Unit 2: Number Systems and Operations

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **9 Weeks**
Status: **Published**

Module 3: Understand Addition and Subtraction of Rational Numbers

Unit Rationale

In this unit, students explore the addition and subtraction of rational numbers, which include both positive and negative integers, fractions, and decimals. Rational numbers form the basis for much of the mathematics students will encounter in later grades, as well as for real-world problem-solving. This unit is essential because it helps students develop fluency in operations with rational numbers, providing a foundation for more advanced topics such as algebra, geometry, and proportional reasoning.

Understanding how to perform operations with rational numbers is crucial for students' ability to navigate everyday situations, from budgeting money to calculating distances or temperatures. By mastering addition and subtraction of rational numbers, students gain the skills to solve real-world problems that require accurate and flexible number operations. Additionally, this unit prepares students for algebraic concepts such as solving equations with integers, fractions, and decimals, making it a critical skill for success in higher mathematics.

This unit is intentionally placed early in the 7th grade curriculum to ensure that students have a solid foundation in working with rational numbers before they encounter more complex operations. Through hands-on practice, visual aids, and real-world examples, students will be able to build their understanding of rational numbers and apply this knowledge confidently in both mathematical contexts and everyday life.

Key Concepts Addressed in This Unit:

- **Understanding Rational Numbers:** Identification and classification of rational numbers, including both positive and negative integers, fractions, and decimals.
- **Addition and Subtraction of Rational Numbers:** Understanding and applying the rules for adding and subtracting integers, fractions, and decimals.
- **Real-World Application:** Using the operations of addition and subtraction of rational numbers to solve practical problems related to money, temperature, distance, and more.

This unit also encourages students to practice critical Standards for Mathematical Practice such as:

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP4:** Model with mathematics.

By the end of this unit, students will be proficient in adding and subtracting rational numbers in a variety of formats, laying the groundwork for future mathematical success and a deeper understanding of the world around them.

Essential Questions

- **What are rational numbers, and how do we identify them in different forms (fractions, decimals, and integers)?**
- **How do the properties of addition and subtraction change when working with rational numbers, including both positive and negative values?**
- **How can we apply addition and subtraction of rational numbers to solve real-world problems involving money, temperature, distance, or other situations?**
- **What strategies can we use to accurately add or subtract fractions, decimals, and integers with rational values?**
- **How do we determine the correct operation (addition or subtraction) and ensure the result makes sense in a given context?**
- **How do the rules for adding and subtracting negative numbers compare to those for adding and subtracting positive numbers?**
- **Why is it important to understand addition and subtraction of rational numbers when solving algebraic expressions and equations?**
- **How do visual models (such as number lines or diagrams) help in understanding the addition and subtraction of rational numbers?**

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.

- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Add or Subtract a Positive Integer on a Number Line

Student Learning Intentions (WALT)

- **We are learning to** add and subtract positive integers using a number line.
- **We are learning to** represent addition and subtraction of positive integers visually on a number line.

Student Success Criteria (I Can Statements)

- **I can** represent addition and subtraction of positive integers on a number line.
- **I can** accurately add and subtract positive integers using a number line to show the correct direction (right for addition, left for subtraction).
- **I can** solve problems involving addition and subtraction of positive integers using the number line method.

Instructional Strategies and Activities

1. **Introduction to the Number Line**
    - Begin by reviewing the concept of a number line and its parts. Discuss the direction of positive integers (to the right) and negative integers (to the left).
    - Demonstrate how to use a number line to solve basic addition and subtraction problems involving positive integers.

2. **Guided Practice**
    - Present problems like **3 + 2** or **5 - 3**, and model how to solve them using a number line. Use physical or digital tools to demonstrate, emphasizing the process of moving to the right for addition and to the left for subtraction.
3. **Collaborative Practice**
    - Have students work in pairs or small groups to solve similar problems on a number line. Provide a number line chart for students to use, or let them create their own on paper.
4. **Independent Practice**
    - After practicing together, allow students to work independently on a set of problems that involve both addition and subtraction of positive integers. These problems can include word problems to make the task more engaging.
5. **Class Discussion**
    - Discuss the strategies students used to solve the problems. Highlight different approaches and strategies. Focus on how moving right on the number line for addition and left for subtraction helps students visualize the process.

Formative Assessments

1. **Exit Ticket**:
    - At the end of the lesson, provide a short formative assessment (exit ticket) that includes a couple of addition and subtraction problems with positive integers on the number line. Students will need to draw the number line and show their work.
2. **Partner Check-In**:
    - Have students pair up and check each other's work, discussing any differences in their approach. This promotes peer learning and provides immediate feedback.
3. **Quick Check**:
    - Ask students to solve a few problems on a number line during the lesson as a quick check for understanding. You can walk around and provide instant feedback.

Instructional Materials and Resources

- **Number line handouts** (physical or digital)
- **Markers or pens** for students to use to mark their number lines
- **Whiteboard or interactive display** to model number line examples
- **Practice worksheets** with a variety of addition and subtraction problems involving positive integers

Reflections and Suggested Modifications

- **For Struggling Students**: Provide additional practice with smaller numbers and scaffold the process of moving along the number line. Use physical objects (such as counters or beads) to demonstrate the movement visually.
- **For Advanced Students**: Challenge them by introducing more complex word problems or asking them to solve addition and subtraction problems using larger positive integers.
- **Differentiation**: Provide number lines with varying levels of difficulty. Some students might benefit from a number line with more clearly marked intervals, while others might need fewer marks to encourage them to focus on the concept of movement along the number line.

This lesson encourages the use of visual representation to solidify the understanding of integer operations and provides hands-on practice in a collaborative environment, helping students grasp the concept of addition and subtraction of positive integers.

Lesson 2: Add or Subtract a Negative Integer on a Number Line

Student Learning Intentions (WALT)

- **We are learning to** add and subtract negative integers using a number line.
- **We are learning to** represent addition and subtraction of negative integers visually on a number line.

Student Success Criteria (I Can Statements)

- **I can** add and subtract negative integers using a number line.
- **I can** represent addition and subtraction of negative integers visually on a number line by moving to the left for subtraction and to the right for addition.
- **I can** solve problems involving negative integers using the number line method.

Instructional Strategies and Activities

1. **Introduction to Negative Integers on a Number Line**
    - Begin by reviewing the number line concept, emphasizing that positive integers move to the right and negative integers move to the left. Introduce how the number line extends in both directions.
    - Demonstrate how to add and subtract negative integers by starting at a given point on the number line and moving in the appropriate direction. For example, **-3 + (-2)** means starting at **-3** and moving left two units.
2. **Guided Practice**

- Model a few problems, like **-4 + (-3)** or **-2 - (-5)**, on the number line. Show how to move to the left for addition of negative integers, and move to the right for subtracting negative integers.

- Emphasize the importance of the direction on the number line when working with negative numbers and how subtracting a negative is like adding a positive.

3. **Collaborative Practice**

    - Have students work in pairs or small groups to solve similar problems on a number line. Provide each group with a number line and a set of problems. Encourage them to explain their reasoning to each other as they work through the problems.

4. **Independent Practice**

    - After practicing with a partner, let students solve problems independently on a worksheet that includes both addition and subtraction of negative integers using a number line. Allow students to draw the number line to show their work.

5. **Class Discussion**

    - Discuss the strategies students used to solve the problems. Ask them to explain how they decided whether to move left or right on the number line. Discuss common misconceptions, such as confusion between subtracting negative numbers versus adding them.

Formative Assessments

1. **Exit Ticket**:

    - Give each student a short formative assessment (exit ticket) with a couple of addition and subtraction problems involving negative integers on a number line. Students will need to draw the number line and show their work.

2. **Partner Check-In**:

    - Have students pair up to check each other's work, discussing any differences or challenges they encountered when moving on the number line.

3. **Quick Check**:

    - Ask students to solve a few problems on the number line during the lesson. Walk around the room to observe and provide immediate feedback on their approach and understanding.

