are not only critical for success in algebra but also in science, economics, and everyday problem-solving—where understanding how two variables interact can inform decisions and predictions.

Essential Questions

- What does it mean for two quantities to have a linear relationship?
- How can you determine if a relationship is linear from a table, graph, or equation?
- What does the slope represent in a linear relationship?
- How is the y-intercept interpreted in different contexts?
- How are tables, graphs, equations, and verbal descriptions connected in representing linear relationships?
- What is the difference between proportional and non-proportional linear relationships?
- How can linear relationships be used to model and solve real-world problems?

Pre-Assessments

- Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)

- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.

- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning

- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)

- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks

- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Explain Slope with Similar Triangles

Student Learning Intentions (WALT):

- We are learning to define and calculate the slope of a line.
- We are learning to explain how slope relates to similar triangles.
- We are learning to use the concept of slope to solve problems involving right triangles and linear relationships.

Student Success Criteria (I can statements):

- I can define slope as a ratio of the rise over the run between two points on a line.
- I can explain how similar triangles help us understand the concept of slope.
- I can calculate the slope of a line from two points or from a graph.

- I can use similar triangles to find the slope of a line and apply it to real-world problems.
-

Instructional Strategies and Activities:

Introduction to Slope and Similar Triangles (15 minutes)

1. Direct Instruction:

- Begin by introducing the concept of slope as the ratio of vertical change (rise) to horizontal change (run) between two points on a line.

- Define the formula for slope:

$$\text{slope}(m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{slope}(m) = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

- Discuss how slope is a measure of how steep a line is, with a positive slope indicating an upward trend and a negative slope indicating a downward trend.

2. Relating Slope to Similar Triangles:

- Draw two right triangles on the board with the same angles, but different sizes (the triangles are similar).
- Emphasize that similar triangles have corresponding angles that are congruent and their corresponding sides are proportional.
- Explain that the slope of a line can be thought of as the ratio of the vertical side (rise) to the horizontal side (run) in a right triangle formed by the line. Since the triangles are similar, the ratio of rise to run is constant along the line.

3. Example (Visual Demonstration):

- Draw a right triangle on a coordinate plane, showing how the slope can be calculated from the rise and run.
 - Show another triangle that is similar to the first one. Explain that because the triangles are similar, the ratio of the rise over run will be the same for both triangles.
-

Guided Practice (20 minutes)

1. Example Problems:

- **Example 1:**

Given two points on the coordinate plane: $A(1, 2)$ and $B(4, 6)$, find the slope of the line that passes through these points.

Solution:

$$m = \frac{6 - 2}{4 - 1} = \frac{4}{3}$$

Discuss how the change in y (rise) is 4 and the change in x (run) is 3, which gives the slope of the line.

○ **Example 2:**

Draw two similar right triangles on the board. The first triangle has a rise of 3 units and a run of 4 units. The second triangle has a rise of 6 units and a run of 8 units. Show that the slope (rise/run) is the same for both triangles:

$$m = \frac{3}{4} = \frac{6}{8} = \frac{3}{4}$$

Discuss how the corresponding sides of the triangles are proportional and that the slope is constant.

2. Interactive Visualization:

- If possible, use an interactive graphing tool (like GeoGebra or Desmos) to show how adjusting the points on a line changes the triangles formed by the rise and run, and how the slope remains constant for similar triangles.
- Allow students to manipulate the points to see the relationship between the triangles and the slope.

Independent Practice (15 minutes)

1. Worksheet:

- Provide a worksheet where students calculate the slope of lines from various pairs of points on the coordinate plane.
- Include questions where students draw and label right triangles based on given lines and calculate the slope by finding the rise and run.
- Example problems might include:
 - Given the points $C(2,5)$ and $D(7,11)$, calculate the slope of the line.
 - Draw a line through points $E(0,0)$ and $F(3,4)$, then calculate the slope.

