Unit 4: Transform and Construct Geometric Figures

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **9 Weeks**
Status: **Published**

Module 8: Transformation and Congruence

Unit Rationale

Understanding transformations is fundamental to developing a strong sense of spatial reasoning and geometric thinking. In this unit, students explore how figures move within a plane through translations, reflections, rotations, and dilations. These transformations form the building blocks for recognizing when two figures are congruent (having the same shape and size) or similar (having the same shape but different sizes).

Students will build on their prior experiences with shapes and coordinate grids to develop a formal understanding of how figures can be manipulated without changing their essential properties. By learning to describe and perform transformations, students strengthen their ability to reason logically about geometric relationships and develop precision in mathematical communication.

Recognizing congruence through transformations provides students with a dynamic, visual approach to understanding geometry. Instead of relying solely on static measurements, students learn to think about how figures correspond and how their properties are preserved or altered through movement. This approach is essential for preparing students for more advanced geometry topics in high school, including proof, similarity, and trigonometry.

Throughout the unit, students will:

- Explore rigid transformations (translations, reflections, and rotations) that preserve distance and angle measures.
- Understand congruence as a sequence of rigid transformations.
- Apply transformations to solve problems on and off the coordinate plane.
- Use precise mathematical language to describe and justify transformations.

This unit emphasizes several key **Standards for Mathematical Practice**, particularly:

- **MP2: Reason abstractly and quantitatively** — by interpreting and describing movements in mathematical terms.
- **MP4: Model with mathematics** — by applying transformations to real-world contexts and using diagrams to support reasoning.
- **MP6: Attend to precision** — by using clear vocabulary and notation when describing transformations.
- **MP7: Look for and make use of structure** — by recognizing patterns in how figures change under transformations.

By the end of the unit, students will not only be able to perform and describe transformations but also make

deeper connections between motion, symmetry, and congruence. These skills are critical not only in mathematics but in fields such as engineering, computer graphics, design, and architecture, where understanding and manipulating shapes are essential.

Essential Questions

- How can transformations help us describe how figures move in a plane?
- What does it mean for two figures to be congruent?
- How can sequences of transformations be used to show that two figures are congruent?
- How do translations, reflections, and rotations affect the size, shape, and position of a figure?
- In what ways can we use the coordinate plane to model and describe transformations precisely?
- Why is it important to describe transformations using accurate mathematical language?
- How do transformations connect to real-world movements and designs?

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Investigate Transformations

Student Learning Intentions or We are learning to … (WALT):

- We are learning to define and identify transformations (translation, reflection, and rotation).
- We are learning to apply transformations to geometric figures and analyze how these transformations affect their position and orientation.

Student Success Criteria … "I can statements":

- I can define and describe the three types of transformations: translation, reflection, and rotation.
- I can apply each type of transformation to geometric figures on a coordinate plane.
- I can analyze how each transformation changes the position and orientation of a figure.

Instructional Strategies and Activities:

Introduction to Transformations (10 minutes)

- **Direct Instruction**: Introduce the three types of transformations:
    - **Translation**: Sliding a figure without rotating or flipping it.
    - **Reflection**: Flipping a figure over a line (creating a mirror image).
    - **Rotation**: Turning a figure around a fixed point (e.g., 90°, 180°, 270°).
- **Visual Models**: Demonstrate each transformation using a Smartboard or projector with visual aids. Draw examples of each transformation on the coordinate plane to make the concepts clearer.

Guided Practice (15 minutes)

- **Example 1**: **Translation**
    - Show a triangle with coordinates (e.g., A(1, 2), B(4, 2), C(2, 4)) and have students translate it 3 units to the right and 2 units down.
- **Example 2**: **Reflection**
    - Reflect a figure over the x-axis and explain how the coordinates change.
- **Example 3**: **Rotation**
    - Rotate a square 90° counterclockwise and show the changes in coordinates.

Ask students questions like:

- How do you know which way to move a figure when translating it?
- What happens to the coordinates of a figure during a reflection?
- How do you determine the new location of a point after a rotation?

Independent Practice (15 minutes)

- Students work on a set of problems, including:
    - **Translation**: Translate a figure 2 units up and 3 units left.
    - **Reflection**: Reflect a rectangle over the y-axis.
    - **Rotation**: Rotate a triangle 180° counterclockwise around the origin.
- Provide a **Transformation Worksheet** with a mix of problems requiring translation, reflection, and rotation.

Peer Collaboration (10 minutes)

- Have students pair up and share their solutions. They should explain the steps they took to apply the transformations.
- Encourage students to check each other's work and discuss any differences in their answers or strategies.

Formative Assessments:

- **Exit Ticket (5 minutes)**:
  Have students complete a short exit ticket with the following:

  1. Draw and label a shape, then translate it 3 units to the left and 4 units up.
  2. Reflect a figure over the x-axis and describe the changes in the coordinates.
  3. Rotate a shape 90° counterclockwise around the center.

- **Observation and Questioning**:
  During guided practice and peer collaboration, observe students' problem-solving strategies. Ask them to explain how they arrived at their answers to ensure they understand the process.

- **Quick Check-in**:
  Periodically ask students to describe what happens during each transformation and check for understanding.

Instructional Materials and Resources:

- **Whiteboard/Smartboard**: For demonstrating transformations with visual aids.
- **Worksheets/Problem Sets**: A set of practice problems involving real-life transformation examples.
- **Interactive Tools (Optional)**: Online graphing tools like Desmos to manipulate and visualize transformations.
- **Handouts/Reference Sheets**: A step-by-step guide on how to perform each type of transformation (translation, reflection, rotation).

Reflections and Suggested Modifications:

Reflection:

- Reflect on how students engaged with the lesson. Did they find the transformations concept easy to grasp? Were they able to work through the problems successfully?
- Did the use of visual models and real-world examples help students make connections to the material?

Suggested Modifications:

- **Differentiation**:
  - **For Struggling Students**: Provide extra support with simpler problems, such as working through transformations on smaller, more manageable shapes. Use manipulatives, such as paper

cutouts or tracing paper, to help visualize reflections.

- **For Advanced Students**: Challenge them with more complex problems that involve applying multiple transformations in sequence or having them explore other transformations (e.g., dilation or shear).

- **Group Work**: Pair students with varying abilities together to support one another during independent practice. Implement a "think-pair-share" strategy where students first think through a problem individually and then discuss it with a partner.

Lesson 2: Explore Translations

Student Learning Intentions or We are learning to … (WALT):

- We are learning to define and identify translations as a type of geometric transformation.
- We are learning to apply translations to geometric figures and determine how they affect the position of the figures on the coordinate plane.

Student Success Criteria … "I can statements":

- I can define translation as a transformation that slides a figure without changing its shape or orientation.
- I can apply translation to a geometric figure on the coordinate plane by moving it a specified distance in the horizontal and/or vertical direction.
- I can describe the effect of a translation on the position of a figure using coordinates.

Instructional Strategies and Activities:

Introduction to Translations (10 minutes)

- Direct Instruction: Introduce translation as a type of transformation where a figure is "slid" from one position to another without changing its size, shape, or orientation.
  - Explain that translation involves moving a figure along a straight path, either horizontally, vertically, or both.
  - Show a simple example on the coordinate plane. For instance, move a point A(1, 3) 3 units to the right and 2 units up to the new position A'(4, 5).
- Visual Demonstration:

- On the Smartboard, demonstrate how translation moves figures along the coordinate plane. Use different shapes (e.g., triangles, squares) and translate them both horizontally and vertically.
- Emphasize that the shape and orientation of the figure do not change, only the position.

Guided Practice (15 minutes)

- Example 1:
  - Translate a point (e.g., B(2, 4)) 5 units right and 3 units down.
  - Show students how to apply the translation to the point and plot the new position on the coordinate plane.
  - Discuss the process: How do you determine the new coordinates after translation?
- Example 2:
  - Translate a triangle with vertices at (1, 2), (4, 5), and (2, 6) 2 units left and 4 units up.
  - Show the resulting translated triangle on the coordinate plane, ensuring students see how all points are moved the same way.
- Questions to Ask:
  - What happens to the coordinates when we translate a point or figure?
  - How do you know how far to move a figure in the horizontal and vertical directions?

Independent Practice (20 minutes)

- Students will now work on translating different shapes on the coordinate plane. Provide a Translation Worksheet that includes:
  - Translating points, lines, and polygons (e.g., squares, triangles).
  - Instructions to translate shapes by different amounts (both horizontally and vertically).
- Students will:
  - Translate points 3 units right and 2 units down.
  - Translate a rectangle 4 units left and 1 unit up.
  - Translate a triangle 5 units right and 3 units down.
- Support: As students work, circulate the room to answer questions and provide guidance if students are struggling to understand the translation process.

Peer Collaboration (10 minutes)

- After completing the worksheet, have students pair up and check each other's translations.
    - Each student should explain the steps they took to translate their shapes to their partner.
    - Students should also verify each other’s work and ensure that the translations are accurate.
- Discussion Prompt:
    - What patterns do you notice when translating figures?
    - How does translation affect the shape and size of the figure?