Instructional Materials and Resources

- **Number line handouts** (physical or digital)
- **Markers or pens** for students to draw their number lines
- **Whiteboard or interactive display** to model examples
- **Practice worksheets** with problems involving addition and subtraction of negative integers

Reflections and Suggested Modifications

- **For Struggling Students**: Provide additional one-on-one or small group support. You might use colored pencils to help students distinguish between positive and negative movements on the number line. Offer more practice with simpler numbers.
- **For Advanced Students**: Challenge them by introducing problems that involve a mix of negative and positive integers in one problem, such as **-4 + 2 - (-3)**, to deepen their understanding of operations with negative numbers.
- **Differentiation**: Offer number lines with different levels of complexity. Some students may benefit from a number line with larger, clearly marked intervals, while others may need a more blank or simplified version to focus on the concept of moving in the correct direction.

This lesson allows students to develop a solid understanding of adding and subtracting negative integers with visual and hands-on methods. The use of number lines helps clarify the movement left and right, which is essential to grasping the concept of negative integer operations.

Lesson 3: Use a Number Line to Add or Subtract Rational Numbers

Student Learning Intentions (WALT)

- **We are learning to** add and subtract rational numbers (fractions, decimals, and integers) using a number line.
- **We are learning to** represent the addition and subtraction of rational numbers visually on a number line.

Student Success Criteria (I Can Statements)

- **I can** add and subtract rational numbers (fractions, decimals, and integers) using a number line.
- **I can** correctly identify where to start on the number line for different rational numbers.
- **I can** show my work by visually representing rational number addition or subtraction on a number line.
- **I can** explain my reasoning when using a number line to add or subtract rational numbers.

Instructional Strategies and Activities

1. **Introduction to Rational Numbers on a Number Line**
    - Begin by reviewing the concept of rational numbers (numbers that can be expressed as fractions, decimals, or integers). Explain that rational numbers can be positive or negative and can be represented on a number line.
    - Demonstrate how to position fractions, decimals, and integers on a number line. For example,

show how **1/2**, **0.25**, and **-3** are placed on the number line.

- Explain how the number line can be used to add and subtract rational numbers by visually moving from one point to another based on the operation (adding moves to the right, subtracting moves to the left).

2. **Guided Practice**

   - Model a couple of examples where both fractions and decimals are added and subtracted on a number line. For example:

     - **2 + 1/2**: Start at 2 and move 1/2 unit to the right.

     - **3 - 0.75**: Start at 3 and move 0.75 units to the left.

   - Emphasize the importance of finding the correct starting point on the number line and moving in the correct direction based on whether the operation is addition or subtraction.

3. **Collaborative Practice**

   - Students will work in pairs or small groups. Provide each group with a set of problems involving the addition and subtraction of rational numbers (fractions, decimals, and integers). Have them solve these problems by drawing number lines and showing their work.

   - Encourage students to explain their thinking to each other and make sure they are positioning the fractions, decimals, and integers correctly on the number line before moving to the next step.

4. **Independent Practice**

   - After practicing with partners, students will complete a set of independent problems. The worksheet should include a mix of fractions, decimals, and integers to reinforce the concept of adding and subtracting rational numbers on a number line.

   - Allow students to draw their number lines and solve the problems step-by-step.

5. **Class Discussion**

   - Bring the class together to discuss their strategies. Ask students to share how they positioned fractions and decimals on the number line and how they determined the direction to move (left for subtraction, right for addition).

   - Address any misunderstandings and provide additional examples if necessary.

Formative Assessments

1. **Exit Ticket**:

   - At the end of the lesson, give each student a short exit ticket with a couple of problems involving the addition or subtraction of rational numbers on a number line. Ask students to draw their number lines and show their work.

2. **Partner Check-In**:

- During the collaborative practice, ask students to check each other's work, discuss the number line drawings, and verify the direction of movement for addition and subtraction of rational numbers.

3. **Quick Check**:
    - Walk around the room during independent practice and provide feedback on students' understanding. Observe how well they use the number line to represent their calculations.

Instructional Materials and Resources

- **Number line handouts** (physical or digital) with clear markings for fractions, decimals, and integers
- **Markers, pencils, or pens** for students to draw their number lines
- **Whiteboard or interactive display** for demonstrating examples
- **Practice worksheets** containing problems with fractions, decimals, and integers
- **Calculators** (optional, for decimal operations)

Reflections and Suggested Modifications

- **For Struggling Students**: Provide additional practice with simpler problems using only integers before introducing fractions and decimals. You can also provide a number line with pre-marked intervals to help students focus on the direction of movement rather than on positioning the numbers.
- **For Advanced Students**: Challenge them with problems that require mixed operations or problems where they need to add or subtract multiple rational numbers (e.g., **3/4 + 2 - 1/8**). These can be solved step-by-step on a number line.
- **Differentiation**: Offer number lines with varying levels of complexity. For example, some students may benefit from number lines with smaller increments, while others may need a more detailed number line with both positive and negative rational numbers clearly labeled.

This lesson provides a visual, hands-on way for students to understand the addition and subtraction of rational numbers. Using the number line as a tool helps students conceptualize the movement of rational numbers, reinforcing their understanding of these operations.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
    - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:

    - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

- **Visuals**:
    - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.

- **Front-Loading Vocabulary**:
    - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
    - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.

- **Checking for Understanding**:
    - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.

- **Extra Time**:
    - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.

- **Oral Reading**:
    - The teacher will read work aloud to students, which can include class work, tests, and literature circles.

- **Timers**:
    - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
    - The teacher organizes information into manageable units to ensure students with 504 plans are

not overwhelmed by excessive detail.

- **Checking for Understanding**:
    - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
    - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
    - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
    - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
    - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.
- **Peer Support**:
    - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
    - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
    - Teachers should check in with at-risk students regularly, even for brief periods, to offer support

and guidance as needed.

- **Contracts**:
    - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
    - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.
- **Tests/Assessments**:
    - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:
    - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
    - Fist to five: "How well do you understand what we talked about today?"
    - Fist to five: "How well did I teach this today?"
- **Classroom Activity**:
    - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:

- Color-coding ideas
- Learning vocabulary in student languages
- Visual and kinesthetic learning
- Representations of learning without words

- **Classroom Activity**:
  - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
  - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
- **Classroom Activity**:
  - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
  - "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
  - Learn about different bases and numerical ideas:
    - Base 2 (binary) and connections to computer programming
    - How the Yoruba of Nigeria used base 20
    - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
  - "How might you all go about this?"

- "What do you notice?"

- **Classroom Activity**:
  - Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
  - "How does this connect to what you've learned in the past?"
  - "How can you use that knowledge today?"
- **Classroom Activity**:
  - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
  - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"
- **Classroom Activity**:
  - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
  - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:

- "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
  - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
  - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
  - "Let's get into partners and do a think-pair-share. We will incorporate everyone's ideas and try to synthesize them."
- **Classroom Activity**:
  - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
  - "Come up with at least two answers that might solve this problem."
- **Classroom Activity**:
  - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
  - **Classroom Activity**:
    - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or

procedurally accepted

- **Verbal Example**:
    - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
    - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:
    - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:
    - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
    - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).
- **Climate Change Example**:
    - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MA.K-12.1	Make sense of problems and persevere in solving them.
MA.K-12.2	Reason abstractly and quantitatively.
MA.K-12.3	Construct viable arguments and critique the reasoning of others.
MA.K-12.4	Model with mathematics.
MA.K-12.5	Use appropriate tools strategically.
MA.K-12.6	Attend to precision.
MA.K-12.7	Look for and make use of structure.
MA.K-12.8	Look for and express regularity in repeated reasoning.
MATH.9-12.A.REI	Reasoning with Equations and Inequalities

21st Century Life and Career

CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Integration of Career Readiness. Life Literacies and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP11	Use technology to enhance productivity.

Integration of Computer Science and Design ThinkingNew Section

CS.9-12.8.1.12.AP.1 Design algorithms to solve computational problems using a combination of original and existing algorithms.

