

# Unit 02: Limits of Continuity

Content Area: **Template**  
Course(s):  
Time Period: **Full Year**  
Length: **FY**  
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## Standards Alignment

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### New Jersey Student Learning Standards

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MA.F-IF	Interpreting Functions
MA.A-SSE	Seeing Structure in Expressions
MA.A-SSE.A	Interpret the structure of expressions
MA.F-IF.A	Understand the concept of a function and use function notation
MA.A-SSE.A.1	Interpret expressions that represent a quantity in terms of its context.
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$ . The graph of $f$ is the graph of the equation $y = f(x)$ .
MA.A-SSE.A.1a	Interpret parts of an expression, such as terms, factors, and coefficients.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.A-SSE.A.1b	Interpret complicated expressions by viewing one or more of their parts as a single entity.
MA.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$ .
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.A-SSE.B	Write expressions in equivalent forms to solve problems
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.A-SSE.B.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines.
MA.A-SSE.B.3b	Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
MA.A-SSE.B.3c	Use the properties of exponents to transform expressions for exponential functions.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or

	as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C	Analyze functions using different representations
MA.A-SSE.B.4	Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.A-APR	Arithmetic with Polynomials and Rational Expressions
MA.A-APR.A	Perform arithmetic operations on polynomials
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.A-APR.B	Understand the relationship between zeros and factors of polynomials
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.A-APR.B.3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.A-APR.C	Use polynomial identities to solve problems
MA.A-APR.C.4	Prove polynomial identities and use them to describe numerical relationships.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.F-BF	Building Functions
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.A-APR.D	Rewrite rational expressions
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.A-APR.D.6	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$ , where $a(x)$ , $b(x)$ , $q(x)$ , and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
MA.A-APR.D.7	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.

MA.F-BF.A.1b	Combine standard function types using arithmetic operations.
MA.A-CED	Creating Equations
MA.A-CED.A	Create equations that describe numbers or relationships
MA.F-BF.A.1c	Compose functions.
MA.A-CED.A.1	Create equations and inequalities in one variable and use them to solve problems.
MA.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
MA.F-BF.A.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MA.A-CED.A.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
MA.F-BF.B	Build new functions from existing functions
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$ , $kf(x)$ , $f(kx)$ , and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
MA.F-BF.B.4	Find inverse functions.
MA.A-CED.A.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
MA.F-BF.B.4a	Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse.
MA.A-REI	Reasoning with Equations and Inequalities
MA.F-BF.B.4b	Verify by composition that one function is the inverse of another.
MA.A-REI.A	Understand solving equations as a process of reasoning and explain the reasoning
MA.A-REI.A.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
MA.F-BF.B.4c	Read values of an inverse function from a graph or a table, given that the function has an inverse.
MA.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
MA.F-BF.B.4d	Produce an invertible function from a non-invertible function by restricting the domain.
MA.A-REI.B	Solve equations and inequalities in one variable
MA.F-BF.B.5	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
MA.F-LE	Linear and Exponential Models
MA.A-REI.B.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
MA.A-REI.B.4	Solve quadratic equations in one variable.
MA.F-LE.A	Construct and compare linear and exponential models and solve problems
MA.F-LE.A.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MA.A-REI.B.4a	Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

MA.F-LE.A.1a	Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
MA.A-REI.B.4b	Solve quadratic equations by inspection (e.g., for $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$ .
MA.F-LE.A.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MA.A-REI.C	Solve systems of equations
MA.A-REI.C.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
MA.F-LE.A.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MA.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MA.A-REI.C.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
MA.A-REI.C.7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
MA.F-LE.A.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
MA.F-LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab$ to the $ct$ power = $d$ where $a$ , $c$ , and $d$ are numbers and the base $b$ is 2, 10, or $e$ ; evaluate the logarithm using technology.
MA.F-LE.B	Interpret expressions for functions in terms of the situation they model
MA.A-REI.C.8	Represent a system of linear equations as a single matrix equation in a vector variable.
MA.A-REI.C.9	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension $3 \times 3$ or greater).
MA.F-LE.B.5	Interpret the parameters in a linear or exponential function in terms of a context.
MA.A-REI.D	Represent and solve equations and inequalities graphically
MA.F-TF	Trigonometric Functions
MA.A-REI.D.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
MA.F-TF.A	Extend the domain of trigonometric functions using the unit circle
MA.F-TF.A.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
MA.A-REI.D.11	Explain why the $x$ -coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$ ; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
MA.F-TF.A.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
MA.A-REI.D.12	Graph the solutions to a linear inequality in two variables as a half plane (excluding the

boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

MA.F-TF.A.3	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$ , $\pi/4$ and $\pi/6$ , and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$ , $\pi + x$ , and $2\pi - x$ in terms of their values for $x$ , where $x$ is any real number.
MA.F-TF.A.4	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
MA.F-TF.B	Model periodic phenomena with trigonometric functions
MA.F-TF.B.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
MA.F-TF.B.6	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
MA.F-TF.B.7	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
MA.F-TF.C	Prove and apply trigonometric identities
MA.F-TF.C.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ given $\sin(\theta)$ , $\cos(\theta)$ , or $\tan(\theta)$ and the quadrant of the angle.
MA.F-TF.C.9	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

## **Integration of Career Readiness, Life Literacies and Key Skills**

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CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

## **Technology / Integration of Computer Science and Design Thinking**

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TECH.8.1.12	Educational Technology: All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.
TECH.8.1.12.A	Technology Operations and Concepts: Students demonstrate a sound understanding of technology concepts, systems and operations.
TECH.8.1.12.A.1	Create a personal digital portfolio which reflects personal and academic interests,

	achievements, and career aspirations by using a variety of digital tools and resources.
TECH.8.1.12.A.2	Produce and edit a multi-page digital document for a commercial or professional audience and present it to peers and/or professionals in that related area for review.
TECH.8.1.12.A.3	Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.
TECH.8.1.12.A.4	Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.
TECH.8.1.12.B	Creativity and Innovation: Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
TECH.8.1.12.B.2	Apply previous content knowledge by creating and piloting a digital learning game or tutorial.
TECH.8.1.12.B.CS1	Apply existing knowledge to generate new ideas, products, or processes.
TECH.8.1.12.F	Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.
TECH.8.1.12.F.1	Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
TECH.8.2.12	Technology Education, Engineering, Design, and Computational Thinking - Programming: All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.
TECH.8.2.12.A	The Nature of Technology: Creativity and Innovation: Technology systems impact every aspect of the world in which we live.
TECH.8.2.12.A.1	Propose an innovation to meet future demands supported by an analysis of the potential full costs, benefits, trade-offs and risks, related to the use of the innovation.

## **Interdisciplinary Connections: NJSL for ELA, Social Studies, Science and/or Math Section**

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### **Capacities of the Literate Individual**

#### **Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language**

They demonstrate independence.

They build strong content knowledge.

They respond to the varying demands of audience, task, purpose, and discipline.

They comprehend as well as critique.

They value evidence

They use technology and digital media strategically and capably.

LA.K-12.NJSLSA.R	Reading
MATH.K-12.1	Make sense of problems and persevere in solving them Key Ideas and Details
LA.RL.11-12.1	Cite strong and thorough textual evidence and make relevant connections to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
MATH.K-12.2	Reason abstractly and quantitatively
LA.K-12.NJSLSA.R1	Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
LA.RL.11-12.2	Determine two or more themes or central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to produce a complex account; provide an objective summary of the text.
LA.K-12.NJSLSA.R2	Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
MATH.K-12.3	Construct viable arguments and critique the reasoning of others Craft and Structure
MATH.K-12.4	Model with mathematics
LA.RL.11-12.4	Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including words with multiple meanings or language that is particularly fresh, engaging, or beautiful. (e.g., Shakespeare as well as other authors.)
LA.K-12.NJSLSA.R4	Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
MATH.K-12.5	Use appropriate tools strategically Integration of Knowledge and Ideas
LA.K-12.NJSLSA.R7	Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.
MATH.K-12.6	Attend to precision
LA.K-12.NJSLSA.R8	Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
LA.RL.11-12.8	(Not applicable to literature)
MATH.K-12.7	Look for and make use of structure Range of Reading and Level of Text Complexity
MATH.K-12.8	Look for and express regularity in repeated reasoning
LA.K-12.NJSLSA.R10	Read and comprehend complex literary and informational texts independently and proficiently with scaffolding as needed.
LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.
LA.K-12.NJSLSA.W	Writing
LA.RI.11-12.2	Determine two or more central ideas of a text, and analyze their development and how

