

Unit 05: Theory and Techniques of Integration

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **FY**
Status: **Published**

Standards Alignment

New Jersey Student Learning Standards

MA.F-IF	Interpreting Functions
MA.A-SSE	Seeing Structure in Expressions
MA.A-SSE.A	Interpret the structure of expressions
MA.F-IF.A	Understand the concept of a function and use function notation
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.A-SSE.B.3c	Use the properties of exponents to transform expressions for exponential functions.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or

	as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C	Analyze functions using different representations
MA.A-SSE.B.4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.A-APR	Arithmetic with Polynomials and Rational Expressions
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.A-APR.B	Understand the relationship between zeros and factors of polynomials
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.A-APR.C	Use polynomial identities to solve problems
MA.A-APR.C.4	Prove polynomial identities and use them to describe numerical relationships.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.F-BF	Building Functions
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.A-APR.D	Rewrite rational expressions
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.

MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.A-CED	Creating Equations
MA.A-CED.A	Create equations that describe numbers or relationships
MA.F-BF.A.1c	Compose functions.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
MA.F-BF.B	Build new functions from existing functions
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-BF.B.4	Find inverse functions.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.F-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse.
MA.A-REI	Reasoning with Equations and Inequalities
MA.F-BF.B.4b	Verify by composition that one function is the inverse of another.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.F-BF.B.4c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MA.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MA.F-BF.B.4d	Produce an invertible function from a non-invertible function by restricting the domain.
MA.A-REI.B	Solve equations and inequalities in one variable
MA.F-BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
MA.F-LE	Linear and Exponential Models
MA.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MA.A-REI.B.4	Solve quadratic equations in one variable.
MA.F-LE.A	Construct and compare linear and exponential models and solve problems
MA.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MA.A-REI.B.4a	Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

MA.F-LE.A.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .
MA.F-LE.A.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MA.A-REI.C	Solve systems of equations
MA.A-REI.C.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
MA.F-LE.A.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MA.A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
MA.A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
MA.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MA.F-LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to ab to the ct power = d where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
MA.A-REI.C.8	Represent a system of linear equations as a single matrix equation in a vector variable.
MA.A-REI.C.9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
MA.A-REI.D	Represent and solve equations and inequalities graphically
MA.F-TF	Trigonometric Functions
MA.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
MA.F-TF.A	Extend the domain of trigonometric functions using the unit circle
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.A-REI.D.11	Explain why the x -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.A-REI.D.12	Graph the solutions to a linear inequality in two variables as a half plane (excluding the

boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B	Model periodic phenomena with trigonometric functions
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.F-TF.C	Prove and apply trigonometric identities
MA.F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Integration of Career Readiness, Life Literacies and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Technology / Integration of Computer Science and Design Thinking

TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
TECH.8.1.12.A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
TECH.8.1.12.A.1	Create a personal digital portfolio which reflects personal and academic interests,

	achievements, and career aspirations by using a variety of digital tools and resources.
TECH.8.1.12.A.2	Produce and edit a multi-page digital document for a commercial or professional audience and present it to peers and/or professionals in that related area for review.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.A.4	Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.
TECH.8.1.12.B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
TECH.8.1.12.B.2	Apply previous content knowledge by creating and piloting a digital learning game or tutorial.
TECH.8.1.12.C	Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
TECH.8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.2.12	Technology Education, Engineering, Design, and Computational Thinking - Programming: All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.
TECH.8.2.12.A	The Nature of Technology: Creativity and Innovation: Technology systems impact every aspect of the world in which we live.
TECH.8.2.12.A.1	Propose an innovation to meet future demands supported by an analysis of the potential full costs, benefits, trade-offs and risks, related to the use of the innovation.

Interdisciplinary Connections: NJSL for ELA, Social Studies, Science and/or Math Section

MATH.K-12.1	Make sense of problems and persevere in solving them Key Ideas and Details
LA.RL.11-12.1	Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
MATH.K-12.2	Reason abstractly and quantitatively
LA.K-12.NJSLSA.R1	Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
LA.K-12.NJSLSA.R2	Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
LA.RL.11-12.2	Determine two or more themes or central ideas of a text and analyze their development

	over the course of the text, including how they interact and build on one another to produce a complex account; provide an objective summary of the text.
MATH.K-12.3	Construct viable arguments and critique the reasoning of others Craft and Structure
LA.RL.11-12.4	Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including words with multiple meanings or language that is particularly fresh, engaging, or beautiful. (e.g., Shakespeare as well as other authors.)
LA.K-12.NJSLSA.R4	Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically Integration of Knowledge and Ideas
MATH.K-12.6	Attend to precision
LA.K-12.NJSLSA.R8	Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
LA.RL.11-12.8	(Not applicable to literature)
MATH.K-12.7	Look for and make use of structure Range of Reading and Level of Text Complexity
LA.K-12.NJSLSA.R10	Read and comprehend complex literary and informational texts independently and proficiently with scaffolding as needed.
MATH.K-12.8	Look for and express regularity in repeated reasoning
LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
LA.K-12.NJSLSA.W	Writing Text Types and Purposes
LA.RI.11-12.2	Determine two or more central ideas of a text, and analyze their development and how they interact to provide a complex analysis; provide an objective summary of the text.
LA.K-12.NJSLSA.W1	Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
LA.K-12.NJSLSA.W2	Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
LA.RI.11-12.4	Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines faction in Federalist No. 10).
LA.K-12.NJSLSA.W4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
LA.K-12.NJSLSA.W6	Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.
LA.RI.11-12.10b	By the end of grade 12, read and comprehend literary nonfiction at grade level text-complexity or above.
LA.W.11-12.1	Write arguments to support claims in an analysis of substantive topics or texts, using valid