CS.9-12.8.1.12.AP.5 Decompose problems into smaller components through systematic analysis, using constructs such as procedures, modules, and/or objects.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

LA.RH.9-10.8	Assess the extent to which the reasoning and evidence in a text support the author's claims.
LA.RST.9-10.5	Analyze the relationships among concepts in a text, including relationships among key terms (e.g., force, friction, reaction force, energy).
LA.RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.
LA.RST.9-10.8	Determine if the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.

Module 4: Add and Subtract Rational Numbers

Unit Rationale

Understanding the addition and subtraction of rational numbers is fundamental to developing proficiency in arithmetic and algebraic reasoning. In this unit, students will explore how to add and subtract both positive and negative rational numbers, including integers, fractions, and decimals. This knowledge is crucial not only for success in 7th-grade mathematics but also for building the foundation for more advanced topics in algebra and real-world applications.

This unit is strategically placed in the curriculum to enhance students' fluency with rational numbers and prepare them for the next step in their mathematical development. Students will learn to perform operations with rational numbers using multiple methods, such as number lines, manipulatives, and algorithms. This will give them a deep understanding of how to handle these numbers efficiently in both abstract and applied settings.

The relevance of adding and subtracting rational numbers extends beyond the classroom. These operations are used in everyday situations such as budgeting, measurements, temperature changes, and financial transactions. By mastering these skills, students will be able to solve real-world problems that involve comparing quantities, calculating gains and losses, or measuring distances, all of which are essential for informed decision-making in both personal and professional contexts.

Throughout the unit, students will focus on critical Standards for Mathematical Practice, including:

- **MP2**: Reason abstractly and quantitatively.
- **MP4**: Model with mathematics.
- **MP6**: Attend to precision.
- **MP7**: Look for and make use of structure.

By the end of this unit, students will be able to confidently add and subtract rational numbers and apply these skills to solve problems in various mathematical contexts. They will also have the necessary tools to approach

more complex mathematical concepts involving rational numbers, such as proportional reasoning, equations, and inequalities, setting the stage for future success in mathematics.

Essential Questions

- **How can the properties of addition and subtraction help us work with both positive and negative rational numbers?**
- **What strategies can we use to add or subtract fractions and decimals with different signs?**
- **In what real-world situations do we need to add or subtract rational numbers, and how can we apply these skills?**
- **How does the number line help us visualize and understand the addition and subtraction of rational numbers?**
- **What patterns or rules exist when adding and subtracting integers, fractions, and decimals?**
- **How can we use estimation and approximation to check the reasonableness of our answers when adding or subtracting rational numbers?**
- **How do the operations of addition and subtraction of rational numbers connect to other areas of mathematics, such as solving equations or working with ratios?**
- **Why is it important to understand the concept of absolute value when adding and subtracting rational numbers?**
- **How does the structure of rational numbers help us determine the correct process for performing operations?**
- **What happens when we add or subtract rational numbers with the same or different denominators, and how does this affect the outcome?**

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
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- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
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3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Compute Sum of Rational Numbers

Student Learning Intentions (WALT)

- **WALT (We Are Learning To):**
    - Compute the sum of rational numbers.
    - Apply the rules for adding integers, fractions, and decimals to find the sum of rational numbers.

Student Success Criteria ("I Can" Statements)

- **I can:**
    - Identify rational numbers and determine the correct method for adding them (integers, fractions, or decimals).
    - Accurately compute the sum of two or more rational numbers using the appropriate steps.
    - Solve real-world problems involving the addition of rational numbers.

Instructional Strategies and Activities

1. Warm-Up (5–10 minutes)

- Begin with a review of rational numbers: fractions, decimals, and integers.
- Quick review of how to identify whether a number is rational (i.e., it can be written as a fraction of two integers).
- Ask students to solve a few simple problems involving adding integers, such as:
  -3 + 5
  7 + (-4)

2. Guided Practice (10–15 minutes)

- **Demonstration:** Show how to compute the sum of rational numbers on a number line.
  - **Example 1 (Adding Integers):**
    -4 + 6
    Use the number line to illustrate moving 4 units to the right from -4, resulting in +2.
  - **Example 2 (Adding Fractions):**
    1/4 + 3/4
    Explain how fractions with a common denominator are added by simply adding the numerators:
    1/4 + 3/4 = 4/4 = 1
  - **Example 3 (Adding Decimals):**
    2.3 + 5.4
    Align the decimals and add as usual, showing the importance of lining up the decimal points:
    2.3 + 5.4 = 7.7

3. Independent Practice (15–20 minutes)

- Provide students with a mix of problems where they add integers, fractions, and decimals. These should involve both positive and negative numbers.
  - Example problems:
    -2 + 5
    3/5 + 1/2
    7.2 + (-4.5)
    -1/3 + 2/3
- Students work independently, with the teacher providing support as needed.

4. Real-World Application (10 minutes)

- Present a real-world scenario involving rational numbers and addition.
  Example:
  "A bank account has a balance of -$20. A deposit of $50 is made. What is the new balance?"

- Students will apply their understanding of rational numbers to compute the sum in this real-world context.

After solving, discuss how rational numbers are used to model real-world situations like finances.

5. Reflection/Exit Ticket (5 minutes)

- Ask students to reflect on the day's lesson by answering:
  "Explain how adding rational numbers is similar to or different from adding whole numbers."
- Use an exit ticket where students solve one addition problem involving rational numbers and submit it as they leave.

Formative Assessments

1. **Exit Ticket:**
   - One problem that asks students to add two rational numbers (e.g., *2/3 + (-1/4)* or *-5.2 + 3.1*).
   - Students explain their strategy in solving the problem.
2. **Observations during Guided Practice:**
   - Teacher walks around to observe student understanding during the guided practice and provides individualized support.
3. **Peer Review:**
   - After completing independent practice, students pair up to check each other's work, explaining their methods to ensure mutual understanding.

Instructional Materials and Resources

- **Whiteboard/Projector** for examples and guided practice.
- **Number line** to visually demonstrate adding integers and fractions.
- **Worksheet** with problems for independent and group practice, including a mix of integer, fraction, and decimal addition.
- **Real-World Application Worksheet** to connect rational number addition to everyday scenarios (e.g., banking, shopping).
- **Exit Ticket Template** for quick assessment.

Reflections and Suggested Modifications

- **Differentiation:**
    - Provide extra support for students struggling with fractions by using visual aids or breaking down the steps further.
    - For advanced students, challenge them with problems that involve mixed fractions or decimals with more places after the decimal point.
- **Classroom Setup:**
    - Consider using math manipulatives (like fraction strips or decimal grids) for hands-on practice for students who need more tactile learning opportunities.
- **Reflection:**
    - After this lesson, check if students are able to solve addition problems with a variety of rational numbers. If many students are struggling with adding fractions, consider reviewing the concept of finding a common denominator.

Lesson 2: Compute Differences of Rational Numbers

Student Learning Intentions (WALT)

- **WALT (We Are Learning To):**
    - Compute the difference of rational numbers.
    - Apply the rules for subtracting integers, fractions, and decimals to find the difference of rational numbers.

Student Success Criteria ("I Can" Statements)

- **I can:**
    - Identify rational numbers and determine the correct method for subtracting them (integers, fractions, or decimals).
    - Accurately compute the difference between two rational numbers using the appropriate steps.
    - Solve real-world problems involving the subtraction of rational numbers.