2. Exit Ticket:

- Ask students to answer two questions on an exit ticket:
 - What does the slope of a line represent in terms of rise and run?
 - Explain how similar triangles help us understand the concept of slope.

Peer Collaboration (10 minutes)

1. Think-Pair-Share:

- In pairs, have students explain how to find the slope of a line using similar triangles.
 - Each pair will present their explanation and one worked example to the class, showing how they calculated the slope using the rise/run concept and how similar triangles help illustrate this relationship.
-

Formative Assessments:

1. Observation and Questioning:

- Walk around the classroom during guided practice and independent practice. Ask questions like:
 - “How did you find the rise and run for the triangle?”
 - “How does the slope relate to the shape and size of the triangle?”

2. Exit Ticket Review:

- Review exit tickets to assess student understanding of the slope concept and their ability to connect it to similar triangles.
-

Instructional Materials and Resources:

- **Whiteboard/Smartboard:** For demonstrating slope calculations and drawing triangles.
 - **Graphing Tools (Optional):** Interactive graphing tools like GeoGebra or Desmos for visualizing slope and similar triangles.
 - **Worksheets:** A set of problems for practicing slope calculation and drawing right triangles.
 - **Rulers and Graph Paper:** For accurate graphing and measurement during practice.
-

Reflections and Suggested Modifications:

● Reflection:

- After the lesson, reflect on how well students understood the relationship between slope and similar triangles. Did they grasp the proportionality aspect of slope through triangles?

- Were students able to apply the slope formula correctly and explain their reasoning?

- **Suggested Modifications:**

- **For struggling students:** Provide additional visual aids and simpler examples of right triangles to practice calculating slope. Use more concrete examples with grid paper to help visualize the rise and run.
- **For advanced students:** Offer more challenging problems involving lines with negative slopes or non-horizontal/vertical lines, and have students explore slope in the context of real-world applications, such as architecture or engineering.

Lesson 2: Derive $y=mx$

Student Learning Intentions (WALT):

- We are learning to derive the equation of a line, specifically $y=mx$, from the slope formula.
- We are learning to understand the meaning of the slope-intercept form of a line.
- We are learning to interpret and apply the equation $y=mx$ in real-world contexts.

Student Success Criteria (I can statements):

- I can derive the equation of a line $y=mx$ from the slope formula.
 - I can explain the meaning of the slope m in the equation $y=mx$.
 - I can apply the equation $y=mx$ to find missing values in problems involving linear relationships.
 - I can interpret the slope-intercept form of a line and explain its components.
-

Instructional Strategies and Activities:

Introduction to Deriving $y = mx$ (15 minutes)

1. Direct Instruction:

- Start by reviewing the concept of **slope** from Lesson 1. Remind students that the slope of a line is defined as:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where m is the slope, and (x_1, y_1) and (x_2, y_2) are two

points on the line.

- Discuss the **slope-intercept form** of a line, which is:

$$y = mx + b$$

where:

- m is the slope of the line,
 - b is the y-intercept (the value of y when $x = 0$).
- For this lesson, focus on the specific case of the line passing through the origin, where $b = 0$. This simplifies the equation to:

$$y = mx$$

Explain that this equation represents a line with a slope m that passes through the origin $(0, 0)$.

2. Deriving the Equation $y = mx$:

- **Step 1:** Start with the basic form of the slope formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- **Step 2:** If the line passes through the origin, let one of the points be $(0, 0)$ and the other be (x, y) . The formula now becomes:

$$m = \frac{y - 0}{x - 0} = \frac{y}{x}$$

- **Step 3:** Rearrange this equation to solve for y :

$$y = mx$$

This is the equation of a line that passes through the origin with slope m .

Guided Practice (20 minutes)

1. Example 1:

- Consider a line with slope $m = 2$ that passes through the origin. Using the equation $y = mx$, write the equation of the line:

$$y = 2x$$

- Plot this line on a graph and verify that it passes through the origin and rises 2 units for every 1 unit it moves horizontally.