Formative Assessments:

- Exit Ticket (5 minutes):
  Ask students to answer the following questions on a short exit ticket:
    1. Translate the point (3, 7) 4 units right and 2 units down.
    2. Translate a square with vertices (1, 1), (1, 4), (4, 1), and (4, 4) 3 units left and 1 unit up.
- Observation and Questioning:
  Observe students during independent and peer work. Ask them to explain how they are applying translations to the coordinates. Check that students can accurately translate shapes and describe the process.

Instructional Materials and Resources:

- Whiteboard/Smartboard: For demonstrating translations and showing visual examples.
- Worksheets/Problem Sets: A set of problems that involve translating points, lines, and polygons.
- Graph Paper: Students can use graph paper to help accurately plot and translate shapes.
- Interactive Tools (Optional): Use online tools like Desmos or GeoGebra to visualize translations and check answers dynamically.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how students engaged with the concept of translation. Did they understand the idea of

sliding a figure without changing its orientation?

- Were students able to perform translations on the coordinate plane with accuracy?

Suggested Modifications:

- Differentiation:
    - For Struggling Students: Provide additional one-on-one support or simplified examples to help students who are struggling with the concept. Consider using physical manipulatives (like paper cutouts of shapes) that students can move around to reinforce the idea of translation.
    - For Advanced Students: Challenge them with more complex translation problems, such as translating shapes with negative coordinates or applying translations to figures that span multiple quadrants.
- Group Work: If students are still unsure, pair them with classmates who have mastered the concept. Encourage students to help one another explain how to translate shapes on the coordinate plane.

Lesson 3: Explore Reflections

Student Learning Intentions or We are learning to … (WALT):

- We are learning to define and identify reflections as a type of geometric transformation.
- We are learning to apply reflections to geometric figures across different lines of reflection, such as the x-axis, y-axis, and the line y = x.
- We are learning to understand how reflections preserve the size and shape of figures but change their orientation.

Student Success Criteria … "I can statements":

- I can define reflection as a transformation that flips a figure over a line, creating a mirror image.
- I can apply a reflection to a figure across the x-axis, y-axis, and the line y = x, and identify the resulting image.
- I can describe how reflections affect the orientation and position of a figure on the coordinate plane.

Instructional Strategies and Activities:

Introduction to Reflections (10 minutes)

- **Direct Instruction**: Introduce **reflection** as a geometric transformation where a figure is flipped over a line, creating a mirror image. The line over which the figure is reflected is called the **line of reflection**.
    - Discuss how reflections preserve the size and shape of the figure but alter its orientation. The image is a mirror image of the original figure.
    - Show the concept of reflection using simple figures on the coordinate plane. For example, reflect a point (3, 2) across the x-axis to (3, -2), and reflect a point (4, -3) across the y-axis to (-4, -3).
- **Visual Demonstration**:
    - Draw a triangle and reflect it over the x-axis and y-axis on the coordinate plane. Show how each point of the triangle is flipped to create a mirror image.
    - Discuss how reflections preserve distances and angles but reverse orientations (e.g., flipping the "front" to the "back").

Guided Practice (15 minutes)

- **Example 1**:
    - Reflect the point (2, 5) over the x-axis. Show how the y-coordinate changes sign while the x-coordinate remains the same: (2, 5) → (2, -5).
- **Example 2**:
    - Reflect the point (-3, 4) over the y-axis. The x-coordinate changes sign, but the y-coordinate stays the same: (-3, 4) → (3, 4).
- **Example 3**:
    - Reflect the point (4, 3) over the line y = x. The coordinates swap positions: (4, 3) → (3, 4).
- **Interactive Q&A**:
  Ask students questions during the guided practice:
    - What happens to the coordinates when we reflect over the x-axis?
    - What changes when reflecting over the y-axis or the line y = x?

Independent Practice (20 minutes)

- Students will now practice reflecting points across various lines of reflection on their own. Provide a **Reflection Worksheet** with the following tasks:
    - Reflect the points (2, 6), (3, -4), (-5, -2) over the x-axis.

- Reflect the points (-4, 7), (0, -3), (6, 2) over the y-axis.
- Reflect the points (1, 2), (-2, 5), (3, -1) over the line y = x.

- **Differentiation**: For students who may struggle, provide extra support and visual aids, such as a graphing tool or graph paper, to help them see the transformation. Use simpler points at first to ensure understanding.

Peer Collaboration (10 minutes)

- After completing the reflection problems, students will work in pairs to check and compare their answers.
  - Each student will explain how they reflected the points and describe any patterns they notice in the transformations.
  - Encourage students to collaborate and discuss what happens when reflecting points over different lines.
- **Discussion Prompt**:
  - How does the reflection over the x-axis differ from the reflection over the y-axis or y = x?
  - What happens to the image when we reflect across different lines?

Formative Assessments:

- **Exit Ticket (5 minutes)**:
  Have students complete a short exit ticket with the following questions:
  1. Reflect the point (5, -2) over the x-axis.
  2. Reflect the point (3, 4) over the line y = x.
  3. Explain in words how a reflection changes a figure.
- **Observation and Questioning**:
  Walk around during independent and peer work to observe how students apply the concept of reflection. Ask students to explain their thought process and reasoning.
- **Quick Check-in**:
  Periodically check for understanding by asking questions like, “What is the effect of reflecting a figure over the y-axis? What does the new image look like?”

Instructional Materials and Resources:

- **Whiteboard/Smartboard**: For modeling reflections and showing visual examples.
- **Graph Paper**: To assist students in plotting and reflecting points accurately.
- **Reflection Worksheet**: A set of practice problems with points to reflect over the x-axis, y-axis, and the line $y = x$.
- **Interactive Tools (Optional)**: Use online graphing tools (e.g., Desmos or GeoGebra) to help visualize reflections dynamically and provide immediate feedback.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how well students understood the concept of reflections. Were they able to reflect points across different lines accurately? Did they understand the idea of orientation changing while maintaining size and shape?
- Did students struggle more with specific lines of reflection, such as $y = x$, compared to the x-axis and y-axis?

Suggested Modifications:

- **Differentiation**:
    - **For Struggling Students**: Provide extra practice with simple figures and review the steps for reflecting over each line (e.g., x-axis, y-axis). Use graph paper for more visual support.
    - **For Advanced Students**: Challenge them to reflect more complex shapes (like triangles, squares, or polygons) and then describe how the shape's orientation and position change. Provide problems involving multiple transformations, such as translating a figure and then reflecting it.
- **Group Work**: If needed, pair stronger students with those who may need additional help. In a group setting, students can explain their work and teach each other.

Lesson 4: Explore Rotations

Student Learning Intentions or We are learning to … (WALT):

- We are learning to define and understand rotations as a type of geometric transformation.
- We are learning to apply rotations to geometric figures on the coordinate plane around a specific point of rotation.
- We are learning to understand how rotating a figure by 90°, 180°, or 270° affects the position and

orientation of the figure, while preserving its size and shape.

Student Success Criteria … "I can statements":

- I can define rotation as a transformation that turns a figure around a fixed point.
- I can rotate a figure by 90°, 180°, and 270° around the origin (0, 0) on the coordinate plane.
- I can describe how rotations preserve the size and shape of the figure but change its orientation.
- I can apply the rules of rotation to rotate points and shapes accurately.

Instructional Strategies and Activities:

Introduction to Rotations (10 minutes)

- Direct Instruction: Introduce rotation as a geometric transformation where a figure is turned around a fixed point, called the center of rotation. The most common center of rotation is the origin (0, 0) on the coordinate plane.
    - Explain that rotations preserve the size and shape of a figure, but change its orientation. The amount of rotation is described in degrees: 90°, 180°, and 270°.
    - Show a visual example by rotating a simple shape (e.g., a triangle) by 90° clockwise around the origin.
- Visual Demonstration:
    - Use a simple coordinate plane on the board to demonstrate how rotating a point changes its coordinates:
        - A 90° clockwise rotation transforms a point (x, y) to (y, -x).
        - A 180° rotation transforms a point (x, y) to (-x, -y).
        - A 270° clockwise rotation transforms a point (x, y) to (-y, x).
    - Plot a point, rotate it, and show how its position changes, emphasizing the change in orientation.

Guided Practice (15 minutes)

- Example 1 (90° Rotation):
    - Rotate the point (2, 3) by 90° clockwise. Using the rotation rule (x, y) → (y, -x), the point

becomes (3, -2).

  - Show another example: Rotate the point (-1, 4) by 90° clockwise, and calculate (-1, 4) → (4, 1).

- Example 2 (180° Rotation):
  - Rotate the point (2, 3) by 180°. Using the rotation rule (x, y) → (-x, -y), the point becomes (-2, -3).
  - Rotate the point (-4, -1) by 180°, which transforms (-4, -1) to (4, 1).
- Example 3 (270° Rotation):
  - Rotate the point (2, 3) by 270°. Using the rotation rule (x, y) → (-y, x), the point becomes (-3, 2).
  - Rotate the point (-4, 1) by 270°, which transforms (-4, 1) to (-1, -4).
- Interactive Q&A:
  Ask students questions during the guided practice to ensure understanding:
  - What happens to the coordinates when we rotate a point 90°?
  - How does a 180° rotation differ from a 90° rotation?
  - What do you notice when a figure is rotated multiple times?