Instructional Strategies and Activities

1. Warm-Up (5–10 minutes)

- Begin with a review of rational numbers: integers, fractions, and decimals.
- Quick review of how to identify whether a number is rational (i.e., it can be written as a fraction of two integers).
- Ask students to solve a few simple problems involving subtracting integers, such as:
  6 - 3
  -4 - (-7)

2. Guided Practice (10–15 minutes)

- **Demonstration:** Show how to compute the difference of rational numbers using a number line.
  - **Example 1 (Subtracting Integers):**
    7 - (-3)
    Use the number line to illustrate subtracting a negative number: moving 3 units to the right from 7, resulting in 10.
  - **Example 2 (Subtracting Fractions):**
    3/4 - 1/4
    Explain how fractions with a common denominator are subtracted by simply subtracting the numerators:
    3/4 - 1/4 = 2/4 = 1/2
  - **Example 3 (Subtracting Decimals):**
    6.5 - 2.3
    Align the decimals and subtract as usual:
    6.5 - 2.3 = 4.2

3. Independent Practice (15–20 minutes)

- Provide students with a mix of problems where they subtract integers, fractions, and decimals. These should involve both positive and negative numbers.
  - Example problems:
    5 - (-3)
    3/5 - 1/2
    7.1 - 4.6
    -2/3 - 1/3
- Students work independently, with the teacher providing support as needed.

4. Real-World Application (10 minutes)

- Present a real-world scenario involving rational numbers and subtraction.
  Example:
  "A temperature was 8°C in the morning. By the afternoon, the temperature dropped to -3°C. What is the difference in temperature?"

- Students will apply their understanding of rational numbers to compute the difference in this real-world context.

After solving, discuss how rational numbers are used to model real-world situations like temperature changes or bank account balances.

5. Reflection/Exit Ticket (5 minutes)

- Ask students to reflect on the day's lesson by answering:
  "How does subtracting rational numbers compare to adding them?"
- Use an exit ticket where students solve one subtraction problem involving rational numbers and submit it as they leave.

Formative Assessments

1. **Exit Ticket:**
   - One problem that asks students to subtract two rational numbers (e.g., *5/6 - 2/3* or *-6.3 - 3.2*).
   - Students explain their strategy in solving the problem.
2. **Observations during Guided Practice:**
   - Teacher walks around to observe student understanding during the guided practice and provides individualized support.
3. **Peer Review:**
   - After completing independent practice, students pair up to check each other's work, explaining their methods to ensure mutual understanding.

Instructional Materials and Resources

- **Whiteboard/Projector** for examples and guided practice.
- **Number line** to visually demonstrate subtracting integers and fractions.
- **Worksheet** with problems for independent and group practice, including a mix of integer, fraction, and decimal subtraction.
- **Real-World Application Worksheet** to connect rational number subtraction to everyday scenarios (e.g., temperature, finances).
- **Exit Ticket Template** for quick assessment.

Reflections and Suggested Modifications

- **Differentiation:**
    - Provide extra support for students struggling with fractions by using visual aids or breaking down the steps further.
    - For advanced students, challenge them with problems that involve mixed fractions or decimals with more places after the decimal point.
- **Classroom Setup:**
    - Consider using math manipulatives (like fraction strips or decimal grids) for hands-on practice for students who need more tactile learning opportunities.
- **Reflection:**
    - After this lesson, check if students are able to solve subtraction problems with a variety of rational numbers. If many students are struggling with subtracting fractions, consider reviewing the concept of finding a common denominator.

Lesson 3: Understand and Compute Products and Quotients of Rational Numbers

Student Learning Intentions (WALT)

- **WALT (We Are Learning To):**
    - Understand how to multiply and divide rational numbers.
    - Apply the rules for multiplying and dividing integers, fractions, and decimals.

Student Success Criteria ("I Can" Statements)

- **I can:**
    - Identify the rules for multiplying and dividing rational numbers (integers, fractions, and decimals).
    - Accurately compute the products and quotients of rational numbers using the appropriate steps.
    - Solve real-world problems that involve the multiplication and division of rational numbers.

Instructional Strategies and Activities

1. Warm-Up (5–10 minutes)

- Begin by reviewing multiplication and division of integers.
    - **Example Multiplication:**
      -4 × 3 = -12
      -3 × -2 = 6
    - **Example Division:**
      12 ÷ -3 = -4
      -6 ÷ -2 = 3
- Remind students that rational numbers include integers, fractions, and decimals.

2. Guided Practice (10–15 minutes)

- **Multiplying Integers:**
    - Use the rule for multiplying integers (same signs = positive, different signs = negative).
    - **Example:**
      -5 × 2 = -10
      4 × -3 = -12
- **Multiplying Fractions:**
    - Multiply the numerators and denominators.
    - **Example:**
      3/4 × 2/5 = (3 × 2) / (4 × 5) = 6/20 = 3/10
- **Multiplying Decimals:**
    - Multiply as if they were whole numbers, then count the total number of decimal places.
    - **Example:**
      0.6 × 0.2 = 0.12
    - Emphasize that the decimal places need to be counted.
- **Dividing Integers:**
    - Use the rule for dividing integers (same signs = positive, different signs = negative).
    - **Example:**
      -8 ÷ 4 = -2
      12 ÷ -3 = -4
- **Dividing Fractions:**
    - Invert the second fraction and multiply.
    - **Example:**

3/4 ÷ 2/5 = 3/4 × 5/2 = (3 × 5) / (4 × 2) = 15/8

- **Dividing Decimals:**
    - Convert the divisor into a whole number by moving the decimal point, then divide as usual.
    - **Example:**
      4.5 ÷ 0.3 = 15

3. Independent Practice (15–20 minutes)

- Provide students with a variety of problems that require multiplying and dividing rational numbers, including integers, fractions, and decimals.
    - **Example Problems:**
      -3 × 4
      5/6 ÷ 2/3
      0.5 × 0.2
      -2 ÷ 5
      1/2 ÷ -3/4
- Have students work independently and provide support as needed.

4. Real-World Application (10 minutes)

- Present a real-world scenario that involves multiplying and dividing rational numbers.
  Example:
  "A recipe calls for 3/4 of a cup of sugar. If you want to make 2 batches, how much sugar will you need?"
  Solution: Multiply:
  3/4 × 2 = 6/4 = 1 1/2 cups of sugar.
    - Present another scenario involving division.
      Example:
      "You have 6.75 meters of rope and need to cut it into 0.5-meter pieces. How many pieces can you cut?"
      Solution: Divide:
      6.75 ÷ 0.5 = 13.5, so you can cut 13 pieces, with 0.25 meters left over.

5. Reflection/Exit Ticket (5 minutes)

- Ask students to reflect on the day's lesson by answering:
  "What is the difference between multiplying and dividing rational numbers compared to adding and subtracting them?"
- Exit ticket: Students solve one multiplication or division problem involving rational numbers and submit it as they leave.

Formative Assessments

1. **Exit Ticket:**
    - One problem that asks students to compute the product or quotient of two rational numbers (e.g., $-5/6 \times 3/2$ or $0.75 \div 0.25$).
2. **Observations during Guided Practice:**
    - Walk around to monitor student progress and provide assistance, particularly with identifying the correct operations for each type of rational number.
3. **Peer Review:**
    - After independent practice, students pair up to check each other's answers and discuss the strategies used to solve the problems.

Instructional Materials and Resources

- **Whiteboard/Projector** to model and demonstrate problems.
- **Worksheet** with a variety of multiplication and division problems involving integers, fractions, and decimals.
- **Real-World Application Worksheet** to reinforce practical scenarios where rational number operations are used.
- **Exit Ticket Template** for quick assessment at the end of the lesson.

Reflections and Suggested Modifications

- **Differentiation:**
    - Provide visual aids, such as fraction strips or decimal grids, for students who may need more support in understanding how to multiply and divide fractions and decimals.
    - Offer additional practice for advanced students with more complex problems, such as those involving mixed numbers or multi-step solutions.
- **Classroom Setup:**
    - Ensure students have access to calculators for checking their decimal division and multiplication if needed, especially if they are not yet comfortable with manual computation.
    - Use online tools or interactive math games for extra practice during independent or group work.
- **Reflection:**

- After this lesson, evaluate if students have gained a clear understanding of how to multiply and divide rational numbers. If necessary, review any steps that caused confusion (e.g., converting decimals or inverting fractions) during the next lesson or small group work.

