Unit 05: Series (8 weeks)

Content Area: **Template**
Course(s):
Time Period: **Full Year**
Length: **FY**
Status: **Published**

Standards Alignment

New Jersey Student Learning Standards

MA.F-IF	Interpreting Functions
MA.F-IF.A	Understand the concept of a function and use function notation
MA.F-IF.A.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
MA.F-IF.A.2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
MA.F-IF.A.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
MA.F-IF.B	Interpret functions that arise in applications in terms of the context
MA.F-IF.B.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
MA.F-IF.B.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
MA.F-IF.B.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
MA.F-IF.C	Analyze functions using different representations
MA.F-IF.C.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
MA.F-IF.C.7a	Graph linear and quadratic functions and show intercepts, maxima, and minima.
MA.F-IF.C.7b	Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
MA.F-IF.C.7c	Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7d	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
MA.F-IF.C.7e	Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
MA.F-IF.C.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
MA.F-IF.C.8a	Use the process of factoring and completing the square in a quadratic function to show

	zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
MA.F-IF.C.8b	Use the properties of exponents to interpret expressions for exponential functions.
MA.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
MA.F-BF	Building Functions
MA.F-BF.A	Build a function that models a relationship between two quantities
MA.F-BF.A.1	Write a function that describes a relationship between two quantities.
MA.F-BF.A.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context.
MA.F-BF.B	Build new functions from existing functions
MA.F-BF.B.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Integration of Career Readiness, Life Literacies and Key Skills

CRP.K-12.CRP1	Act as a responsible and contributing citizen and employee.
CRP.K-12.CRP2	Apply appropriate academic and technical skills.
CRP.K-12.CRP3	Attend to personal health and financial well-being.
CRP.K-12.CRP4	Communicate clearly and effectively and with reason.
CRP.K-12.CRP5	Consider the environmental, social and economic impacts of decisions.
CRP.K-12.CRP6	Demonstrate creativity and innovation.
CRP.K-12.CRP7	Employ valid and reliable research strategies.
CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP9	Model integrity, ethical leadership and effective management.
CRP.K-12.CRP10	Plan education and career paths aligned to personal goals.
CRP.K-12.CRP11	Use technology to enhance productivity.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Technology / Integration of Computer Science and Design Thinking

TECH.8.2.12.E	Computational Thinking: Programming: Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.
TECH.8.2.12.E.1	Demonstrate an understanding of the problem-solving capacity of computers in our world.

Interdisciplinary Connections: NJSLS for ELA, Social Studies, Science and/or Math Section

Capacities of the Literate Individual
Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, & Language

They demonstrate independence.

They build strong content knowledge.
They respond to the varying demands of audience, task, purpose, and discipline.

They comprehend as well as critique.
They value evidence

They use technology and digital media strategically and capably.

MATH.K-12.1	Make sense of problems and persevere in solving them
LA.K-12.NJSLSA.R	Reading
	Key Ideas and Details
MATH.K-12.2	Reason abstractly and quantitatively
LA.K-12.NJSLSA.R1	Read closely to determine what the text says explicitly and to make logical inferences and relevant connections from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
MATH.K-12.3	Construct viable arguments and critique the reasoning of others
MATH.K-12.4	Model with mathematics
MATH.K-12.5	Use appropriate tools strategically
MATH.K-12.6	Attend to precision
MATH.K-12.7	Look for and make use of structure
MATH.K-12.8	Look for and express regularity in repeated reasoning
LA.RI.11-12.1	Accurately cite strong and thorough textual evidence, (e.g., via discussion, written response, etc.), to support analysis of what the text says explicitly as well as inferentially, including determining where the text leaves matters uncertain.

Integration of Diversity, Equity and Inclusion; Climate Change; Informational and Media LiteracyNew Section

see Crosswalks

21st Century Life and Careers

CRP.K-12.CRP8	Utilize critical thinking to make sense of problems and persevere in solving them.
CRP.K-12.CRP12	Work productively in teams while using cultural global competence.

Stage I: Desired Results

Transfer/Overview/Rationale

Transfer / Overview / Rationale

Unit Rationale

The purpose of this unit…

The technique of using power series to approximate an arbitrary function near a specific value allows for an important connection to the tangent line problem and is a natural extension that help acheive a better approximation. The concept of approximation is a common theme and power series provide a unifying, comprehensive conclusion.