Independent Practice (20 minutes)

- Students will now practice rotating points by 90°, 180°, and 270° on their own. Provide a Rotation Worksheet with the following tasks:
  - Rotate the points (3, 2), (1, -4), (-2, 5), and (4, -1) by 90° clockwise.
  - Rotate the points (2, 3), (4, -5), (-3, 7), and (-1, 2) by 180°.
  - Rotate the points (1, 2), (-3, 4), (2, -3), and (5, 6) by 270° clockwise.
- Differentiation:
  - For students who need additional support, provide a chart with the rules for rotating points (90°, 180°, and 270°). Use graph paper to help visualize the rotations.

Peer Collaboration (10 minutes)

- After completing the rotation problems, students will work in pairs to check and compare their answers.
  - Each student will explain how they rotated the points and describe any patterns they notice in

the transformations.

    - Encourage students to discuss which rotations result in similar or opposite orientations for the figure.

- Discussion Prompt:
    - How does rotating a figure 180° compare to rotating it 90° or 270°?
    - How can we predict the new coordinates after a rotation?

Formative Assessments:

- Exit Ticket (5 minutes):
  Have students complete a short exit ticket with the following questions:
    1. Rotate the point (2, 5) by 90° clockwise.
    2. Rotate the point (-3, -4) by 180°.
    3. Explain in words how a 270° rotation affects a figure.
- Observation and Questioning:
  Walk around during independent practice to observe how students apply the rotation rules. Ask students to explain how they arrived at their answers.
- Quick Check-in:
  Periodically check for understanding by asking students how they would rotate a point or figure by 90°, 180°, or 270° and why the rules work that way.

Instructional Materials and Resources:

- Whiteboard/Smartboard: For modeling rotations and showing visual examples.
- Graph Paper: To assist students in plotting and rotating points accurately.
- Rotation Worksheet: A set of practice problems with points to rotate by 90°, 180°, and 270°.
- Interactive Tools (Optional): Use online graphing tools (e.g., Desmos or GeoGebra) to help visualize rotations dynamically and provide immediate feedback.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how well students understood the concept of rotations. Were they able to apply the rules for

90°, 180°, and 270° rotations accurately? Did they understand how rotations change the position of points but preserve the shape?

- Did students struggle more with a particular rotation? For example, did they find it harder to rotate 270° compared to 90°?

Suggested Modifications:

- Differentiation:
    - For Struggling Students: Provide extra practice with simpler points and help students visualize rotations using graph paper. Consider using digital graphing tools that can dynamically show the transformations.
    - For Advanced Students: Challenge them by providing more complex figures (e.g., polygons or shapes) to rotate. Encourage them to explore rotations around points other than the origin.
- Group Work: Pair stronger students with those who may need additional help. Encourage peer explanations and support during the rotation practice.

Lesson 5: Understand and Recognize Congruent Figures

Student Learning Intentions or We are learning to … (WALT):

- We are learning to define congruent figures and understand their properties.
- We are learning to identify congruent figures based on size, shape, and orientation.
- We are learning to recognize and apply transformations (translations, rotations, reflections) to determine if two figures are congruent.

Student Success Criteria … "I can statements":

- I can define congruent figures as shapes that have the same size and shape, but may be in different positions or orientations.
- I can recognize congruent figures based on matching corresponding sides and angles.
- I can apply transformations (translations, rotations, reflections) to determine if two figures are congruent.
- I can justify why two figures are congruent using specific transformations.

Instructional Strategies and Activities:

Introduction to Congruence (10 minutes)

- **Direct Instruction**: Introduce the concept of **congruent figures** as shapes that have the same size and shape, even if they are rotated, reflected, or translated. Explain that congruence means that the figures can be made to overlap perfectly through a sequence of transformations.
    - Emphasize that corresponding sides and angles in congruent figures are equal.
    - Show a pair of congruent shapes and demonstrate that one shape can be transformed into the other through translation, rotation, or reflection.
- **Visual Demonstration**:
    - Use the whiteboard to draw two triangles: one in its original position and another one that has been reflected, rotated, or translated. Label the corresponding sides and angles, and discuss how they match up.
    - Show how transformations can help identify congruent figures by changing the position or orientation but preserving size and shape.

Guided Practice (15 minutes)

- **Example 1 (Recognizing Congruence)**:
    - Present two triangles with the following coordinates: Triangle A (2, 1), (4, 5), (6, 3) and Triangle B (1, 2), (3, 6), (5, 4).
    - Ask students to compare the triangles and check if they are congruent by verifying that the corresponding sides and angles are equal. If not, discuss how transformations might be applied.
- **Example 2 (Using Transformations to Prove Congruence)**:
    - Provide two squares: one on a coordinate plane and the other in a rotated position. Demonstrate how rotating the square can make the two shapes congruent.
    - Ask students to determine if the two shapes are congruent using transformations. Discuss how to check if the corresponding sides and angles match.
- **Interactive Q&A**:
  During the guided practice, ask questions such as:
    - What makes these two figures congruent?
    - How can we show that two shapes are congruent using transformations?
    - If one figure is rotated or reflected, does the size or shape change?

Independent Practice (20 minutes)

- Provide students with a **Congruence Worksheet** with the following tasks:
    - Compare two sets of figures and decide if they are congruent. If they are, explain which transformation (translation, rotation, or reflection) could be used to show congruence.
    - Example 1: A triangle and its reflection across the y-axis. Are they congruent? If so, explain how.
    - Example 2: A rectangle and its rotation by 90°. Are they congruent? If so, explain how.
    - Example 3: A quadrilateral and a translated version of the same shape. Are they congruent? If so, explain why.
- **Differentiation**:
    - For students who need additional support, provide simpler shapes (e.g., squares, triangles) and help them apply transformations step by step. Allow students to use graph paper to visualize the transformations.
    - For advanced students, provide more complex shapes or multiple transformations (e.g., a rotation followed by a reflection) and ask them to determine congruence.

Peer Collaboration (10 minutes)

- After completing the independent practice, have students pair up and compare their answers.
    - Each student will explain why they think the figures are congruent and the transformations used to verify this.
    - Encourage students to use precise geometric language, referring to corresponding sides and angles.
- **Discussion Prompt**:
    - Can a shape be congruent to another even if it's turned or flipped? Why or why not?
    - What transformations could be used to make one figure overlap with another?
    - How do we know that the two shapes are congruent based on their angles and sides?

Formative Assessments:

- **Exit Ticket (5 minutes)**:
  After the lesson, have students complete an exit ticket with 2-3 problems:

1. Given two rectangles, explain how you would determine if they are congruent.
2. Given two triangles, one rotated and the other reflected, determine if they are congruent and justify your answer.

- **Observation and Questioning**:
  Observe students during the independent practice and peer collaboration. Ask probing questions to assess their understanding of congruence and transformations.

- **Quick Check-in**:
  Periodically check for understanding by asking students to describe what makes two figures congruent and how they would use transformations to prove congruence.

Instructional Materials and Resources:

- **Whiteboard/Smartboard**: For demonstrating transformations and showing visual examples of congruence.
- **Graph Paper**: To help students visualize and plot transformations accurately.
- **Congruence Worksheet**: A set of problems where students determine if figures are congruent based on transformations and explain their reasoning.
- **Interactive Tools (Optional)**: Use online graphing tools (e.g., Desmos or GeoGebra) to demonstrate transformations and visually compare congruent figures.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how well students understood the concept of congruence and transformations. Were they able to identify congruent figures accurately?
- Did students understand the significance of corresponding sides and angles in determining congruence? Were they able to explain their reasoning effectively?

Suggested Modifications:

- **Differentiation**:
  - **For Struggling Students**: Provide additional practice with simpler shapes and offer more visual support by using graph paper or digital tools. Focus on comparing basic transformations (e.g., translation or reflection) and gradually move to more complex shapes.
  - **For Advanced Students**: Challenge students by providing irregular shapes and asking them to apply multiple transformations to determine congruence. Encourage them to explore real-world applications of congruence (e.g., engineering, art).

- **Group Work**: Pair students with varying levels of understanding to foster peer support and discussion. Consider using a “think-pair-share” strategy, where students think individually, discuss with a partner, and then share their findings with the class.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
    - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
    - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals**:
    - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary**:
    - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
    - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
    - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.
- **Extra Time**:
    - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.

- **Oral Reading**:
    - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
    - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
    - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
    - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
    - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
    - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
    - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
    - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check

back to ensure they haven't forgotten.

- **Peer Support**:
    - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
    - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
    - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
    - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
    - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.
- **Tests/Assessments**:
    - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:
    - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
    - Fist to five: "How well do you understand what we talked about today?"

- Fist to five: "How well did I teach this today?"