Lesson 4: Write Rational Numbers as Decimals

Student Learning Intentions (WALT)

- **WALT (We Are Learning To):**
  - Understand how to convert rational numbers (fractions) into decimal form.
  - Apply methods for converting fractions to decimals, both terminating and repeating, and recognize patterns.

Student Success Criteria ("I Can" Statements)

- **I can:**
  - Convert a fraction into a decimal using long division or a calculator.
  - Identify whether the decimal form of a fraction is terminating or repeating.
  - Recognize and express repeating decimals using a bar notation.

Instructional Strategies and Activities

1. Warm-Up (5–10 minutes)

- Begin with a review of fractions and decimals. Ask students what they already know about converting between these two forms.
  - For example:
    What is 1/2 as a decimal?
    What is 3/4 as a decimal?
- Discuss the difference between terminating and repeating decimals, providing examples.
  - **Terminating Decimal:** 0.5, 0.75, 1.25
  - **Repeating Decimal:** 1/3 = 0.333..., 2/3 = 0.666...

2. Guided Practice (10–15 minutes)

- **Method 1: Long Division (for fractions that do not easily simplify)**

- Teach students to divide the numerator by the denominator to find the decimal equivalent.

- **Example:** Convert 1/4 to a decimal.
  1 ÷ 4 = 0.25 (Terminating decimal)

- **Method 2: Repeating Decimals**

  - Discuss how some fractions, such as 1/3, result in a repeating decimal.

  - **Example:** Convert 1/3 to a decimal.
    1 ÷ 3 = 0.333...

  - Teach students to use a bar notation to represent repeating decimals: *0.333...* becomes *0.3¯\overline{3}3*

- **Method 3: Using a Calculator (if allowed)**

  - For more complex fractions, show students how to use a calculator to convert fractions to decimals.

  - **Example:** Convert 7/8 to a decimal.
    7 ÷ 8 = 0.875 (Terminating decimal)

  - Have students practice a few conversions on their own using a calculator.

3. Independent Practice (15–20 minutes)

- Provide a set of problems that require students to convert fractions into decimals, both terminating and repeating.

  - **Examples:**
    Convert 2/5 to a decimal.
    Convert 5/6 to a decimal.
    Convert 1/7 to a decimal.
    Convert 3/8 to a decimal.

- Students will also identify if the decimal is terminating or repeating and use bar notation for repeating decimals.

4. Real-World Application (10 minutes)

- Provide students with real-world problems that involve converting fractions to decimals.
  Example 1: "A recipe calls for 3/4 cup of sugar. What is this amount as a decimal?"
  Solution:
  3 ÷ 4 = 0.75
  Example 2: "If a movie lasts 2/3 of an hour, how long is it in minutes?"
  Solution:
  2 ÷ 3 = 0.666..., which is approximately 40 minutes
  Answer: 40 minutes (repeating decimal, 0.6¯\overline{6}6)

5. Reflection/Exit Ticket (5 minutes)

- Ask students to reflect on the following:
  "How can knowing how to convert fractions to decimals help you in everyday life?"

- Exit Ticket: Have students solve the following problem:
  Convert 5/12 to a decimal (use long division or a calculator).

Formative Assessments

1. **Exit Ticket:**
    - One problem requiring students to convert a fraction to a decimal (e.g., *Convert 7/9 to a decimal*).
    - This checks understanding of the conversion process and the ability to recognize repeating decimals.
2. **Observations during Guided Practice:**
    - Walk around to monitor student progress as they convert fractions to decimals. Provide individual feedback as necessary, especially if students are struggling with repeating decimals.
3. **Peer Review:**
    - After independent practice, have students pair up to compare answers and discuss the process used to convert fractions into decimals. This will allow them to refine their understanding by explaining the steps to a peer.

Instructional Materials and Resources

- **Whiteboard/Projector** for visual demonstrations of long division.
- **Fraction-to-Decimal Conversion Worksheet** with problems ranging from simple to more complex fractions.
- **Calculator** (if available) to help check answers and practice conversions for more difficult fractions.
- **Real-World Application Worksheet** to help students see how fractions and decimals are used outside of math class.

Reflections and Suggested Modifications

- **Differentiation:**
    - For students who struggle with division, use visual aids like fraction bars or a number line to show the relationship between fractions and their decimal equivalents.
    - Offer additional support for students with difficulty recognizing repeating decimals by working

through more examples together.

- **Extension:**
    - For students who understand the material well, have them explore more complex fractions and work with larger or smaller numbers, such as *11/13* or *9/25*, and explain when a fraction will result in a repeating decimal.
- **Classroom Setup:**
    - Consider using an interactive whiteboard or tablet-based app to show fraction-to-decimal conversions in real time, allowing students to engage with the material visually.
- **Reflection:**
    - After the lesson, evaluate if students understand the concept of terminating vs. repeating decimals. If there is confusion, revisit long division methods or provide additional practice with repeating decimals using bar notation.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
    - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
    - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals**:
    - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary**:
    - The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
    - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
    - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.
- **Extra Time**:
    - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.
- **Oral Reading**:
    - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
    - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
    - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
    - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
    - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
    - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they

have learned.

- **Modify/Change Activities**:
    - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
    - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.
- **Peer Support**:
    - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
    - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
    - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
    - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
    - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension tasks instead of reading themselves.
- **Tests/Assessments**:
    - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:

- Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
  - Fist to five: "How well do you understand what we talked about today?"
  - Fist to five: "How well did I teach this today?"
- **Classroom Activity**:
  - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
  - Color-coding ideas
  - Learning vocabulary in student languages
  - Visual and kinesthetic learning
  - Representations of learning without words
- **Classroom Activity**:
  - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
  - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"

- **Classroom Activity**:
    - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
    - "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
    - Learn about different bases and numerical ideas:
        - Base 2 (binary) and connections to computer programming
        - How the Yoruba of Nigeria used base 20
        - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
    - "How might you all go about this?"
    - "What do you notice?"
- **Classroom Activity**:
    - Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
    - "How does this connect to what you've learned in the past?"
    - "How can you use that knowledge today?"
- **Classroom Activity**:

- Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
  - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"
- **Classroom Activity**:
  - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
  - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
  - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
  - "Reflect on your day so far. What math have you already used today?"
- **Classroom Activity**:
  - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
    - "Let's get into partners and do a think-pair-share. We will incorporate everyone's ideas and try to synthesize them."
- **Classroom Activity**:
    - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
    - "Come up with at least two answers that might solve this problem."
- **Classroom Activity**:
    - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
    - **Classroom Activity**:
        - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
    - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
    - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:

- After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:
  - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
  - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).
- **Climate Change Example**:
  - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MATH.7.NS.A	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers

Integration of Career Readiness. Life Literacies and Key Skills

CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP6	Demonstrate creativity and innovation.

21st Century Life and Career

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP3	Attend to personal health and financial well-being.

Integration of Computer Science and Design Thinking

CS.CS	Computing Systems
CS.K-2.DA	Data & Analysis

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

SOC.K-12.7	Taking Informed Action
SCI.9-12.5.1.12.A.1	Refine interrelationships among concepts and patterns of evidence found in different central scientific explanations.
SCI.9-12.5.1.12.C	Scientific knowledge builds on itself over time.

Module 5: Applying Properties to Operations

Unit Rationale

Understanding and applying properties of operations is a cornerstone of mathematical reasoning, enabling students to perform calculations efficiently and solve complex problems. This unit provides students with the foundational knowledge necessary to apply properties such as the distributive, associative, and commutative properties of operations to simplify expressions, solve equations, and manipulate mathematical statements. These properties are essential tools for working with integers, rational numbers, and algebraic expressions, forming the basis for more advanced concepts in algebra and beyond.

This unit is designed to deepen students' understanding of how operations behave and how they can be used strategically in mathematical problem-solving. Students will explore the impact of these properties on both numerical and algebraic expressions, recognizing when and how to apply them in different contexts. By the end of the unit, students will have the ability to simplify expressions, solve equations, and perform operations more flexibly and efficiently.