Meaning

Essential Questions

Essential Questions

- How do you define convergence and divergence of an infinite series, and how can the definition be applied to analyze some common convergent and divergent infinite series?
- How can you test when an infinite series converges, and how do these test make sense?
- How are Taylor polynomials used to approximate functions?
- What is a Taylor series and what are tnhe consequences of Taylor's theorem?

Enduring Understanding/Indicators of Understanding

Enduring Understanding/Indicators of Understanding

- The sum of an infinite number of real numbers may converge.
- A function can be represented by an associated power series over the interval of convergence for the power series.

Acquisition (Student Learning Objectives)

Knowledge

Students will know…

- The nth partial sum is defined as the sum of the first n terms of a sequence
- An infinite series of numbers converges to a real number S if and only if the limit of its sequence of partial sums exists and equals S
- Common series of numbers include geometric series, the harmonic series, and p-series
- A series may be absolutely convergent, conditionally convergent, or divergent
- If a series converges absolutely, then it converges
- In addition to examining the limit of the sequence of partial sums of the series, methods for determining whether a series of numbers
- The sum of a geometric series
- If an alternating series converges by the alternating series test, then the alternating series error bound can be used to estimate how close a partial sum is to the value of the infinite series
- If a series converges absolutely, then any series obtained from it by regrouping or rearranging the terms has the

same value
- The coefficient of the nth degree in a Taylor polynomial
- Taylor polynomials for a function f centered at x=a can be used to approximate function values of f near x=a
- In many cases, as the degree of a Taylor polynomial increases, the nth degree polynomial will converge to the original function over some interval
- The Lagrange error bound can be used to bound the error of a Taylor polynomial approximation to a function
- In some situations where the signs of a Taylor polynomial are alternating, the alternating series error bound can be used to bound the error of a Taylor polynomial approximation to the function
- The Maclaurin series for sinx, cosx, and e^x provide the foundation for constructing the Maclaurin series for other functions
- The Maclaurin series for 1/(1-x) is a geometric series
- A Taylor polynomial for f(x) is a partial sum of the Taylor series for f(x)
- A power series for a given function can be derived by various methods
- If a power series converges, it either converges at a single point or has an interval of convergence
- The ratio test can be used to determine the radius of convergence of a power series
- If a power series has a positive radius of convergence, then the power series is the Taylor series of the function to which it converges over the open interval
- The radius of convergence of a power series obtained by term by term differentiation or integration is the same as the radius of convergence of the original power series.

Skills

Student will be skilled at …

- Determine whether a series converges or diverges
- Determine or estimate the sum of a series
- Construct and use Taylor polynomials
- Write a power series representing a given function
- Determine the radius and interval of convergence of a power series

Stage 3: Learning Plan

Resource and Mentor Texts

Finney, Ross L., Franklin D. Demana, Bert K. Waits, and Daniel Kennedy. *Calculus-Graphical, Numerical, Algebraic*.

Menlow Park: Scott Forseman Addison Wesley, 2012.

Cade, Sharon, Rhea Caldwell, and Jeff Lucia. Fast Track to a 5: *Preparing for the AP Calculus Am and Calculus BC Examinations*. Evanston: McDougal Littell, 2006.

Lederman, David. *Multiple-Choice & Free-response Questions in Preparation for the AP Calculus (AB) Examination*. New York: D&S Marketing Systems, Inc., 2004.

Lifshitz, Maxine. *AP Calculus AB/BC: Preparing for the Advanced Placement Examinations*. New York: Amsco School Publications, Inc., 2004.

Barron's AP Calculus prep book

http://jamesrahn.com/pages/calculus%20ap/calculus_labs.htm

http://apcentral.collegeboard.com/apc/public/courses/teachers_corner/2178.html

https://www.khanacademy.org/

http://webspace.ship.edu/msrenault/GeoGebraCalculus/GeoGebraCalculusApplets.html

Formative Assessment Strategies

- Announced quizzes
- Short unannounced quizzes
- Take home AP problem packets
- Teacher observations
- Homework (quick check, collect, quiz)
- Exit tickets
- Teacher observations
- Student work on the board
- Kahoot
- Teacher guided questions
- Students submit picture of quiz/problem solution to google classroom, then as a class we review the answer so they can correct their work and don't have to wait until next class to review.