- **Classroom Activity**:
  - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
  - Color-coding ideas
  - Learning vocabulary in student languages
  - Visual and kinesthetic learning
  - Representations of learning without words
- **Classroom Activity**:
  - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
  - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
- **Classroom Activity**:
  - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
  - "Why do you think we call it Pythagorean’s theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
  - Learn about different bases and numerical ideas:

        - Base 2 (binary) and connections to computer programming
        - How the Yoruba of Nigeria used base 20
        - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
    - "How might you all go about this?"
    - "What do you notice?"
- **Classroom Activity**:
    - Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
    - "How does this connect to what you've learned in the past?"
    - "How can you use that knowledge today?"
- **Classroom Activity**:
    - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
    - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"
- **Classroom Activity**:
    - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
    - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
    - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
    - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
    - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
    - "Let’s get into partners and do a think-pair-share. We will incorporate everyone’s ideas and try to synthesize them."
- **Classroom Activity**:
    - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
    - "Come up with at least two answers that might solve this problem."

- **Classroom Activity**:
    - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
    - **Classroom Activity**:
        - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
    - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
    - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:
    - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:
    - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
    - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).

- **Climate Change Example**:

    - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MATH.9-12.A.REI	Reasoning with Equations and Inequalities

21st Century Life and Career

CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Integration of Career Readiness. Life Literacies and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

CRP.K-12.CRP11	Use technology to enhance productivity.

Integration of Computer Science and Design ThinkingNew Section

CS.9-12.8.1.12.AP.1 Design algorithms to solve computational problems using a combination of original and existing algorithms.

CS.9-12.8.1.12.AP.5 Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

CS.K-2.AP	Algorithms & Programming
CS.K-2.DA	Data & Analysis

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

LA.RH.9-10.8	Assess the extent to which the reasoning and evidence in a text support the author's claims.
LA.RST.9-10.5	Analyze the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).
LA.RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
LA.RST.9-10.8	Determine if the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.

Module 9: Draw and Analyze Two-Dimensional Figures

Unit Rationale

In Module 9, students explore the properties of two-dimensional geometric figures, focusing on both drawing and analyzing them. This module introduces key concepts such as angles, parallelism, symmetry, congruence, and similarity, helping students develop a deep understanding of how these properties interact within two-dimensional shapes. The goal is for students to be able to both construct and analyze figures accurately and to recognize and describe the relationships between different types of two-dimensional shapes.

Understanding how to draw and analyze two-dimensional figures is foundational for developing spatial reasoning, a skill that is critical for success in higher-level mathematics, engineering, art, architecture, and everyday problem-solving. This module builds upon previous knowledge of basic shapes and extends it by encouraging students to manipulate, categorize, and reason about geometric shapes in a more sophisticated manner. By the end of this module, students will be able to construct geometric figures with precision, identify their properties, and use this understanding to solve real-world problems.

In addition to geometric content, the module reinforces several key Standards for Mathematical Practice:

- MP1: Make sense of problems and persevere in solving them.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.

These skills are fundamental for fostering both the analytical and practical aspects of mathematics, encouraging students to think logically, communicate mathematically, and develop a systematic approach to problem-solving.

Throughout this unit, students will:

- Learn how to draw and construct geometric figures using tools such as rulers, protractors, and compasses.
- Analyze two-dimensional figures to identify properties such as angles, lines of symmetry, congruence, and parallelism.
- Solve real-world problems involving two-dimensional shapes, fostering a deeper connection between mathematical concepts and real-life applications.

By mastering the skills in this module, students will gain the tools needed to engage with more advanced geometric concepts and develop a solid foundation for further mathematical learning.

Essential Questions

- How can we accurately draw and construct two-dimensional geometric figures?
- What properties do different two-dimensional figures share, and how can we identify and analyze them?
- How do we determine whether two figures are congruent or similar, and what are the implications of these relationships?
- In what ways do angles, symmetry, and parallelism influence the properties of two-dimensional figures?
- How can two-dimensional figures be used to solve real-world problems in fields such as art, architecture, and design?
- How do the properties of two-dimensional figures affect their ability to tile, tessellate, or fill spaces in patterns?

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Draw Shapes with Given Conditions

Student Learning Intentions (WALT):

- We are learning to draw two-dimensional shapes based on given conditions.
- We are learning to use geometric properties to construct specific shapes (e.g., triangles, quadrilaterals, polygons).

- We are learning to apply geometric reasoning to ensure our drawn shapes meet the specified criteria.

Student Success Criteria (I can statements):

- I can draw a shape when given specific conditions such as side lengths, angles, or symmetry.
- I can identify and apply properties of shapes (e.g., sum of interior angles, parallel sides, equal lengths).
- I can check that the shapes I draw meet the given criteria and verify their accuracy.

Instructional Strategies and Activities:

1. **Introduction to Drawing Shapes (Direct Instruction)**:
    - Begin the lesson with a brief review of basic geometric shapes and their properties (e.g., triangles, quadrilaterals, polygons).
    - Provide a list of conditions or criteria for drawing specific shapes (e.g., an equilateral triangle with sides of length 5, a rectangle with specific side lengths and angles).
    - Demonstrate the process of drawing shapes step by step on the board. For example:
        - For a triangle with specific angle measures, use a protractor to measure angles and a ruler for side lengths.
        - For a quadrilateral, discuss how to apply the properties of parallel sides, right angles, or symmetry.
2. **Guided Practice**:
    - Give students a set of specific conditions for various shapes. For example:
        - Draw a square with a side length of 6 cm.
        - Draw an isosceles triangle with two sides of 4 cm and the included angle of 60°.
        - Draw a rectangle where one side is 5 cm and the other is 8 cm, and the angles are right angles.
    - Walk students through each example as they follow along, checking their work and providing feedback as needed.
    - Ask students to explain the reasoning behind each step. For example, when drawing a square, students should explain that all sides are equal and angles are 90°.
3. **Independent Practice**:
    - Have students complete a worksheet or a set of tasks where they must draw shapes based on

given conditions. For example:

- Draw a rhombus with diagonals of 6 cm and 8 cm.
- Draw a trapezoid with one pair of parallel sides and a given angle.
- Draw a regular pentagon with all sides equal.

- Provide rulers, protractors, and graph paper to support students' work.
- Encourage students to check their work by verifying the conditions (e.g., sum of angles, side lengths).

4. **Peer Collaboration**:
   - Pair students to work together and check each other's drawings.
   - Have them provide constructive feedback on whether the shapes meet the given conditions.
   - After reviewing their partner's drawing, each student should explain what they checked for (e.g., equal sides, correct angles).

Formative Assessments:

1. **Exit Ticket**:
   - At the end of the lesson, students will complete a brief exit ticket where they draw one shape based on specific conditions (e.g., "Draw a parallelogram with opposite sides equal and an angle of 60°").
   - Students must explain the properties of the shape they drew.
2. **Observation and Questioning**:
   - During independent and collaborative work, observe students' process for drawing shapes. Ask questions to check their understanding, such as:
     - How did you determine the side lengths or angles?
     - What properties of the shape are important for ensuring accuracy?
     - Can you explain why your shape fits the given conditions?

Instructional Materials and Resources:

- **Geometric Tools**: Rulers, protractors, compasses, and graph paper for accurate drawings.
- **Visual Aids**: Use diagrams or Smartboard visuals to demonstrate how to construct shapes step by step.

- **Worksheet**: A set of tasks for students to complete, focusing on drawing shapes based on given conditions.
- **Peer Review Checklist**: A checklist for students to use when reviewing each other's work, ensuring that all conditions for the shape are met.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how students performed in applying geometric properties to their drawings. Did they struggle with any particular shapes or conditions?
- Consider the effectiveness of the peer review process. Were students able to help each other and check for accuracy?
- Assess whether some students needed more support with the use of geometric tools (e.g., protractors or rulers).

Suggested Modifications:

- **Differentiation**: For students who find drawing shapes challenging, provide additional scaffolding, such as templates or more detailed step-by-step guides.
- **Enrichment**: For students who master the basics quickly, offer more complex conditions, such as drawing shapes with given symmetries or applying transformations to basic shapes.
- **Collaborative Work**: Use pair or small-group activities for students to support each other, especially those struggling with the geometric tools.

Lesson 2: Draw and Construct Triangles Given Side Lengths

Student Learning Intentions (WALT):

- We are learning to draw triangles given specific side lengths.
- We are learning to understand the properties of triangles and how to construct them using geometric tools.
- We are learning to verify that the triangles we construct meet the given side length conditions.

Student Success Criteria (I can statements):

- I can use a ruler to measure and draw side lengths accurately.

- I can construct a triangle using three given side lengths.
- I can verify that the triangle I construct satisfies the triangle inequality theorem (the sum of the lengths of any two sides must be greater than the length of the third side).
- I can explain how to check if a triangle can be formed from the given side lengths.

Instructional Strategies and Activities:

1. **Introduction to Triangles (Direct Instruction)**:
    - Begin with a brief review of the properties of triangles, focusing on the relationship between side lengths and angles.
    - Introduce the **Triangle Inequality Theorem**: The sum of the lengths of any two sides of a triangle must be greater than the length of the third side. Discuss why this is important when constructing triangles.
    - Show students the process for constructing a triangle given three side lengths. Emphasize the importance of accurate measurements and the proper use of a ruler and compass.
        - Example: For side lengths of 5 cm, 7 cm, and 9 cm, students will:
            1. Draw one side (e.g., 7 cm).
            2. Place the compass on one endpoint and draw an arc with radius 5 cm.
            3. Place the compass on the other endpoint and draw an arc with radius 9 cm.
            4. Mark the intersection of the arcs; this is the third vertex of the triangle.
2. **Guided Practice**:
    - Provide students with a set of triangle side lengths to construct. For example:
        - Draw a triangle with side lengths of 6 cm, 8 cm, and 10 cm.
        - Draw a triangle with side lengths of 4 cm, 5 cm, and 7 cm.
    - Work through the first example together, demonstrating the use of a compass to find the third vertex.
    - Discuss how to check if the side lengths meet the triangle inequality theorem before starting construction.
    - Ask questions to check for understanding:
        - How do you know that the side lengths will form a triangle?
        - What happens if the side lengths do not satisfy the triangle inequality theorem?