2. Example 2:

- Given a slope of $m = -3$, write the equation of the line and explain its behavior:
 $y = -3x$
- Discuss the meaning of the negative slope (the line decreases as x increases).
- Plot this line on the graph to show that it passes through the origin and falls 3 units for every 1 unit it moves horizontally.

3. Interactive Visualization (Optional):

- Use an online graphing tool like **GeoGebra** or **Desmos** to plot lines with different slopes (positive, negative, and zero). Show how changing the slope m affects the steepness and direction of the line.

Independent Practice (15 minutes)

1. Worksheet:

- Provide a worksheet where students calculate the equation of the line for various slopes.
- Example problems include:
 - Find the equation of the line with a slope of 4 that passes through the origin.
 - Write the equation of the line with a slope of $\frac{1}{2}$ that passes through the origin.
 - Find the equation of the line with a slope of $-\frac{2}{3}$ that passes through the origin.

2. Problem-Solving Application:

- Provide real-world problems where students need to find the equation of a line given the slope and a point on the line (which could be the origin for this lesson):
 - A car travels 3 miles per hour. Write the equation for the distance traveled over time.
 - A hiker climbs 5 feet for every 2 feet of horizontal movement. Write the equation for the height of the hiker as a function of horizontal distance.

Peer Collaboration (10 minutes)

1. Think-Pair-Share:

- Have students pair up to discuss the meaning of the equation $y = mx$ and its components.
- One student explains how to derive the equation from the slope formula, while the other asks

questions and provides feedback.

- Each pair will then share a real-world context in which a line with a given slope might be used.
-

Formative Assessments:

1. Exit Ticket:

- Ask students to write the equation of a line with slope $m=5$ that passes through the origin. This will assess their understanding of how to derive the equation $y=mx$.
- Include a second question: "If a line passes through the origin and has a slope of -2 , what happens to the line's direction compared to a line with a positive slope?"

2. Observation and Questioning:

- During independent practice, circulate around the classroom and ask students questions like:
 - "How did you find the slope?"
 - "What does the slope tell you about the steepness or direction of the line?"
 - "Can you describe what happens to the graph when the slope changes?"
-

Instructional Materials and Resources:

- **Whiteboard/Smartboard:** For demonstrating the derivation of the equation and graphing the lines.
 - **Graphing Tools (Optional):** GeoGebra or Desmos for visualizing the relationship between slope and line.
 - **Worksheets:** Problems for practicing the derivation and application of $y=mx$.
 - **Graph Paper:** For students to plot points and lines accurately.
-

Reflections and Suggested Modifications:

• Reflection:

- After the lesson, reflect on how well students understood the concept of deriving the equation $y=mx$ and the significance of the slope in real-world contexts. Did they make connections between the algebraic form and the graphical representation?
- Were students able to apply the formula correctly when given the slope?

• Suggested Modifications:

- **For struggling students:** Provide additional practice with smaller steps, such as first identifying the slope and then graphing simple lines.
- **For advanced students:** Challenge them with more complex lines, such as those with non-zero y-intercepts, to expand their understanding of the slope-intercept form ($y=mx+by = mx + by=mx+b$).

Lesson 3: Derive and Interpret $y = mx + b$

Student Learning Intentions (WALT):

- We are learning to derive the slope-intercept form of a line, $y=mx+by = mx + by=mx+b$, from the slope formula.
- We are learning to interpret the slope m and the y-intercept b in the equation $y=mx+by = mx + by=mx+b$.
- We are learning to apply $y=mx+by = mx + by=mx+b$ to solve real-world problems and make predictions.