The placement of this unit is intentional, occurring after students have developed a basic understanding of arithmetic operations. It serves as a bridge to algebraic thinking, preparing students for more advanced topics such as solving multi-step equations and working with polynomials. The real-world applications of this unit will help students see the relevance of these properties in everyday life, such as when calculating discounts, splitting costs, or simplifying financial problems.

Real-World Relevance

Throughout this unit, students will see how the properties of operations apply to practical scenarios, such as calculating areas, distributing quantities, and making comparisons in business and finance. By connecting the abstract properties to real-world situations, students will gain a deeper understanding of why these concepts are important and how they make problem-solving easier. This context-based approach ensures that students not only grasp the theoretical significance of these properties but also understand their utility in everyday problem-solving.

Key Mathematical Practices

Throughout this unit, students will engage with the following **Standards for Mathematical Practice**:

- **MP2: Reason abstractly and quantitatively.**
- **MP7: Look for and make use of structure.**
- **MP8: Look for and express regularity in repeated reasoning.**

By the end of this unit, students will be able to use the properties of operations to simplify complex expressions, solve equations, and approach problems in a more structured and strategic way. The skills they develop in this unit will not only strengthen their foundational math skills but also prepare them for algebraic and higher-level mathematics, laying a solid foundation for future academic success.

Focus of the Unit

- Recognizing and applying the distributive, associative, and commutative properties.
- Simplifying expressions using these properties to make calculations easier.
- Applying these properties to solve equations and real-world problems.
- Developing strategies for handling increasingly complex mathematical scenarios by leveraging properties of operations.

By the end of the unit, students will have developed a deeper, more flexible understanding of how to apply properties of operations in various mathematical and real-world contexts, building their problem-solving and algebraic skills for future academic challenges.

Essential Questions

- How do the properties of operations (commutative, associative, and distributive) help simplify and solve mathematical expressions and real-world problems?
- In what ways can the distributive property make complex calculations easier to solve?
- How can understanding the commutative and associative properties help in reorganizing or grouping terms to simplify expressions?
- How do the properties of operations apply to real-world contexts like budgeting, shopping, or splitting quantities?

Pre-Assessments

Benchmark assessments are given within the first semester using HMH Into Math.

1. Readiness Check (Diagnostic Assessment)
- Found at the beginning of each module/unit.
- Assesses prerequisite skills necessary for success in the upcoming lessons.
- Usually includes a mix of multiple-choice and short answer items.
- Great for determining small-group needs or identifying which students might benefit from additional support.

2. Diagnostic Assessments in Ed: Your Friend in Learning
- Online assessments tied to Into Math.
- Adaptive in nature (depending on your district's setup) and aligned with the lesson standards.
- Can provide recommendations for intervention or enrichment based on results.

3. Module Quizzes (Pre-Use)
- While designed for post-instruction, some teachers use the Module Quiz or Mid-Module Checkpoint as a pre-assessment to gauge student background knowledge.
- Use selectively, focusing on concepts that build directly on prior grades' standards.

4. Lesson-Specific Checks
- Some lessons include "Are You Ready?" sections or warm-ups that can double as informal pre-assessments.
- Often appear in the Teacher Edition or digital platform and can be used as bell-ringers or exit tickets.

Instructional Plan

Lesson 1: Apply Properties to Multi-Step Problems with Rational Numbers

Student Learning Intentions (WALT):

- We are learning to apply the properties of operations (commutative, associative, distributive) to solve multi-step problems with rational numbers.
- We are learning to break down complex problems involving rational numbers into manageable steps using properties of operations.

Student Success Criteria (I can statements):

- I can use the properties of operations (commutative, associative, and distributive) to simplify

expressions with rational numbers.

- I can solve multi-step problems involving rational numbers by applying the appropriate property to each step.
- I can justify my reasoning when solving problems with rational numbers.

Instructional Strategies and Activities:

1. **Direct Instruction & Review**:
    - Begin by reviewing the key properties of operations (commutative, associative, distributive) using both integer and rational number examples.
    - Provide explicit examples of how to break down multi-step problems into smaller, simpler parts using these properties.
2. **Guided Practice**:
    - Walk students through a set of problems step-by-step, emphasizing how to identify which property to apply.
    - Use examples that require combining rational numbers through addition, subtraction, multiplication, and division, and applying the properties appropriately in each situation.
3. **Independent Practice**:
    - Provide students with a set of multi-step problems that require the use of properties of operations to solve. Encourage students to work independently and apply what they have learned.
    - Use scaffolding techniques such as graphic organizers (e.g., step-by-step breakdown of each operation).
4. **Interactive Practice**:
    - Use a number line or visual aids to demonstrate how properties can help simplify calculations (e.g., moving between positive and negative rational numbers, showing how to apply distributive property with fractions).
    - Utilize math apps or interactive tools for students to practice applying the properties in different scenarios.
5. **Collaborative Learning**:
    - In pairs or small groups, students can solve a challenging multi-step problem, explaining their reasoning and discussing the properties used. This will help reinforce understanding and encourage collaborative problem-solving.

Formative Assessments:

1. **Exit Ticket**:
    - At the end of the lesson, ask students to solve a brief multi-step problem and explain which properties they used to solve it.
    - This will help assess individual understanding and areas that need reinforcement.
2. **Quick Poll/Thumbs Up/Thumbs Down**:
    - During guided practice, ask students to signal understanding with a thumbs up/down after each step of the problem-solving process to check for comprehension.
3. **Peer Review**:
    - Have students swap their work with a peer to check for correct application of the properties. They can provide feedback on whether the correct property was applied in the right order.

Instructional Materials and Resources:

- Whiteboard and markers
- Student notebooks and graph paper for visual aids and calculations
- Interactive math tools (e.g., online apps or digital math notebooks)
- Handouts with example problems, including a step-by-step guide for using properties of operations
- Number line visual for understanding addition and subtraction with rational numbers

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Reflections and Suggested Modifications:

- **Reflection**: After the lesson, reflect on how well students were able to apply the properties to solve multi-step problems. Were they able to break down complex problems into smaller steps? Did they show an understanding of the different properties and their application?
- **Modifications**:
    - For students who need more support, provide additional practice problems with simpler steps before moving on to more complex ones.
    - For advanced students, incorporate problems with more than two rational numbers, requiring them to apply multiple properties in a single expression.
    - Consider using a "think-pair-share" activity to encourage discussion and peer-to-peer learning.

Lesson 2: Solve Multi-Step Problems with Rational Numbers in Context

Student Learning Intentions (WALT):

- We are learning to solve multi-step problems involving rational numbers in real-world contexts.
- We are learning to apply the properties of operations and understand how to manage rational numbers within word problems.

Student Success Criteria (I can statements):

- I can identify the steps needed to solve a multi-step problem involving rational numbers.
- I can apply properties of operations (commutative, associative, distributive) to simplify and solve problems.
- I can explain my reasoning and show how I arrived at my solution.
- I can model real-world scenarios and translate them into mathematical problems involving rational numbers.

Instructional Strategies and Activities:

1. Contextualizing the Problem:
    - Begin with an introduction to real-world problems that involve multi-step solutions (e.g., budgeting, shopping, temperature changes, or distance traveled).
    - Highlight the importance of breaking down problems into smaller, manageable steps. Provide a scenario where rational numbers are involved, such as calculating the total cost of an item after discounts and taxes, or figuring out temperature changes in a day.
2. Modeling the Process:
    - Walk students through an example of a multi-step problem. For instance:
        - A shopping scenario where a customer buys several items with prices given as rational numbers, applies a discount, then calculates tax, and finally determines the total amount spent.
        - A temperature problem where temperatures are increased or decreased by rational numbers, and students have to find the final temperature after several changes.
    - Emphasize how each step involves operations with rational numbers, and demonstrate how to apply the correct properties to simplify and solve.
3. Guided Practice:

- Provide students with a set of contextual problems similar to the one modeled. Allow them to work in pairs or small groups to discuss the steps needed to solve the problem.
- Focus on helping students identify the order of operations, and reinforce the use of properties of operations to solve the problems step-by-step.