they interact to provide a complex analysis; provide an objective summary of the text.

#### Text Types and Purposes

LA.K-12.NJSLSA.W1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

LA.K-12.NJSLSA.W2 Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.

#### Production and Distribution of Writing

LA.K-12.NJSLSA.W4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.

LA.K-12.NJSLSA.W6 Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

LA.RI.11-12.10b By the end of grade 12, read and comprehend literary nonfiction at grade level text-complexity or above.

LA.W.11-12.1 Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.

LA.K-12.NJSLSA.SL Speaking and Listening

LA.W.11-12.1.A Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences claim(s), counterclaims, reasons, and evidence.

LA.W.11-12.1.B Develop claim(s) and counterclaims avoiding common logical fallacies and using sound reasoning and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases.

#### Comprehension and Collaboration

LA.K-12.NJSLSA.SL1 Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.

LA.K-12.NJSLSA.SL2 Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.

LA.K-12.NJSLSA.SL5 Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.

LA.K-12.NJSLSA.SL6 Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

LA.W.11-12.2.D Use precise language, domain-specific vocabulary, and techniques such as metaphor, simile, and analogy to manage the complexity of the topic.

LA.W.11-12.2.F Provide a concluding paragraph or section that supports the argument presented (e.g., articulating implications or the significance of the topic).

LA.K-12.NJSLSA.L Language

#### Knowledge of Language

LA.K-12.NJSLSA.L3 Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

LA.W.11-12.4 Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)

LA.W.11-12.6 Use technology, including the Internet, to produce, share, and update individual or shared

writing products in response to ongoing feedback, including new arguments or information.

LA.SL.11-12.1	Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
LA.SL.11-12.1.A	Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
LA.SL.11-12.1.D	Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task.
LA.SL.11-12.2	Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, qualitatively, orally) evaluating the credibility and accuracy of each source.
LA.SL.11-12.5	Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
LA.SL.11-12.6	Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate.
LA.L.11-12.3	Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

## **Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media Literacy**

see Crosswalks

## **21st Century Life and Careers**

### **Stage I: Desired Results**

### **Transfer/Overview/Rationale**

#### **Transfer / Overview / Rationale**

Unit Rationale

The purpose of this unit...

**Differential and Integral Calculus is based on the combination of the concepts of limits and continuity. Without**

these two key concepts, calculus would not be possible. Throughout this unit, we will look at the key theories of limits and continuity and delve deeper to realise the possibilities that lie ahead in the world of differential and integral calculus.

## Meaning

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## Essential Questions

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### Essential Questions

- How does an understanding of limits in this unit help to analyze functions graphically and analytically?
- How do the tools of calculus help to better explain the behavior displayed by functions graphed with the aid of technology?
- What is the difference between the value of a limit and the value of a function?
- Why is it necessary for both the function and its limit to exist at a given point to ensure that the function is continuous there?
- Why is continuity important in the study of calculus?

## **Enduring Understanding/Indicators of Understanding**

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### Enduring Understanding/Indicators of Understanding

- With the aid of technology, graphs of functions are often easily produced. The emphasis shifts to the interplay between the geometric and analytical information and on the use of calculus to predict and explain the observed behavior of a function.
- The limit of a function can exist and be calculated even though a function fails to exist at a specified location.
- Limits can be used to describe unbounded behavior in functions.
- Continuity is fundamental to calculus and ensures that every point along a graph of a function exists.