3. **Independent Practice**:
    - Give students additional side length sets and ask them to construct the triangles independently. Examples might include:
        - A triangle with side lengths 5 cm, 8 cm, and 12 cm.
        - A triangle with side lengths 10 cm, 10 cm, and 14 cm.
        - A triangle with side lengths 7 cm, 9 cm, and 16 cm (Note: Discuss why this set of side lengths does not form a valid triangle).
    - Encourage students to check their work by verifying the triangle inequality theorem for each set of side lengths.
    - Circulate around the classroom to provide support and answer questions as needed.
4. **Peer Collaboration**:
    - Have students work in pairs to check each other's triangle constructions. They should verify that the side lengths meet the triangle inequality theorem and that the triangles have been accurately drawn.
    - Each student explains the construction process to their partner, which reinforces their understanding of the steps involved.
    - Use a peer checklist to help students assess each other's work:
        - Are the side lengths measured correctly?
        - Does the triangle satisfy the triangle inequality theorem?
        - Is the construction neat and accurate?

Formative Assessments:

1. **Exit Ticket**:
    - At the end of the lesson, students will complete an exit ticket with the following tasks:
        - Construct a triangle given side lengths (e.g., 5 cm, 6 cm, and 7 cm).
        - Explain whether the triangle inequality theorem is satisfied for a given set of side lengths (e.g., 3 cm, 6 cm, and 10 cm) and justify their answer.
2. **Observation and Questioning**:
    - During independent and collaborative practice, observe students as they work on constructing triangles. Ask questions like:
        - How did you decide where to place the third vertex?

        - Can you check if the side lengths satisfy the triangle inequality theorem?
        - How would you correct your triangle if it didn't meet the conditions?

3. **Quick Check-in**:
    - Ask individual students or pairs to describe the steps they took to construct their triangles and explain how they ensured the side lengths met the triangle inequality theorem.

Instructional Materials and Resources:

- **Geometric Tools**: Rulers, compasses, and graph paper for constructing accurate triangles.
- **Visual Aids**: Diagrams showing the step-by-step process of constructing triangles from given side lengths.
- **Worksheets**: A set of problems that provide students with side lengths and ask them to construct triangles and verify if the side lengths satisfy the triangle inequality theorem.
- **Peer Review Checklist**: A checklist to guide students as they assess their partner's triangle construction.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how students engaged with the triangle construction activity. Did they understand the steps involved in creating a triangle from given side lengths?
- Assess if students had difficulty applying the triangle inequality theorem. Were they able to identify when the side lengths did not form a valid triangle?
- Consider how well students collaborated with their peers. Did they help each other check the accuracy of their triangles?

Suggested Modifications:

- **Differentiation**: For students who struggle with the geometric tools, provide additional scaffolding, such as pre-measured side lengths or a template for the construction process.
- **Enrichment**: Challenge advanced students with sets of side lengths that form non-typical triangles (e.g., scalene triangles) or ask them to create triangles with certain properties (e.g., right-angled triangles).
- **Group Work**: Pair students of varying abilities to foster collaboration. Have them work together on the more complex triangle constructions.

Lesson 3: Draw and Construct Triangles Given Angle Measures

Student Learning Intentions (WALT):

- We are learning to construct triangles given specific angle measures.
- We are learning to understand the properties of angles in triangles and how to use geometric tools to construct them.
- We are learning to verify that the triangles we construct meet the given angle conditions.

Student Success Criteria (I can statements):

- I can use a protractor to measure and draw angles accurately.
- I can construct a triangle using three given angle measures.
- I can verify that the sum of the angles in a triangle equals 180°.

Instructional Strategies and Activities:

1. **Introduction to Angle Measures in Triangles (Direct Instruction)**:
    - Begin with a review of the **angle sum property of triangles**: the sum of the interior angles of any triangle is always 180°.
    - Introduce the concept of constructing triangles when the angle measures are given. Discuss how knowing just the angles of a triangle allows for the construction of the shape, but we need to be aware of the type of triangle (e.g., acute, right, or obtuse).
    - Provide an example where you give three angle measures, such as 50°, 60°, and 70°. Show how to use a protractor to draw each angle and complete the triangle.
2. **Step-by-Step Construction of a Triangle Given Angles (Guided Practice)**:
    - Guide the students through the construction process using a triangle with angles of 40°, 75°, and 65°.
        - **Step 1**: Start by drawing the base of the triangle with any length (e.g., 6 cm).
        - **Step 2**: Using a protractor, measure and draw the first angle (e.g., 40°) at one endpoint of the base.
        - **Step 3**: From the other endpoint of the base, measure and draw the second angle (e.g., 75°).

        - **Step 4**: The third angle will automatically be determined because the sum of the interior angles must be 180°. The third angle should be 65°.
        - **Step 5**: Complete the triangle by drawing the final sides to connect the angles.
    - Discuss how to check that the sum of the three angles equals 180° and that the triangle is valid.
3. **Independent Practice**:
    - Give students a set of problems with specific angle measures and ask them to construct triangles. For example:
        - Construct a triangle with angle measures of 45°, 55°, and 80°.
        - Construct a triangle with angle measures of 30°, 60°, and 90°.
        - Construct a triangle with angle measures of 25°, 85°, and 70°.
    - Encourage students to check their work by verifying the angle sum property (i.e., the sum should be 180°).
    - Allow students to use protractors and rulers to accurately measure and draw their triangles.
4. **Peer Collaboration**:
    - Have students work in pairs to check each other's triangle constructions. They should verify that the angles have been measured correctly and that the triangle is valid.
    - Each student explains the construction process to their partner, reinforcing their understanding of the steps involved.
    - Use a peer checklist to guide this process:
        - Are the angles measured correctly?
        - Does the sum of the angles equal 180°?
        - Are the sides of the triangle drawn accurately?

Formative Assessments:

1. **Exit Ticket**:
    - At the end of the lesson, students will complete an exit ticket with the following tasks:
        - Construct a triangle given the angles 50°, 60°, and 70°.
        - Explain whether the triangle you constructed satisfies the angle sum property and why.
2. **Observation and Questioning**:
    - As students work on constructing triangles, observe their use of protractors and rulers. Ask

questions like:

- How did you ensure that your angles were measured accurately?
- What happens if the sum of the angles doesn't equal 180°?
- Can you explain the process you used to construct the triangle?

3. **Quick Check-in**:
   - Periodically check for understanding by asking individual students or pairs:
     - What would happen if you changed one of the angles?
     - How can you tell if your triangle construction is accurate?

Instructional Materials and Resources:

- **Geometric Tools**: Protractors, rulers, and graph paper for accurate angle measurement and construction.
- **Visual Aids**: Diagrams showing the step-by-step process of constructing triangles given angles.
- **Worksheets**: A set of problems providing angle measures and asking students to construct triangles based on those angles.
- **Peer Review Checklist**: A checklist to help students assess their partner's triangle construction.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how students engaged with the angle measurement process. Did they understand the relationship between the angles and the triangle's shape?
- Consider how well students checked the angle sum property and verified the validity of their triangles. Were they able to identify any mistakes?
- Assess whether students struggled with using the protractor. Did they need additional practice or support?

Suggested Modifications:

- **Differentiation**: For students who struggle with angle measurements, provide pre-drawn base angles or simplified triangle examples with large angles that are easier to measure.
- **Enrichment**: Challenge advanced students to explore different types of triangles (e.g., equilateral, isosceles, or scalene) and construct them with given angle measures that correspond to those triangle

types.

- **Group Work**: For peer collaboration, group students with varying abilities to encourage mutual support. Pair students who can help each other verify the correctness of angle measures and the triangle construction.

Lesson 4: Draw and Analyze Shapes to Solve Problems

Student Learning Intentions (WALT):

- We are learning to draw and analyze different geometric shapes.
- We are learning to apply geometric principles to solve real-world problems involving shapes.
- We are learning to interpret geometric shapes to find solutions to practical problems.

Student Success Criteria (I can statements):

- I can accurately draw geometric shapes based on given information (e.g., lengths, angles, and properties).
- I can analyze the properties of shapes to solve problems.
- I can apply geometric reasoning to find the solution to real-world problems.

Instructional Strategies and Activities:

1. **Introduction to Geometric Shapes and Their Properties (Direct Instruction)**:
    - Start by reviewing basic geometric shapes such as triangles, squares, rectangles, circles, and polygons. Discuss their properties (e.g., the number of sides, angles, symmetry).
    - Explain that geometry can be used to solve real-world problems by applying the properties of these shapes. For example, a problem might involve finding the area of a garden, the perimeter of a room, or the volume of a box.
    - Introduce the idea of problem-solving by drawing geometric shapes to scale based on provided information (e.g., length of sides, angles, or other given data).
2. **Step-by-Step Example (Guided Practice)**:
    - Present a real-world problem where students need to draw and analyze a geometric shape to solve it. For example:
        - *Problem*: A rectangular garden has a length of 10 meters and a width of 5 meters. What

is the perimeter and area of the garden?