Student Success Criteria (I can statements):

- I can derive the equation of a line in slope-intercept form, $y=mx+by = mx + by=mx+b$.
 - I can explain the meaning of the slope m and the y-intercept b in the equation.
 - I can use the equation $y=mx+by = mx + by=mx+b$ to write the equation of a line when given a slope and a point on the line.
 - I can apply $y=mx+by = mx + by=mx+b$ to solve problems in real-world contexts, making predictions and interpreting results.
-

Instructional Strategies and Activities:

Introduction to Deriving and Interpreting $y=mx+by = mx + by=mx+b$ (15 minutes)

1. Direct Instruction:

- Begin by reviewing the equation for the slope of a line, $m=\frac{y_2 - y_1}{x_2 - x_1}$, from earlier lessons.
- Introduce the **slope-intercept form** of the equation of a line:

$$y=mx+by = mx + by=mx+b$$

where:

- m is the **slope** of the line (the rate of change).
- b is the **y-intercept** (the point where the line crosses the y-axis, or when $x=0$).
- Explain that this equation can represent any straight line on a coordinate plane and that the slope and y-intercept describe the line's steepness and position.

2. Derivation of the Slope-Intercept Form:

- Start with the point-slope form of the equation of a line:

$$y - y_1 = m(x - x_1)$$

where m is the slope and (x_1, y_1) is a point on the line.

- If we know the line crosses the **y-axis** (i.e., when $x=0$), substitute $x=0$ into the equation:

$$y - y_1 = m(0 - x_1)$$

This simplifies to the form:

$$y - y_1 = m(-x_1)$$

where $b = -y_1$, the y-intercept.

- Explain that the equation $y = mx + b$ describes any straight line, where m controls how steep the line is, and b tells us where the line intersects the y-axis.

Guided Practice (20 minutes)

1. Example 1:

- Given the slope $m=3$ and the point $(0,2)$ on the line (i.e., the line intersects the y-axis at $y=2$), derive the equation of the line.
 - Using $y = mx + b$, substitute $m=3$ and $b=2$:
$$y = 3x + 2$$
 - Discuss how the slope $m=3$ means the line rises 3 units for every 1 unit it moves to the right, and the y-intercept $b=2$ tells us the line crosses the y-axis at $y=2$.

2. Example 2:

- Given a slope $m=-2$ and a point $(4,0)$ on the line, find the equation of the line.
 - First, use the point-slope form:

$$y - y_1 = m(x - x_1) \quad y - y_1 = m(x - x_1) \quad y - y_1 = m(x - x_1)$$

Substitute $m = -2$, $x_1 = 4$, and $y_1 = 0$:

$$y - 0 = -2(x - 4) \quad y - 0 = -2(x - 4)$$

Simplify:

$$y = -2x + 8$$

- Explain that the slope $m = -2$ means the line falls 2 units for every 1 unit it moves to the right, and the line crosses the y-axis at $y = 8$ when $x = 0$.

Interactive Visualization (Optional)

• Graphing the Lines:

- Use an interactive graphing tool (such as GeoGebra or Desmos) to graph lines with various slopes and y-intercepts.
- Show how changing the slope m affects the steepness of the line, and changing the y-intercept b shifts the line up or down without affecting its slope.

Independent Practice (15 minutes)

1. Worksheet:

- Provide students with problems that involve finding the equation of a line given a slope and a point. Examples include:
 - Find the equation of a line with slope $m = 4$ passing through $(0, -5)$.
 - Find the equation of a line with slope $m = -\frac{1}{2}$ passing through $(2, 3)$.
 - Find the equation of a line with slope $m = 0$ passing through $(0, 1)$.

2. Real-World Problem:

- A car starts at a distance of 0 miles and travels at 60 miles per hour. Write the equation that models the distance the car travels after t hours.
 - This is a linear relationship, so the equation is:
$$d = 60t$$
 - The slope is 60, representing the rate of travel (60 miles per hour), and the y-

intercept $b=0$ because the car starts at 0 miles.

Peer Collaboration (10 minutes)

1. Think-Pair-Share:

- Have students pair up to discuss their answers from the independent practice. Each student explains how they derived the equation and how the slope and y-intercept relate to the line's graph.
 - After the discussion, ask each pair to share a real-world situation that could be modeled by a linear equation in the form $y=mx+b$.
-

Formative Assessments:

1. Exit Ticket:

- Ask students to write the equation of a line with a slope of 2 and a y-intercept of -3.
- Include a second question: "What does the slope of 2 tell you about the line?"