## **Acquisition (Student Learning Objectives)**

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### **Knowledge**

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Students will know...

- The Limit of a function is the value that the function approaches as the independent variable approaches a specified number.
- The properties of limits can be used to simplify expressions and determine the value of limits algebraically.
- The properties of limits include the limit of a constant, sum, difference, product, quotient, constant multiple, and power.
- Unbounded behaviour created by the existence of asymptotes can be recognised graphically.
- Horizontal asymptotes exist when a function approaches infinity.

- A function is continuous if its graph can be drawn from beginning to end without lifting pencil from paper.
- Graphs of continuous functions have no breaks or holes
- A function is continuous at any point,  $c$  in the interior of the function if both the function and its limit exist  $x \rightarrow c$  and are equal in value there.
- For a continuous function on the interval  $[a,b]$ , the Intermediate Value Theorem guarantees that the function has a value corresponding to every number along the interval.

## **Skills**

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Student will be skilled at ...

- Calculate limits using algebra.
- Estimate limits from graphs or tables of data.
- Use the properties of limits to simplify expressions and evaluate limits.
- Justify the value of  $\sin$  using a graphing calculator. Use this common limit to simplify expressions and evaluate limits algebraically.
- Recognize unbounded behavior on a graph of a function that has
- asymptotes.
- Describing asymptotic behavior in terms of limits involving infinity.
- Comparing relative magnitudes of functions and their rates of change - For example: Contrasting exponential growth, polynomial growth, and logarithmic growth.
- Determine whether or not a function is continuous intuitively.
- Use the understanding of limits to evaluate the continuity of function at a point algebraically.
- Use the definition of continuity at an endpoint to evaluate the continuity of functions on a given interval.
- Check the validity of the Intermediate Value Theorem intuitively.

## **Stage 3: Learning Plan**

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### **Resource and Mentor Texts**

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- Calculus Textbook (Pearson)
- Desmos online graphing application
- Ti-83 Calculator
- Internet

### **Formative Assessment Strategies**

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- Warm Up Activities
- Homework and class assignments
- Ticket to Leave
- Weekly Quizzes on essential concepts
- White boards (Individual and Group)

Limits Scavenger Hunt - Students will need to determine various limits. Each answer will lead the student to the next problem

### **Learning Activities/Unit of Study**

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- Warm Up--problem of the day related to previous learned skills or bellringers problems
- Review/Check Homework - (group check, partner check, whiteboard check)- Lecture
- Lecture
- Work together to understand and practice the skill - partner work/larger group work to read lesson, and practice skills through "On Your Own" problems incorporated throughout each lesson
- Stations - (Small group instruction, skills practice - scavenger hunts, online games, board work)
- Board/White Board Work - (solve problems/practice skills at board, or at seat with individual white boards)
- Kahoot to reinforce skills
- Review and practice skills using a variety of materials - (text, workbook, chromebook, games, activities,

discussion)

- Student led instruction
- Scavenger hunts--self-checking, out of seats activity
- Jeopardy style review games
- Relay races--each student does one part of a problem, hands it to the next student to check then completes the next part, etc.
- Partner/Group investigation where students must create a formula, method, or strategy to solve a problem.
- Students “as teachers” where they present a method or formula they discovered through investigation
- Khan Academy - A useful site where students can teach themselves or reinforce skills previously learned
- Online games on chromebooks

## **Modifications and/or Accommodations**

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### **Suggested Modifications (ELL, Sp. Ed, Gifted, At-risk of Failure)**

#### **English Language Learners**

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

#### **Special Education Students**

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

## Students with 504 Plans

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## Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

## Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

**Peer Support:** Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

**Alternate or Modified Assignments:** Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

**Increase One to One Time:** When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

**Contracts:** It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

**Hands On:** As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

**Tests/Assessments:** Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

**Seating:** Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.