- **Step 1**: Have students draw a rectangle and label the sides with the given dimensions (10 meters by 5 meters).
- **Step 2**: Ask students to calculate the perimeter (sum of all sides) and area (length × width) of the rectangle.
- **Step 3**: Discuss the solution and explain how the properties of a rectangle (opposite sides are equal and parallel) make the calculations easier.

3. **Real-World Problem Solving (Independent Practice)**:
   - Provide students with various real-world problems that involve geometric shapes. These could include:
     - A triangular roof with sides of length 8 meters, 10 meters, and 12 meters. Find the perimeter and determine if the triangle is scalene, isosceles, or equilateral.
     - A circular table has a radius of 4 meters. Find the area and the circumference of the table.
     - A square room has a side length of 6 feet. How much carpet is needed to cover the floor?
   - Have students draw the shapes to scale, label the dimensions, and then solve for the required quantities (e.g., perimeter, area, or volume).
   - Encourage students to use geometric formulas, such as:
     - **Area of a rectangle**: Area = length × width
     - **Area of a triangle**: Area = (base × height) / 2
     - **Area of a circle**: Area = $\pi \times \text{radius}^2$
     - **Perimeter of a rectangle**: Perimeter = 2 × (length + width)
     - **Perimeter of a triangle**: Perimeter = sum of the lengths of the sides
     - **Circumference of a circle**: Circumference = $2 \times \pi \times$ radius
4. **Peer Collaboration**:
   - Have students pair up and share their solutions to the problems. They should explain how they drew the shapes, used the geometric properties, and arrived at their solutions.
   - Encourage students to check each other's work for accuracy and to ask questions if they don’t understand each other's reasoning.
   - As students collaborate, circulate around the room to observe and offer support where needed.

Formative Assessments:

1. **Exit Ticket**:
    - At the end of the lesson, give students a brief exit ticket with the following task:
        - Draw a shape based on the following conditions: a rectangle with a length of 8 meters and a width of 3 meters. Calculate the perimeter and area of the rectangle.
        - Ask students to explain the method they used to find the area and perimeter.
2. **Observation and Questioning**:
    - During independent practice and peer collaboration, observe students as they work. Ask questions such as:
        - How did you calculate the area/perimeter of this shape?
        - Can you explain why you used this formula for the shape you drew?
        - What is the most important property of this shape that helped you solve the problem?
3. **Quick Check-in**:
    - Walk around the classroom and engage students with quick, informal questions:
        - How would the problem change if the shape had different dimensions?
        - How do the properties of the shape help you with solving the problem?

Instructional Materials and Resources:

- **Geometric Tools**: Rulers, protractors, and compasses for drawing shapes accurately.
- **Worksheets**: A set of real-world problems that involve different shapes, with space for students to draw and solve the problems.
- **Visual Aids**: Diagrams showing different shapes, their properties, and example problem-solving processes.
- **Interactive Tools**: Use digital tools (e.g., interactive whiteboards, geometry software) for students to practice drawing and analyzing shapes in an interactive way.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how well students were able to draw the shapes and use the properties of these shapes to

solve problems. Did they encounter difficulties with any particular type of shape or calculation?

- Consider whether students understood how geometric shapes can be used to solve real-world problems. Were they able to connect the abstract concept of shapes to practical applications?
- Review if students required more practice with drawing or with using formulas for area and perimeter.

Suggested Modifications:

- **Differentiation**: For students who need additional support, provide more scaffolded problems where key information is provided (e.g., drawing a shape with one given side length and asking them to calculate the perimeter). Consider providing shapes with more familiar dimensions.
- **Enrichment**: For advanced students, offer challenges that involve more complex geometric shapes, such as composite figures (e.g., a shape made of a rectangle and a triangle combined) or problems that require converting between different units of measurement.
- **Group Work**: Organize students into small groups and assign each group a different problem to solve. Afterward, have them present their solutions to the class and explain the steps they took to solve the problem.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
    - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
    - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals**:
    - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary**:
    - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
    - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
    - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.
- **Extra Time**:
    - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.
- **Oral Reading**:
    - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
    - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
    - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
    - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
    - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
    - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
    - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
    - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.
- **Peer Support**:
    - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
    - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
    - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
    - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
    - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.
- **Tests/Assessments**:
    - Tests can be administered orally, or broken into smaller sections. Teachers may administer

parts of a test in the morning, after lunch, and on subsequent days if necessary.

- **Seating**:
    - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
    - Fist to five: "How well do you understand what we talked about today?"
    - Fist to five: "How well did I teach this today?"
- **Classroom Activity**:
    - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
    - Color-coding ideas
    - Learning vocabulary in student languages
    - Visual and kinesthetic learning
    - Representations of learning without words
- **Classroom Activity**:
    - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:

    - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"

- **Classroom Activity**:
    - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
    - "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
    - Learn about different bases and numerical ideas:
        - Base 2 (binary) and connections to computer programming
        - How the Yoruba of Nigeria used base 20
        - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
    - "How might you all go about this?"
    - "What do you notice?"
- **Classroom Activity**:
    - Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
    - "How does this connect to what you've learned in the past?"
    - "How can you use that knowledge today?"

- **Classroom Activity**:
    - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
    - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"
- **Classroom Activity**:
    - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
    - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
    - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
    - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
    - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
    - "Let’s get into partners and do a think-pair-share. We will incorporate everyone’s ideas and try to synthesize them."
- **Classroom Activity**:
    - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
    - "Come up with at least two answers that might solve this problem."
- **Classroom Activity**:
    - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
    - **Classroom Activity**:
        - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
    - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:

- "What would a mathematician who is confused ask about this question?"

- **Classroom Activity**:
  - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:
  - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."

- **Classroom Activity**:
  - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).

- **Climate Change Example**:
  - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MATH.7.G	Geometry
MATH.7.G.B	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume

21st Century Life and Career

CRP.K-12.CRP2	Apply appropriate academic and technical skills.

CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Integration of Career Readiness. Life Literacies and Key Skills

PFL.9.1.4.F	Civic Financial Responsibility

Integration of Computer Science and Design Thinking

CS.K-2.8.1.2.DA.4	Make predictions based on data using charts or graphs.
	Computers follow precise sequences of steps that automate tasks.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

	Language: System and structure, effective use, and vocabulary
SOC.K-12.2	Gathering and Evaluating Sources
	Speaking and Listening: Flexible communication and collaboration
SCI.9-12.5.3.12	All students will understand that life science principles are powerful conceptual tools for making sense of the complexity, diversity, and interconnectedness of life on Earth. Order in natural systems arises in accordance with rules that govern the physical world, and the order of natural systems can be modeled and predicted through the use of mathematics.
TECH.8.1.P.E	Research and Information Fluency: Students apply digital tools to gather, evaluate, and use information.

Module 10: Transformations and Similiarity

Unit Rationale

In this module, students explore geometric transformations and similarity, both fundamental concepts in understanding shapes and their properties. Transformations, including translations, rotations, reflections, and dilations, allow students to manipulate geometric figures and understand their behavior under various conditions. Similarly, the concept of similarity provides a framework for comparing shapes and understanding proportional relationships between corresponding sides and angles.

Understanding transformations and similarity is essential not only in mathematics but also in real-world applications, such as art, architecture, engineering, and computer graphics, where precise manipulation and comparison of shapes are required.

This module provides opportunities for students to:

1. **Understand and perform transformations**: Students will apply transformations to geometric figures and analyze how they affect the figures' properties (e.g., size, shape, and orientation).

2. **Explore similarity**: Students will investigate how the properties of similar figures, such as proportional sides and equal angles, lead to an understanding of geometric scaling and real-world phenomena like maps, blueprints, and models.

3. **Connect transformations to symmetry**: Through the study of reflections, rotations, and translations, students will explore symmetry in geometry and apply these concepts in practical scenarios.

By the end of the module, students will:

- Be able to recognize and perform basic transformations: translations, rotations, reflections, and dilations.
- Understand the criteria for similarity between geometric figures and be able to identify similar figures in various contexts.
- Apply their knowledge of transformations and similarity to solve geometric problems and make predictions based on geometric properties.

This module emphasizes the development of spatial reasoning and logical thinking. By engaging in activities that require students to visualize and manipulate shapes, they will strengthen their problem-solving abilities and gain a deeper understanding of the geometry that underpins much of the world around them.

Key Standards for Mathematical Practice:

- **MP4**: Model with mathematics - Students will model geometric transformations and similarity through hands-on activities and problem-solving.
- **MP7**: Look for and make use of structure - Students will identify patterns and properties of shapes under transformations and similarity.
- **MP8**: Look for and express regularity in repeated reasoning - Students will recognize the repeated application of geometric transformations and similarity properties to solve problems efficiently.