2. Observation and Questioning:

- During independent practice, walk around the room and ask students:
 - "What does the slope represent in this context?"
 - "How does the y-intercept affect the position of the line?"
 - "How do you know if your equation is correct?"
-

Instructional Materials and Resources:

- **Whiteboard/Smartboard:** For demonstrating the derivation and graphing of the line.
 - **Graphing Tools (Optional):** GeoGebra or Desmos to visualize the relationship between slope, y-intercept, and the graph.
 - **Worksheets:** Problems to practice writing the equation of a line given a slope and a point.
 - **Real-World Context Cards:** A set of cards with real-world scenarios that can be modeled by linear equations.
-

Reflections and Suggested Modifications:

- **Reflection:**

- After the lesson, reflect on how well students understood the meaning of the slope and y-intercept. Were they able to connect the equation $y=mx+by = mx + by=mx+b$ with the graph of the line?
- Did students demonstrate an understanding of how the slope affects the line's steepness and direction?

- **Suggested Modifications:**

- **For struggling students:** Provide additional support with graphing lines manually to connect the equation to the visual representation.
- **For advanced students:** Challenge them with more complex linear relationships, such as writing equations for lines that do not pass through the origin or working with larger data sets to derive equations.

Lesson 4: Describe and Sketch Nonlinear Relationships

Student Learning Intentions (WALT):

- We are learning to recognize and describe nonlinear relationships in mathematics.
- We are learning to distinguish between linear and nonlinear relationships.
- We are learning to sketch graphs of nonlinear relationships, such as quadratic, exponential, and other curved graphs.
- We are learning to interpret the features of nonlinear graphs, such as the shape of the curve and key points.

Student Success Criteria (I can statements):

- I can identify whether a relationship is linear or nonlinear based on its graph or equation.
- I can describe the general shape of graphs of common nonlinear relationships (quadratic, exponential, etc.).
- I can sketch the graph of a nonlinear relationship using its key features.
- I can interpret key features of nonlinear graphs, such as the vertex of a parabola or the y-intercept of an exponential graph.

Instructional Strategies and Activities:

Introduction to Nonlinear Relationships (15 minutes)

1. Direct Instruction:

- Review the difference between linear and nonlinear relationships.
 - **Linear relationships** create straight-line graphs, with equations in the form $y=mx+by = mx + by=mx+b$.
 - **Nonlinear relationships** create curved graphs, which cannot be represented by a straight line.
- Introduce different types of nonlinear relationships, including:
 - **Quadratic relationships** (e.g., $y=ax^2+bx+c = ax^2 + bx + cy=ax^2+bx+c$)
 - **Exponential relationships** (e.g., $y=a \cdot b^xy = a \cdot b^xy=a \cdot bx$)
 - **Other curves** (e.g., cubic, reciprocal, logarithmic)
- Discuss how these relationships are represented by equations and graphs, emphasizing that nonlinear graphs change direction, either curving upwards or downwards, or growing/shrinking exponentially.

2. Graphical Examples:

- Show and describe the following graph types:
 - **Quadratic graph (parabola):** The graph of a quadratic function, such as $y=x^2y = x^2$, has a "U" shape, with the vertex at the lowest or highest point.
 - **Exponential graph:** The graph of an exponential function, such as $y=2^xy = 2^xy=2x$, shows rapid growth or decay, curving steeply either upwards or downwards.
- Highlight key features in each graph:
 - **Quadratic graph:** The vertex, axis of symmetry, and direction of the parabola (opening upwards or downwards).
 - **Exponential graph:** The y-intercept, growth or decay rate, and horizontal asymptote (if any).

Guided Practice (20 minutes)

1. Example 1 – Quadratic Relationship:

- Given the equation $y=x^2-4x+3y = x^2 - 4x + 3y=x^2-4x+3$, have students identify the key

features of the quadratic graph.