4. Independent Practice:
    - Provide additional problems with varying contexts (e.g., a financial scenario, distance problems, or temperature-related questions) for students to solve independently.
    - Encourage students to use the strategies demonstrated and show their work with clear reasoning for each step.
5. Class Discussion and Reflection:
    - After completing the problems, have a class discussion where students can share their strategies for solving the problems. Focus on the importance of breaking down the problems and applying the properties correctly.
    - Discuss how real-world problems often involve rational numbers and the importance of understanding how to solve multi-step problems in context.

Formative Assessments:

1. Exit Ticket:
    - At the end of the lesson, ask students to solve a brief contextual multi-step problem involving rational numbers. They should explain how they used the properties of operations and their reasoning for each step.
    - This will help assess individual understanding of how to apply operations in real-world contexts.
2. Think-Pair-Share:
    - After the guided practice, ask students to discuss their answers with a partner. Each pair will explain their reasoning and steps to their partner, reinforcing their understanding through peer feedback.
3. Peer Review:
    - Have students exchange their work with a peer to check for understanding and accuracy. Students should check for proper use of properties and whether the steps taken were correct in solving the problem.

Instructional Materials and Resources:

- Whiteboard and markers
- Student notebooks and graph paper for visual aids and calculations
- Handouts with example problems and space for step-by-step solutions
- Word problem cards with real-world scenarios involving rational numbers
- Online math tools (optional for additional practice)

Reflections and Suggested Modifications:

- Reflection: After the lesson, reflect on how well students were able to apply their understanding of rational numbers in real-world contexts. Were they able to break down multi-step problems effectively? Did they understand the reasoning behind the operations and the properties used?
- Modifications:
    - For struggling students: Provide additional scaffolding by giving them smaller, less complex problems or by breaking the problems into even smaller steps. Pair them with a partner for collaborative problem-solving.
    - For advanced students: Challenge them with more complex problems that involve multiple operations with rational numbers, such as problems requiring the use of fractions, decimals, and mixed numbers. Encourage them to solve problems involving percentages or rates as well.
    - Interactive Tools: Consider using digital resources or apps that allow students to model the problems visually (e.g., using graphs or interactive number lines) to further support the understanding of operations with rational numbers.

Lesson 3: Add, Subtract, Factor, and Expand Algebraic Expressions

Student Learning Intentions (WALT):

- We are learning to add, subtract, factor, and expand algebraic expressions.
- We are learning how to apply these operations to simplify and manipulate algebraic expressions.

Student Success Criteria (I can statements):

- I can identify like terms and combine them when adding or subtracting algebraic expressions.
- I can factor algebraic expressions by identifying common factors.
- I can expand algebraic expressions by using the distributive property.
- I can simplify algebraic expressions by combining like terms and applying the distributive property

correctly.

Instructional Strategies and Activities:

1. Introduction to Key Concepts:
    - Start by explaining the core concepts: adding and subtracting algebraic expressions (combining like terms), factoring expressions (finding the greatest common factor), and expanding expressions (using the distributive property).
    - Use a visual approach to show examples of algebraic expressions and how to recognize like terms (e.g., 2x+3x2x + 3x2x+3x can be simplified to 5x5x5x).
    - Define "factoring" as finding common factors that can be factored out of terms in the expression (e.g., factoring 4x+84x + 84x+8 as 4(x+2)4(x + 2)4(x+2)).
2. Modeling the Operations:
    - Adding and Subtracting Expressions: Start with simple examples, like 2x+3x2x + 3x2x+3x or 5x−2x5x - 2x5x−2x, and explain how to combine like terms. Emphasize the importance of only combining terms that have the same variable raised to the same power.
    - Factoring Expressions: Model how to factor expressions by identifying the greatest common factor (GCF) of terms. For example, factoring 6x+96x + 96x+9 as 3(2x+3)3(2x + 3)3(2x+3).
    - Expanding Expressions: Introduce the distributive property with an example like 3(x+4)3(x + 4)3(x+4). Show how to expand it by distributing the 333 to both terms inside the parentheses: 3x+123x + 123x+12.
3. Guided Practice:
    - Provide students with a set of practice problems to work on as a class. Work through them together, step-by-step, allowing students to participate and share their reasoning.
        - Example problems:
            - Add: 4x+5x4x + 5x4x+5x
            - Subtract: 7y−3y7y - 3y7y−3y
            - Factor: 6x+96x + 96x+9
            - Expand: 2(x+3)2(x + 3)2(x+3)
4. Independent Practice:
    - Give students a variety of problems where they need to apply their skills in adding, subtracting, factoring, and expanding algebraic expressions.
    - Example problems for independent practice:

        - Simplify: 5a+7a5a + 7a5a+7a
        - Factor: 12x+1812x + 1812x+18
        - Expand: 4(2x+5)4(2x + 5)4(2x+5)
5. Class Discussion and Reflection:
    - After completing practice problems, engage students in a discussion about the steps involved in adding, subtracting, factoring, and expanding expressions. Ask them to share strategies they used and any challenges they faced.
    - Use student responses to highlight common misconceptions and reinforce important steps.

Formative Assessments:

1. Exit Ticket:
    - Have students complete a short exit ticket that includes one problem each for adding, subtracting, factoring, and expanding expressions. They should explain their steps for each problem.
    - Example:
        - Add: 3x+4x3x + 4x3x+4x
        - Subtract: 8y−3y8y - 3y8y−3y
        - Factor: 10a+1510a + 1510a+15
        - Expand: 2(x+6)2(x + 6)2(x+6)
2. Think-Pair-Share:
    - After practicing factoring and expanding expressions, have students pair up and explain how they would solve a particular problem. Each student should share their strategy with their partner before discussing the solution as a class.
3. Peer Review:
    - Pair students together and have them review each other's work on a set of algebraic expression problems. They should check each other's work, focusing on whether like terms were combined correctly, whether the expression was factored properly, and if the distributive property was used correctly during expansion.

Instructional Materials and Resources:

- Whiteboard and markers for modeling and working through problems.

- Student notebooks for writing down notes, examples, and solving problems.
- Algebraic expression worksheets for independent practice.
- Visual aids (e.g., charts showing properties of operations, examples of like terms).
- Online math tools (optional for additional practice and digital engagement).

Reflections and Suggested Modifications:

- Reflection: After the lesson, reflect on how well students were able to grasp the concepts of adding, subtracting, factoring, and expanding expressions. Did they understand the relationship between terms and how to apply the distributive property? Did they struggle with certain parts of the lesson?
- Modifications:
    - For struggling students: Provide additional practice with simpler problems, focusing on one operation at a time. Use visual aids such as color-coding like terms or using diagrams to illustrate the distributive property.
    - For advanced students: Provide more complex expressions that involve multiple steps, or challenge them with problems that require factoring quadratics or expanding binomials.
    - Interactive Tools: Consider using algebraic expression apps or websites that allow students to input and manipulate expressions for more dynamic practice.

Modifications and/or Accommodations

English Language Learners (ELL)

- **Native Language Support**:
    - The teacher provides auditory or written content to students in their native language.
- **Adjusted Speech**:
    - The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.
- **Visuals**:
    - The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subject matter.
- **Front-Loading Vocabulary**:

- The teacher front-loads vocabulary by providing students with a list of important vocabulary words they will need to know for a lesson before it is taught. Including pictures with vocabulary words is also beneficial for students.

Special Education Students

- **Chunking**:
  - The teacher presents information in a way that is easy for students to understand and remember. Chunking organizes information into meaningful units to prevent working memory overload, which can be helpful for students with special needs.
- **Checking for Understanding**:
  - It is important to consistently check for understanding, especially for students who have accommodations, to ensure they comprehend the concepts in a way that makes sense to them.
- **Extra Time**:
  - The teacher provides students with special needs extra time to complete work or answer questions, giving them adequate time to process their thoughts.
- **Oral Reading**:
  - The teacher will read work aloud to students, which can include class work, tests, and literature circles.
- **Timers**:
  - The teacher uses timers to help students manage time when completing tasks, especially for students who struggle to finish tasks within time limits.