This module fosters critical thinking and the application of geometric concepts to practical problems, laying the groundwork for more advanced topics in geometry and real-world problem-solving.

Essential Questions

- **What are geometric transformations, and how do they affect the properties of shapes?**
- **How can we use transformations (translations, rotations, reflections, and dilations) to manipulate geometric figures?**
- **What makes two figures similar, and how can we identify and apply similarity in geometric shapes?**
- **How do transformations help us understand symmetry in geometry and the real world?**
- **How can we apply transformations and similarity to solve real-world problems involving geometric shapes?**
- **In what ways do proportional relationships between the sides of similar figures allow us to make predictions or scale models?**

- **How can transformations and similarity be used in fields like art, architecture, and engineering?**
- **How does the concept of similarity relate to the idea of geometric scaling in real-life scenarios?**

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Investigate Reductions and Enlargements

Student Learning Intentions (WALT):

- We are learning to understand the concepts of reductions and enlargements in geometric figures.
- We are learning to apply the concept of scale factor to reduce or enlarge figures.
- We are learning to solve problems involving reductions and enlargements and their real-world applications.

Student Success Criteria (I can statements):

- I can explain the concept of a reduction and an enlargement.
- I can identify the scale factor in a reduction or enlargement.
- I can apply the scale factor to enlarge or reduce a given figure.
- I can solve real-world problems that involve reductions and enlargements.

Instructional Strategies and Activities:

Introduction to Reductions and Enlargements:

- **Direct Instruction:**
    - Begin by explaining what reductions and enlargements are in geometric terms. A **reduction** is when the size of a figure is made smaller, and an **enlargement** is when the size of a figure is made bigger. Both transformations maintain the shape and proportions of the original figure.
    - Use a simple example, such as enlarging a square or reducing a rectangle, to illustrate the concept.
    - Introduce the **scale factor** as the number that multiplies the dimensions of the original figure to get the dimensions of the new figure. A scale factor greater than 1 corresponds to an enlargement, and a scale factor less than 1 corresponds to a reduction.
- **Visual Models:**
    - Use the board or a projector to show examples of reductions and enlargements. Draw a triangle, then show how its dimensions change with a scale factor of 2 (enlargement) and a scale factor of 0.5 (reduction).
    - Provide a visual of the scale factor and how it relates to the side lengths.

Guided Practice:

- **Activity 1:**
    - Have students work in pairs to draw a rectangle. Provide them with a scale factor, either greater

than or less than 1, and ask them to enlarge or reduce the figure accordingly.

    - Ask students to calculate the new dimensions based on the scale factor (e.g., if the original dimensions are 4 cm by 6 cm and the scale factor is 2, the new dimensions should be 8 cm by 12 cm).

- **Activity 2:**

    - Provide several figures and ask students to determine the scale factor required to transform one figure into another. For example, show a smaller triangle and a larger triangle, and ask students to find the scale factor between the two.

Independent Practice:

- **Worksheet:**

    - Distribute a worksheet with various problems involving reductions and enlargements. Include both numerical and word problems where students are required to:

        - Identify the scale factor in a set of figures.
        - Calculate the new dimensions of figures after applying a given scale factor.
        - Solve real-life problems involving reductions and enlargements (e.g., resizing a blueprint or making a photo larger).

- **Real-World Problem Solving:**

    - Provide a word problem where students need to apply their knowledge of scale factors. Example: "A designer is creating a miniature model of a building with a scale factor of 1:100. If the building is 200 meters tall, how tall is the model?"

Peer Collaboration:

- Have students pair up and review each other's answers to ensure the correct application of scale factors. Students can compare how they approached the problem, discuss any errors, and work together to resolve misunderstandings.

Formative Assessments:

Exit Ticket:

- At the end of the lesson, have students complete a quick exit ticket where they solve one problem involving reductions and one problem involving enlargements. This can be a multiple-choice question or a short-answer question where students show their work.

Observation and Questioning:

- During independent practice, circulate the room and observe students' problem-solving strategies. Ask

questions such as:

- "How did you find the new dimensions?"
- "What happens to the shape when you apply a scale factor of less than 1?"

Quick Check-in:

- During peer collaboration, ask students to explain in their own words what a scale factor is and how it affects the size of a shape.

Instructional Materials and Resources:

- **Whiteboard/Smartboard:** For visual modeling of reductions and enlargements.
- **Rulers and Graph Paper:** For students to draw and measure their figures accurately.
- **Worksheets/Problem Sets:** A set of practice problems involving real-world reductions and enlargements.
- **Interactive Tools (Optional):** Online scale factor tools or apps that allow students to manipulate the dimensions of shapes interactively.

Reflections and Suggested Modifications:

Reflection:

- Reflect on students' understanding of the relationship between scale factors and the size of geometric figures. Did they grasp how a scale factor works? Were they able to apply the concept to real-world scenarios?
- Consider whether students struggled with calculating scale factors or with understanding the concept of reduction versus enlargement.

Suggested Modifications:

- **Differentiation:**
  - For students who struggle with the concept of scale factor, provide hands-on activities using physical objects (e.g., creating scale models with cardboard or measuring objects and comparing them).
  - Use visual aids such as color-coded diagrams to highlight the original and transformed figures.
- **Enrichment:**
  - For students who grasp the concept quickly, introduce more complex problems that involve

multiple transformations (e.g., first a reduction, then an enlargement) or work with scale factors in three-dimensional shapes.

- Challenge students to create their own real-world problems involving reductions and enlargements and exchange them with their peers for solving.

Lesson 2: Explore Dilations

Student Learning Intentions (WALT):

- We are learning to understand the concept of dilations in geometry.
- We are learning to apply dilation to geometric figures using a given center and scale factor.
- We are learning to analyze the effects of dilations on the size and shape of figures.

Student Success Criteria (I can statements):

- I can define dilation and explain its properties.
- I can apply a given scale factor to dilate a geometric figure.
- I can identify how dilations affect the size and shape of a figure.
- I can solve problems that involve dilations in both abstract and real-world contexts.

Instructional Strategies and Activities:

Introduction to Dilations:

- **Direct Instruction:**
  - Begin by introducing the concept of **dilation**: a transformation that changes the size of a figure but preserves its shape. The figure is either enlarged or reduced based on a scale factor, but the proportions of the figure remain the same.
  - Define the **center of dilation** as the fixed point in the plane from which the figure is dilated. The figure is enlarged or reduced relative to this center.
  - Explain the concept of **scale factor**:
    - A scale factor greater than 1 results in an **enlargement**.
    - A scale factor between 0 and 1 results in a **reduction**.

        - A scale factor of 1 means the figure remains the same size.
    - Show examples of dilations on the board, demonstrating how a figure changes in size when a scale factor is applied.

- **Visual Models:**
    - Use graph paper to plot a triangle or rectangle. Show how to dilate it using a scale factor of 2 (enlargement) and a scale factor of 0.5 (reduction). Mark the center of dilation and demonstrate the movement of points relative to this center.
    - Display several dilated figures with different scale factors and centers to visually reinforce the concept.

Guided Practice:

- **Activity 1:**
    - Have students work in pairs to create their own dilation problems. They will choose a figure, a center of dilation, and a scale factor. Then, they will apply the dilation and sketch the dilated figure. Ask them to identify how the size and shape of the figure have changed.
- **Activity 2:**
    - Provide several figures on the board and ask students to calculate the new coordinates of the points after a dilation. Start with a simple figure and a center at the origin, then gradually increase complexity by changing the center and scale factor.

Independent Practice:

- **Worksheet:**
    - Give students a worksheet with problems that require them to:
        - Apply a scale factor to dilate geometric figures (both simple and composite shapes).
        - Identify the new coordinates after dilation and plot the dilated figure.
        - Solve real-world problems involving dilations (e.g., resizing an object in a photo or a map).
- **Real-World Problem Solving:**
    - Provide a real-world application problem. For example, “A photographer is resizing a photo using a scale factor of 1.5. If the original photo has dimensions 10 cm by 12 cm, what will the dimensions of the new photo be after dilation?”

Peer Collaboration:

- Have students pair up and share their answers. They should discuss how they found the new coordinates after dilation and explain the reasoning behind their scale factor choices.

- Encourage students to check each other's work for accuracy and clarity in their problem-solving steps.

Formative Assessments:

Exit Ticket:

- At the end of the lesson, have students complete a brief exit ticket with the following questions:
    - What is the definition of dilation?
    - What happens to the size of a figure when the scale factor is greater than 1? Less than 1?
    - Solve a simple dilation problem (e.g., dilate a triangle with a scale factor of 2).

Observation and Questioning:

- As students work through problems, ask questions such as:
    - "How did you find the new position of this point after dilation?"
    - "What effect does the center of dilation have on the transformation?"

Quick Check-in:

- During independent practice, circulate the room and ask students to explain the process they followed to find the new coordinates. This will help ensure that they understand how the scale factor and center of dilation interact.

Instructional Materials and Resources:

- **Whiteboard/Smartboard:** To demonstrate and illustrate dilations visually.
- **Graph Paper:** For students to plot figures and apply dilations accurately.
- **Rulers/Protractors:** For drawing precise figures and ensuring accuracy in dilations.
- **Worksheets/Problem Sets:** A set of practice problems involving dilations in different contexts.
- **Interactive Tools (Optional):** Online graphing tools or apps that allow students to manipulate figures and apply dilations interactively.