- Discuss how to find the **vertex** (by completing the square or using the vertex formula).
- Show how the graph is symmetric and where it crosses the x-axis (the roots).
- Plot key points on the graph (e.g., $x=0, 1, 2, -1$) to create a sketch of the curve.

○ **Discussion Questions:**

- What does the graph look like as x increases or decreases?
- Where is the vertex, and what does it represent in terms of the equation?

2. Example 2 – Exponential Relationship:

- Given the equation $y=2^x$, explain the general shape of an exponential graph.
 - Discuss how the graph increases rapidly as x becomes large.
 - Point out that the graph never reaches the x-axis (asymptote at $y=0$).
 - Plot a few points (e.g., $x=-1, 0, 1, 2$) and sketch the graph.

○ **Discussion Questions:**

- How does the graph change as x becomes positive or negative?
- What happens when $x=0$, and what does it represent?

Independent Practice (15 minutes)

1. Worksheet Activities:

- Provide students with equations of various nonlinear relationships. Have them:
 - Identify the type of relationship (quadratic, exponential, etc.).
 - Identify key features of the graph (vertex, intercepts, asymptotes).
 - Sketch the graph based on these features.

Example Problems:

- $y=x^2-6x+5$ (quadratic)
- $y=3^x$ (exponential)
- $y=-x^3+2x$ (cubic)
- $y=\frac{1}{x}$ (reciprocal)

2. Graphing Calculator or Online Tool (Optional):

- Have students use a graphing calculator or an online tool like Desmos to graph the equations and compare their sketches with the generated graphs. This will allow students to see how their predictions match the actual graphs.
-

Peer Collaboration (10 minutes)

1. Think-Pair-Share:

- Ask students to pair up and discuss their sketches of nonlinear graphs. They should:
 - Share how they identified the type of graph.
 - Discuss any difficulties they had in sketching the graph and how they solved them.
 - Explain the key features of their graphs and how they interpreted them.
 - After discussion, invite pairs to share their insights with the class.
-

Formative Assessments:

1. Exit Ticket:

- Ask students to:
 - Describe a real-world situation that could be modeled by a quadratic function.
 - Sketch the graph of the function and label key features.

2. Observational Questions:

- As students work on the independent practice, circulate around the room and ask:
 - "What are the key features of this graph?"
 - "How did you decide the direction the graph should curve?"
 - "What happens to the graph as xxx increases or decreases?"
-

Instructional Materials and Resources:

- **Whiteboard/Smartboard:** For explaining concepts and demonstrating examples.
- **Graphing Calculators or Online Tools (e.g., Desmos):** For students to visualize nonlinear graphs.

- **Worksheets:** Problems for students to identify, describe, and sketch nonlinear relationships.
 - **Graph Paper:** For sketching graphs by hand.
-

Reflections and Suggested Modifications:

- **Reflection:**
 - After the lesson, reflect on how well students understood the differences between linear and nonlinear relationships. Were they able to identify and sketch nonlinear graphs effectively? Did they understand the key features of the graphs?
- **Suggested Modifications:**
 - **For struggling students:** Provide more direct guidance on identifying key features of nonlinear graphs. Use more visual aids and step-by-step support with sketching.
 - **For advanced students:** Provide more challenging equations or introduce additional nonlinear functions (e.g., logarithmic, trigonometric) for students to explore and graph.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support:**
 - The teacher provides auditory or written content to students in their native language.
 - **Adjusted Speech:**
 - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
 - **Visuals:**
 - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
 - **Front-Loading Vocabulary:**
 - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.
-

Special Education Students

- **Chunking:**

- The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.

- **Checking for Understanding:**

- It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.

- **Extra Time:**

- The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.

- **Oral Reading:**

- The teacher will read work aloud to students, which can include class work, tests, and literature circles.

- **Timers:**

- The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.
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Students with 504 Plans

- **Chunking:**

- The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.