	reasoning and relevant and sufficient evidence.
LA.W.11-12.1.A	Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences claim(s), counterclaims, reasons, and evidence.
LA.K-12.NJSLSA.SL	Speaking and Listening
LA.W.11-12.1.B	Develop claim(s) and counterclaims avoiding common logical fallacies and using sound reasoning and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases.
	Comprehension and Collaboration
LA.K-12.NJSLSA.SL1	Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
LA.K-12.NJSLSA.SL2	Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
LA.K-12.NJSLSA.SL5	Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
LA.K-12.NJSLSA.SL6	Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.
LA.W.11-12.2.D	Use precise language, domain-specific vocabulary, and techniques such as metaphor, simile, and analogy to manage the complexity of the topic.
LA.W.11-12.2.F	Provide a concluding paragraph or section that supports the argument presented (e.g., articulating implications or the significance of the topic).
LA.K-12.NJSLSA.L	Language
	Knowledge of Language
LA.K-12.NJSLSA.L3	Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.
LA.W.11-12.4	Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
LA.W.11-12.6	Use technology, including the Internet, to produce, share, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
LA.SL.11-12.1	Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
LA.SL.11-12.1.A	Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
LA.SL.11-12.1.D	Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task.
LA.SL.11-12.2	Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, qualitatively, orally) evaluating the credibility and accuracy of each source.
LA.SL.11-12.5	Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence

and to add interest.

LA.SL.11-12.6

Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.

LA.L.11-12.3

Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy

see Crosswalks

21st Century Life and Careers

Stage I: Desired Results

Transfer/Overview/Rationale

Transfer / Overview / Rationale

Unit Rationale

The purpose of this unit...

Integral calculus is often referred to as the opposite of differentiation and is associated with finding the area under a curve. In turn it allows us to measure complex shapes that are normally too difficult. In this unit, students will be exploring the techniques and theory of integration with the understanding that it will eventually be applied to finding the area and volume of many complex shapes and solids.

Meaning

Essential Questions

Essential Questions

- How is it possible for the area of an infinite number of rectangles to be finite?
- Why is the Fundamental Theorem of Calculus so important to the subject?
- When is it possible to set up a Riemann Sum as an approximation for area?

Enduring Understanding/Indicators of Understanding

Enduring Understanding/Indicators of Understanding

- The area of an irregularly, shaped object can be approximated using rectangles for which the area can be easily computed.
- The larger the number of rectangles, the more precise the approximation.
- The Fundamental Theorem of Calculus links differentiation and integration while providing powerful tools for evaluating integrals analytically.
- Numerical approximations, Riemann sums, and the definite integral have many applications from diverse disciplines.

Acquisition (Student Learning Objectives)

Knowledge

Students will know...

- Definite integrals can be represented as a limit of Riemann sums
- An integral can be interpreted as the sum of infinitely small quantities. This sum has a limiting value that can be computed with algebra.
- The integral can also be interpreted as the limiting value of the sum of an infinite number of quantities.
- The Fundamental Theorem of Calculus relates the definite integral of a function to its antiderivative.
- The Definite integral of the rate of change of a quantity over an interval is interpreted as the change of the quantity over the interval.
- Antiderivatives can be evaluated by changing the variable of integration using u-substitution resulting in a simpler expression to integrate.
- For definite integrals, the new limits of integration are functions of the substituted variable and must be changing accordingly.
- Whatever applications are chosen, the emphasis is on using the method of setting up an approximation Riemann sum and representing its limit as a definite integral.

Skills

Student will be skilled at ...

- Set up and compute Riemann sums for a variety of problems such as the area under a curve Use numerical approximations to evaluate definite integrals
- Make Riemann sum approximations using left, right, or midpoint as the evaluation points. Use trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically and by tables of values.
- Use of the Fundamental Theorem to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined and to evaluate definite integrals.
- Use the properties of definite integrals and knowledge of basic antiderivatives to evaluate simple integrals
- Perform u-substitution on both indefinite and definite integrals and evaluate, changing limits of integration when appropriate.

Stage 3: Learning Plan

Resource and Mentor Texts

- Calculus Textbook (Pearson)
- Desmos online graphing application
- Ti-83 Calculator
- Internet

Formative Assessment Strategies

- Warm Up Activities
- Homework and class assignments
- Ticket to Leave
- Weekly Quizzes on essential concepts
- White boards (Individual and Group)

Learning Activities/Unit of Study

- Warm Up--problem of the day related to previous learned skills or bellringers problems

- Review/Check Homework - (group check, partner check, whiteboard check)- Lecture
- Lecture
- Work together to understand and practice the skill - partner work/larger group work to read lesson, and practice skills through “On Your Own” problems incorporated throughout each lesson
- Stations - (Small group instruction, skills practice - scavenger hunts, online games, board work)
- Board/White Board Work - (solve problems/practice skills at board, or at seat with individual white boards)
- Kahoot to reinforce skills
- Review and practice skills using a variety of materials - (text, workbook, chromebook, games, activities, discussion)
- Student led instruction
- Scavenger hunts--self-checking, out of seats activity
- Jeopardy style review games
- Relay races--each student does one part of a problem, hands it to the next student to check then completes the next part, etc.
- Partner/Group investigation where students must create a formula, method, or strategy to solve a problem.
- Students “as teachers” where they present a method or formula they discovered through investigation
- Khan Academy - A useful site where students can teach themselves or reinforce skills previously learned
- Online games on chromebooks

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

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Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs

students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.