Reflections and Suggested Modifications:

Reflection:

- Reflect on how well students grasped the concept of dilation. Did they understand how to calculate the new dimensions and coordinates of a figure? Did they recognize the relationship between the scale factor and the size of the figure?
- Evaluate whether students needed more hands-on practice with visualizing dilations and applying them in various contexts.

Suggested Modifications:

- **Differentiation:**
  - For students who need additional support, provide simpler shapes and a fixed center of dilation (e.g., always dilating from the origin) to reduce complexity.
  - Use physical objects, such as paper cutouts, to physically demonstrate dilations and allow students to manipulate the shapes themselves.
- **Enrichment:**
  - For students who understand the concept quickly, have them work on more complex problems, such as dilating composite shapes (e.g., a polygon with several sides) or applying dilations in coordinate geometry problems.
  - Challenge students to find the scale factor when given the original and dilated figure, requiring them to use proportional reasoning.

Lesson 3: Understand and Recognize Similar Figures

Student Learning Intentions (WALT):

- We are learning to understand the concept of similar figures.
- We are learning to identify similar figures based on proportional relationships between corresponding sides and angles.
- We are learning to apply the properties of similar figures to solve problems in geometry.

Student Success Criteria (I can statements):

- I can define similar figures and explain how they are related.
- I can identify similar figures by comparing the ratios of corresponding sides.
- I can determine whether two figures are similar by checking for proportionality of corresponding sides and equality of corresponding angles.

- I can solve real-world problems that involve similar figures.

Instructional Strategies and Activities:

Introduction to Similar Figures:

- **Direct Instruction:**
    - Start by defining **similar figures**: Two figures are similar if they have the same shape but may differ in size. The corresponding angles are equal, and the corresponding sides are proportional (i.e., the ratios of the lengths of corresponding sides are equal).
    - Discuss **scale factor** in relation to similar figures: The scale factor is the ratio of the lengths of corresponding sides of two similar figures. It tells you how much one figure is enlarged or reduced relative to the other.
    - Explain how to recognize similar figures:
        - **Proportional sides:** The ratio of corresponding sides is constant.
        - **Equal angles:** The corresponding angles are congruent.
    - Provide an example: Show two triangles that are similar. Explain how the corresponding angles are equal, and the sides are proportional. Demonstrate how the scale factor is calculated using the sides of the triangles.
- **Visual Models:**
    - Display a pair of similar triangles on the board. Label the corresponding sides and angles. Show how to calculate the scale factor using the side lengths.
    - Use graph paper to demonstrate two similar figures, making sure the ratios of corresponding sides are the same and the angles are congruent.

Guided Practice:

- **Activity 1: Identifying Similar Figures:**
    - Provide students with a set of pairs of figures (e.g., triangles, rectangles) and ask them to determine if they are similar. Guide them to compare corresponding sides and angles, and explain the steps they used to arrive at their answer.
- **Activity 2: Solving Proportions:**
    - Present a pair of similar triangles where the side lengths of one triangle are given, and some side lengths of the second triangle are missing. Have students set up and solve proportions to find the missing sides. Walk them through the process of setting up the proportion and solving it.

Independent Practice:

- **Worksheet:**
    - Provide students with a worksheet that includes problems where they have to:
        - Identify if two figures are similar by comparing corresponding sides and angles.
        - Solve for missing side lengths using proportional relationships in similar figures.
        - Work with word problems that apply the concept of similarity (e.g., finding the height of a tree using similar triangles).
- **Real-World Problem Solving:**
    - Present a real-world scenario: "Two flags are similar in shape, but one is larger than the other. The smaller flag has a width of 5 meters, and the larger flag has a width of 8 meters. What is the scale factor of the enlargement?" Have students solve by finding the scale factor and applying it to other dimensions of the flag.

Peer Collaboration:

- Have students work in pairs to solve a series of problems involving similar figures. After solving the problems, students should explain their reasoning to their partner, ensuring they understand the relationship between corresponding sides and angles.

Formative Assessments:

Exit Ticket:

- At the end of the lesson, have students complete a brief exit ticket with the following questions:
    - What are similar figures, and how do we recognize them?
    - How can you use the proportionality of corresponding sides to solve problems involving similar figures?
    - Solve a short problem where two figures are similar, and students must determine if they are similar and find a missing side.

Observation and Questioning:

- As students work through problems, ask questions such as:
    - "How did you find the scale factor between these two figures?"
    - "What is the relationship between the angles of similar figures?"
    - "Can you explain why the corresponding sides are proportional?"

Quick Check-in:

- During independent practice, circulate the room and check for understanding by asking students to explain how they identified similar figures and how they solved for the missing sides using proportions.

Instructional Materials and Resources:

- **Whiteboard/Smartboard:** For illustrating and solving problems involving similar figures.
- **Graph Paper:** To help students visualize similar figures and measure side lengths accurately.
- **Rulers/Protractors:** For measuring side lengths and angles of figures to ensure accuracy in identifying similar figures.
- **Worksheets/Problem Sets:** A set of practice problems involving similar figures, identifying similarity, and solving for missing side lengths.
- **Interactive Tools (Optional):** Online tools or apps that allow students to manipulate and compare figures to visually understand the relationship between similar figures.

Reflections and Suggested Modifications:

Reflection:

- Reflect on whether students were able to easily identify similar figures by comparing corresponding sides and angles. Did they understand how the scale factor relates to the size of the figures?
- Consider whether students needed more support with setting up and solving proportions, or if they were able to apply these concepts independently.

Suggested Modifications:

- **Differentiation:**
    - For students who need additional support, provide simpler problems with only one pair of corresponding sides given, and gradually increase complexity as they gain confidence.
    - Use physical models (e.g., paper cutouts) for students to manipulate and compare shapes, which may help them better understand the concept of similarity.
- **Enrichment:**
    - For students who grasp the concept quickly, provide problems that involve more complex figures or real-world applications, such as determining the size of an object using similar figures in photography or architecture.

- Challenge students to derive the proportionality relationships when dealing with multiple figures or to apply similarity in three-dimensional contexts.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
  - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
  - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals**:
  - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary**:
  - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
  - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
  - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.
- **Extra Time**:
  - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.

- **Oral Reading**:
    - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
    - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
    - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
    - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
    - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
    - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
    - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
    - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check

back to ensure they haven't forgotten.

- **Peer Support**:
    - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
    - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
    - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
    - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
    - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.
- **Tests/Assessments**:
    - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:
    - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
    - Fist to five: "How well do you understand what we talked about today?"

- Fist to five: "How well did I teach this today?"

- **Classroom Activity**:
  - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
  - Color-coding ideas
  - Learning vocabulary in student languages
  - Visual and kinesthetic learning
  - Representations of learning without words
- **Classroom Activity**:
  - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
  - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
- **Classroom Activity**:
  - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
  - "Why do you think we call it Pythagorean’s theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
  - Learn about different bases and numerical ideas:

        - Base 2 (binary) and connections to computer programming
        - How the Yoruba of Nigeria used base 20
        - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
    - "How might you all go about this?"
    - "What do you notice?"
- **Classroom Activity**:
    - Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
    - "How does this connect to what you've learned in the past?"
    - "How can you use that knowledge today?"
- **Classroom Activity**:
    - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
    - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"
- **Classroom Activity**:
    - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
    - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
    - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
    - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
    - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
    - "Let’s get into partners and do a think-pair-share. We will incorporate everyone’s ideas and try to synthesize them."
- **Classroom Activity**:
    - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
    - "Come up with at least two answers that might solve this problem."

- **Classroom Activity**:
    - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
    - **Classroom Activity**:
        - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
    - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
    - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:
    - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:
    - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
    - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).

- **Climate Change Example**:

    - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MATH.8.G	Geometry
MATH.8.G.A	Understand congruence and similarity using physical models, transparencies, or geometry software
MATH.8.G.A.3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
MATH.8.G.A.4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

21st Century Life and Career

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP7	Employ valid and reliable research strategies.

Integration of Career Readiness, Life Literacies and Key Skills

PFL.9.1.4.B	Money Management
PFL.9.1.4.F	Civic Financial Responsibility

Integration of Computer Science and Design Thinking

	Real world information can be stored and manipulated in programs as data (e.g., numbers, words, colors, images).
	A computing system is composed of software and hardware.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

	Key Ideas and Details
	K-5 To build a foundation for college and career readiness, students must read widely and deeply from among a broad range of high-quality, increasingly challenging literary and informational texts. Through extensive reading of stories, dramas, poems, and myths from diverse cultures and different time periods, students gain literary and cultural knowledge as well as familiarity with various text structures and elements. By reading texts in history/social studies, science, and other disciplines, students build a foundation of knowledge in these fields that will also give them the background to be better readers in all content areas. Students can only gain this foundation when the curriculum is intentionally and coherently structured to develop rich content knowledge within and across grades. Students also acquire the habits of reading independently and closely, which are essential.
SCI.9-12.5.1.12.D	The growth of scientific knowledge involves critique and communication, which are social practices that are governed by a core set of values and norms.