- **Checking for Understanding:**

- Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.

- **Extra Time:**

- Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.
-

Gifted & Talented Strategies

- **Extensions/Enrichments:**

- Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.

- **Modify/Change Activities:**

- Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.
-

Students at Risk of School Failure

- **Directions or Instructions:**

- Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.

- **Peer Support:**

- Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.

- **Alternate or Modified Assignments:**

- Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).

- **Increase One-on-One Time:**

- Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.

- **Contracts:**

- A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.

- **Hands-On Tasks:**

- Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.

- **Tests/Assessments:**

- Tests can be administered orally, or broken into smaller sections. Teachers may administer

parts of a test in the morning, after lunch, and on subsequent days if necessary.

- **Seating:**

- Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Interdisciplinary Connections: NJSL for ELA, Social Studies, Science and/or Math

New Jersey Student Learning Standards: Content Area

MATH.8.EE.B	Understand the connections between proportional relationships, lines, and linear equations
MATH.8.EE.B.5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
MATH.8.EE.B.6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b .

21st Century Life and Career

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP7	Employ valid and reliable research strategies.

Integration of Career Readiness. Life Literacies and Key Skills

PFL.9.1.4.B	Money Management
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Integration of Computer Science and Design Thinking

CS.K-2.DA	Data & Analysis
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Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example:**

- Fist to five: "How well do you understand what we talked about today?"
- Fist to five: "How well did I teach this today?"

- **Classroom Activity:**

- Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.
-

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies:**

- Color-coding ideas
- Learning vocabulary in student languages
- Visual and kinesthetic learning
- Representations of learning without words

- **Classroom Activity:**

- Multilingual Frayer Models for definitions or concepts
-

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example:**

- "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"

- **Classroom Activity:**

- Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.
-

Incorporate the history of mathematics into lessons

- **Verbal Example:**

- "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"

- **Classroom Activity:**

- Learn about different bases and numerical ideas:
 - Base 2 (binary) and connections to computer programming
 - How the Yoruba of Nigeria used base 20
 - How the Mayans conceptualized the number 0 before the first recording of it
-

Solicit student ways of thinking and processing

- **Verbal Example:**

- "How might you all go about this?"
- "What do you notice?"

- **Classroom Activity:**

- Incorporate explorations where students interact with mathematics in a way that allows them to “discover” or experience mathematics.
-

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example:**

- "How does this connect to what you’ve learned in the past?"
- "How can you use that knowledge today?"

- **Classroom Activity:**

- Learning webs that connect content
-

Start with more complex math problems and scaffold as necessary

- **Verbal Example:**

- "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"

- **Classroom Activity:**

- When solving equations, start with the most complex problem, generate ideas for how to solve

it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example:**

- "Show your thinking with words, pictures, symbols."
-

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example:**

- "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"
-

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example:**

- "Reflect on your day so far. What math have you already used today?"

- **Classroom Activity:**

- Community walks to engage with slope.
-

Co-construct knowledge in the classroom

- **Verbal Example:**

- "Let's get into partners and do a think-pair-share. We will incorporate everyone's ideas and try to synthesize them."

- **Classroom Activity:**

- Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.
-

Choose problems that have complex, competing, or multiple answers

- **Verbal Example:**

- "Come up with at least two answers that might solve this problem."

- **Classroom Activity:**

- Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.

- **Classroom Activity:**

- **Deconstructed Multiple Choice:** Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.
-

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example:**

- "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"

- **Classroom Activity:**

- Error Analysis worksheets that highlight what is the right idea behind the mistake.
-

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example:**

- "What would a mathematician who is confused ask about this question?"

- **Classroom Activity:**

- After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.
-

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example:**

- "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."

- **Classroom Activity:**

- Community circles or storytelling circles, incorporating dance, music, song, call and response,

and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).
- **Climate Change Example:**
 - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

LA.K-12.NJSLSA.R

Reading

MATH.8.F

Functions

Range of Writing