Students with 504 Plans

- **Chunking**:
  - The teacher organizes information into manageable units to ensure students with 504 plans are not overwhelmed by excessive detail.
- **Checking for Understanding**:
  - Teachers will continuously check for understanding, ensuring students with accommodations comprehend the lesson content.
- **Extra Time**:
  - Students with 504 plans are given extra time to complete assignments, ensuring they have ample time to process information.

Gifted & Talented Strategies

- **Extensions/Enrichments**:
    - Teachers provide gifted and talented students with enrichment projects that challenge them to deepen their understanding, apply knowledge, or produce something in relation to what they have learned.
- **Modify/Change Activities**:
    - Teachers monitor and adjust activities for students who need more of a challenge. This may involve additional reading, problem-solving, writing, or project work, allowing gifted students to progress at an accelerated rate compared to their peers.

Students at Risk of School Failure

- **Directions or Instructions**:
    - Directions/instructions are provided in limited numbers, both verbally and in simple written format. Teachers may ask students to repeat the instructions to ensure understanding and check back to ensure they haven't forgotten.
- **Peer Support**:
    - Peers can build confidence by helping others. Teachers can set up a system where specific students are assigned to assist at-risk students with clarification before approaching the teacher.
- **Alternate or Modified Assignments**:
    - Teachers should consider modifying assignments for students at risk by simplifying tasks, reducing length, or offering alternative delivery modes (e.g., oral reports instead of written assignments).
- **Increase One-on-One Time**:
    - Teachers should check in with at-risk students regularly, even for brief periods, to offer support and guidance as needed.
- **Contracts**:
    - A working contract helps prioritize tasks and ensures completion. Students and teachers can track progress together by marking off completed tasks with checkmarks or symbols, encouraging accountability.
- **Hands-On Tasks**:
    - Provide concrete, hands-on activities to support at-risk students. This may include using tools like calculators or counters in math or having students use audio recordings for comprehension

tasks instead of reading themselves.

- **Tests/Assessments**:
    - Tests can be administered orally, or broken into smaller sections. Teachers may administer parts of a test in the morning, after lunch, and on subsequent days if necessary.
- **Seating**:
    - Seat students near a helping peer or with quick access to the teacher. For students with hearing or vision issues, seat them at the front for better access to instruction.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

Provide students with opportunities to give feedback to teachers about the classroom and instruction

- **Verbal Example**:
    - Fist to five: "How well do you understand what we talked about today?"
    - Fist to five: "How well did I teach this today?"
- **Classroom Activity**:
    - Exit tickets or surveys asking students to identify how well teachers taught, what helped them learn, what got in the way of their learning, etc.

Treat mathematics as a language that everyone is learning while authentically centering students' home languages

- **Classroom Strategies**:
    - Color-coding ideas
    - Learning vocabulary in student languages
    - Visual and kinesthetic learning
    - Representations of learning without words
- **Classroom Activity**:
    - Multilingual Frayer Models for definitions or concepts

Incorporate true culturally relevant pedagogy, practice, and curriculum

- **Verbal Example**:
    - "What are some of your family traditions that you are proud of? Would you be okay if we brought some of those into the classroom?"
- **Classroom Activity**:
    - Use Ankara fabric to teach mathematical concepts such as tessellations, fractions, area, percentages, etc.

Incorporate the history of mathematics into lessons

- **Verbal Example**:
    - "Why do you think we call it Pythagorean's theorem, when it was used before he was even born? What should we call it instead?"
- **Classroom Activity**:
    - Learn about different bases and numerical ideas:
        - Base 2 (binary) and connections to computer programming
        - How the Yoruba of Nigeria used base 20
        - How the Mayans conceptualized the number 0 before the first recording of it

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Solicit student ways of thinking and processing

- **Verbal Example**:
    - "How might you all go about this?"
    - "What do you notice?"
- **Classroom Activity**:
    - Incorporate explorations where students interact with mathematics in a way that allows them to "discover" or experience mathematics.

Reorganize your classroom teaching around concepts, and teach them more like a web rather than discrete sets of knowledge

- **Verbal Example**:
    - "How does this connect to what you've learned in the past?"
    - "How can you use that knowledge today?"
- **Classroom Activity**:
    - Learning webs that connect content

Start with more complex math problems and scaffold as necessary

- **Verbal Example**:
    - "If we wanted to build a rocket, what are all the things we might need to know before we get started? Along the way, we decided that we want the rocket to reach the moon. What do we need to consider now?"
- **Classroom Activity**:
    - When solving equations, start with the most complex problem, generate ideas for how to solve it, and use the simpler equations as examples to support those ideas.

Offer a variety of ways to demonstrate thinking and knowledge

- **Verbal Example**:
    - "Show your thinking with words, pictures, symbols."

Ask other questions that will demonstrate learning when it is not clear to you how students know the answer

- **Verbal Example**:
    - "If you were working with a fellow mathematician who was absent this day, what might you tell them to help them learn it?"

Learn about, engage with, and incorporate ethnomathematics

- **Verbal Example**:
    - "Reflect on your day so far. What math have you already used today?"

- **Classroom Activity**:
    - Community walks to engage with slope.

Co-construct knowledge in the classroom

- **Verbal Example**:
    - "Let’s get into partners and do a think-pair-share. We will incorporate everyone’s ideas and try to synthesize them."
- **Classroom Activity**:
    - Have students create mathematical definitions in their own words in groups, and bring the groups together to co-construct mathematical definitions as a class.

Choose problems that have complex, competing, or multiple answers

- **Verbal Example**:
    - "Come up with at least two answers that might solve this problem."
- **Classroom Activity**:
    - Challenge standardized test questions by getting the "right" answer, but justify other answers by unpacking the assumptions that are made in the problem.
    - **Classroom Activity**:
        - Deconstructed Multiple Choice: Given a set of multiple-choice answers, students discuss why these answers may have been included. This can also be used to highlight common mistakes.

Identify what is right about the thinking, and highlight the mistake in what is factually or procedurally accepted

- **Verbal Example**:
    - "You recognized that you had to combine the constants 27 and 9, could you explain your thinking?"
- **Classroom Activity**:
    - Error Analysis worksheets that highlight what is the right idea behind the mistake.

Use thoughtful questioning to solicit mathematical thoughts rather than telling

- **Verbal Example**:
    - "What would a mathematician who is confused ask about this question?"
- **Classroom Activity**:
    - After students demonstrate knowledge of a topic, have them play a game where they have to explain their topic to a fellow mathematician and a skeptic. Develop their own reflective questioning/explaining in all three roles.

Create multiple ways of participating that honor myriad ways of thinking and being

- **Verbal Example**:
    - "For this section, feel free to work alone, in pairs, trios, or quads (let them choose)."
- **Classroom Activity**:
    - Community circles or storytelling circles, incorporating dance, music, song, call and response, and other cultural ways of communicating.

Math Climate Change Companion Guide

- **G.MG.A.2 Apply concepts of density based on area and volume in modeling situations** (e.g., persons per square mile, BTUs per cubic foot).
- **Climate Change Example**:
    - Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.

New Jersey Student Learning Standards: Content Area

MATH.7.NS.A	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
MATH.7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number

line diagram.

21st Century Life and Career

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.

Integration of Career Readiness, Life Literacies and Key Skills

PFL.9.1.4.E	Becoming a Critical Consumer
PFL.9.1.4.F	Civic Financial Responsibility

Integration of Computer Science and Design Thinking

CS.CS	Computing Systems
CS.K-2.AP	Algorithms & Programming

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math

SOC.K-12.6	Engaging in Civil Discourse and Critiquing Conclusions
SCI.9-12.5.1.12.A	Students understand core concepts and principles of science and use measurement and observation tools to assist in categorizing, representing, and interpreting the natural and designed world.
SCI.9-12.5.1.12.C	Scientific knowledge builds on itself over time.