Unit 02: Probability (8 weeks)

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **FY**
Status: **Published**

Standards Alignment

New Jersey Student Learning Standards

MA.S-CP	Conditional Probability and the Rules of Probability
MA.S-CP.A	Understand independence and conditional probability and use them to interpret data
MA.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
MA.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
MA.S-CP.A.3	Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
MA.S-CP.A.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
MA.S-CP.A.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.
MA.S-CP.B	Use the rules of probability to compute probabilities of compound events in a uniform probability model
MA.S-CP.B.6	Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.
MA.S-CP.B.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.
MA.S-CP.B.8	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = [P(A)] \times [P(B \mid A)] = [P(B)] \times [P(A \mid B)]$, and interpret the answer in terms of the model.
MA.S-CP.B.9	Use permutations and combinations to compute probabilities of compound events and solve problems.
MA.S-MD	Using Probability to Make Decisions
MA.S-MD.A	Calculate expected values and use them to solve problems
MA.S-MD.A.1	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
MA.S-MD.A.2	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

MA.S-MD.A.3	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value.
MA.S-MD.A.4	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value.
MA.S-MD.B	Use probability to evaluate outcomes of decisions
MA.S-MD.B.5	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
MA.S-MD.B.5a	Find the expected payoff for a game of chance.
MA.S-MD.B.5b	Evaluate and compare strategies on the basis of expected values.
MA.S-MD.B.6	Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
MA.S-MD.B.7	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Integration of Career Readiness, Life Literacies and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Technology / Integration of Computer Science and Design Thinking

TECH.8.2.12	Technology Education, Engineering, Design, and Computational Thinking - Programming: All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.
TECH.8.2.12.C	Design: The design process is a systematic approach to solving problems.
TECH.8.2.12.C.2	Analyze a product and how it has changed or might change over time to meet human needs and wants.
TECH.8.2.12.C.3	Analyze a product or system for factors such as safety, reliability, economic considerations, quality control, environmental concerns, manufacturability, maintenance and repair, and human factors engineering (ergonomics).

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math Section

Capacities of the Literate Individual
Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language

They build strong content knowledge.

LA.K-12.NJSLSA.W	Writing
LA.K-12.NJSLSA.W6	Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.
	Research to Build and Present Knowledge
LA.K-12.NJSLSA.W7	Conduct short as well as more sustained research projects, utilizing an inquiry-based research process, based on focused questions, demonstrating understanding of the subject under investigation.
LA.K-12.NJSLSA.SL1	Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
LA.W.11-12.6	Use technology, including the Internet, to produce, share, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
LA.W.11-12.7	Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
LA.SL.11-12.1.D	Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media LiteracyNew Section

see Crosswalks

21st Century Life and Careers

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.

Stage I: Desired Results

Transfer/Overview/Rationale

Transfer / Overview / Rationale

Unit Rationale

The purpose of this unit…

The purpose of this unit is to explore probability and see how it can be applied to our everyday lives. Probability makes up the basis of statistics. When the FDA tests a new drug, they start by assuming it does not work before they run their study. After gathering data, they look at what the probability is that they would get the results that they got to determine the effectiveness of said drug. The idea that an event with probability less than 1/20 is considered unusual is the building block of every lesson from here until the final test. This unit in particular is most important as the students can take away more life lessons than any other. The idea that gambling is addictive and destructive is no secret yet many still fall into the trap. A basic understanding of probability and expected values, which are explored in this chapter, allow us to make better decisions as to whether a casino game is fair and that maybe it's a better idea to just walk away.

Meaning

Essential Questions

Essential Questions

- **Why is the law of large numbers important to probability and where else can we apply it?**
- **How do random variables change when we combine/transform them and where can the be applied in the real world?**
- **Where do random variables and probability distributions manifest themselves in everyday life?**

Enduring Understanding/Indicators of Understanding

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- **Understanding when 2 events are independent is crucial to making accurate predictions. Unfortunately, many people have lost lots of money believing that, for instance, a roulette ball landing on red 5 times in a row increases the chance of it landing on black the next.**
- **Casinos and most commercial wagers rely upon the concept of expected value to ensure that they come**

out on top in the long run and that you, the consumer, do not.

- **The likelihood of coming across outstanding data values changes if you are looking for them in groups instead of individually.**

Acquisition (Student Learning Objectives)

Knowledge

Knowledge

Students will know…

- **dependence vs independence**
- **mutually exclusive events**
- **bayes theorem**
- **conditional probability**
- **multiplication rule**
- **addition rule**
- **binomial probability**
- **expected value**
- **prob distributions**

Skills

Skills

Student will be skilled at …

- **apply the law of large numbers to situations other than just statistics**
- **judge procedures and casino games based on their expected values and determine if they are fair or not**
- **calculate probabilities for everyday procedures (dice rolles, cards, coins)**
- **classify events as dependent, independent, disjoint, or not disjoint**
- **Calculate probabilities for binomial situations.**

Stage 3: Learning Plan

Resource and Mentor Texts

Resources and Mentor Texts

- **Elemetary Statistics (11th Edition) - School-Issued Textbook**
- **TI-83 Calculator**
- **Online Statistical Calculators (http://www.stattrek.com)**
- **Basketball (for free throw shooting activity)**
- **Play money (for fun fair)**
- **Dice/Cards/Coins/Spinners**

Formative Assessment Strategies

Formative Assessment Strategies

- Do Now--problem of the day related to previous learned skills or bellringers problems

- Review/Check Homework - (group check, partner check, whiteboard check)- Lecture

- Board/White Board Work - (solve problems/practice skills at board, or at seat with individual white boards)

- Kahoot to reinforce skills

- Thumbs up/down/sideways - quick formative assessment to gauge students level of understanding

- Jeopardy style review games

-Relay races--each student does one part of a problem, hands it to the next student to check then completes the next part, etc.

- Scavenger hunts--self-checking, out of seats activity

Learning Activities/Unit of Study

Learning Activities/Unit of Study

-Watch Lucky Movie

-Design fun fair games

-Analyze current existing casino games (roulette, poker)

-Design airplane ticket selling system

-Students discover probabilities through trials

- Lecture

- Work together to understand and practice the skill - partner work/larger group work to read lesson, and practice skills through "On Your Own" problems incorporated throughout each lesson

- Stations - (Small group instruction, skills practice - scavenger hunts, online games, board work)

- Review and practice skills using a variety of materials - (text, workbook, chromebook, games, activities, discussion)

- GIWAR - graphic organizer for analyzing/interpreting/organizing word problems

Student led instruction

- Foldables--creates an organized study guide per chapter

- Socrative--non-multiple choice technology option where students can either “race” or work at the teacher-pace

- Story time--a power point run story where the students are characters and must use a mathematical skill to solve a problem

- Partner/Group investigation where students must create a formula, method, or strategy to solve a problem.

- Students “as teachers” where they present a method or formula they discovered through investigation

- Pear Deck--an interactive online powerpoint where students enter answers, watch videos, and record notes from the information shown on their own device as well as projected.

- Videos by Shmoop to introduce or reinforce concepts in an engaging and comical way

- Math created songs to help reinforce concepts

- Online games on chromebooks (see resources)

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, GIfted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps

prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.