- Use google forms, todays meet, and/or google hangouts for do nows/exit tickets, spot checks, and back channeling
- Chapter tests

Learning Activities/Unit of Study

- Review homework
- Warm Up activity
- Guided notes
- Group work (matching, war, practice problems, exploration, work on problems at the board)
- Calculator explorations
- Foerster calculus explorations
- Stations - (Small group instruction, skills practice - scavenger hunts, online games, board work)
- Kahoot to reinforce skills
- Review and practice skills using a variety of materials - (text, workbook, chromebook, games, activities, discussion)
- Foldables--creates an organized study guide per chapter
- Jeopardy style review games
- Students “as teachers” where they present a method or formula they discovered through investigation

Modifications and/or Accommodations

Suggested Modifications (ELL, Sp. Ed, GIfted, At-risk of Failure)

English Language Learners

Native language support: The teacher provides auditory or written content to students in their native language.

Adjusted Speech: The teacher changes speech patterns to increase student comprehension. This could include facing the students, paraphrasing, clearly indicating the most important ideas, and speaking more slowly.

Visuals: The teacher uses graphics, pictures, visuals, and manipulatives. This helps ELL students better understand and comprehend the subjects at hand.

Front-Loading Vocabulary: The teacher front loads vocabulary. This means providing students with a list of important vocabulary words they will need to know for a book, lesson, etc. prior to the lesson being taught. Including pictures to go with the vocabulary words is also very beneficial for the students.

Special Education Students

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Oral Reading: The teacher will read work orally to students. Class work such as tests and literature circles may need to be read aloud to the student.

Timers: The teacher will use timers as an instructional tool. The use of timers is beneficial for students who have trouble completing tasks. Timers can be helpful so the student is aware of how much time they have to complete an assignment.

Students with 504 Plans

Chunking: The teacher presents information in a way that makes it easy for students to understand and remember. Chunking is based on the presumption that our working memory is easily overloaded by excessive detail. The best way to deliver information is to organize it into meaningful units. Because students with special needs get overloaded easily, chunking is an effective strategy to use with them.

Checking for Understanding: It is important to constantly check for understanding, especially for students who have accommodations. Teachers want to make sure students understand the concepts being covered in a way that makes sense to them.

Extra time: The teacher provides students with special needs extra time to complete work or answer questions. It is important to give students enough time to process their thoughts.

Gifted & Talented Strategies

Extensions/Enrichments: Teachers will provide gifted and talented students with extension/enrichment projects. Students will be challenged to further their understanding, to apply acquired knowledge, and/or to produce something in reference to acquired knowledge.

Modify/Change Activities: Teachers will monitor and modify activities to accommodate those students who need to be challenged further. Additional reading, problem-solving, writing, or project

work is necessary for those students who are ready to move on at a rate more accelerated than their peers. In this way, G & T students are provided the same opportunity for support as special needs students.

Students at Risk of School Failure

Directions or Instructions: Make sure directions and/or instructions are given in limited numbers. Give directions/instructions verbally and in simple written format. Ask students to repeat the instructions or directions to ensure understanding occurs. Check back with the student to ensure he/she hasn't forgotten.

Peer Support: Peers can help build confidence in other students by assisting in peer learning. Many teachers use the 'ask 3 before me' approach. This is fine, however, a student at risk may have to have a specific student or two to ask. Set this up for the student so he/she knows who to ask for clarification before going to you.

Alternate or Modified Assignments: Always ask yourself, "How can I modify this assignment to ensure the students at risk are able to complete it?" Sometimes you'll simplify the task, reduce the length of the assignment or allow for a different mode of delivery. For instance, many students may hand something in, the at-risk student may jot notes and give you the information verbally. Or, it just may be that you will need to assign an alternate assignment.

Increase One to One Time: When other students are working, always touch base with your students at risk and find out if they're on track or needing some additional support. A few minutes here and there will go a long way to intervene as the need presents itself.

Contracts: It helps to have a working contract between you and your students at risk. This helps prioritize the tasks that need to be done and ensure completion happens. Each day write down what needs to be completed, as the tasks are done, provide a checkmark or happy face. The goal of using contracts is to eventually have the student come to you for completion sign-offs.

Hands On: As much as possible, think in concrete terms and provide hands-on tasks. This means a child doing math may require a calculator or counters. The child may need to tape record comprehension activities instead of writing them. A child may have to listen to a story being read instead of reading it him/herself.

Tests/Assessments: Tests can be done orally if need be. Break tests down in smaller increments by having a portion of the test in the morning, another portion after lunch and the final part the next day.

Seating: Seat students near a helping peer or with quick access to the teacher. Those with hearing or sight issues need to be close to the instruction which often means near the front.